INDUCTIVE COMMERCIAL ARITHMETIC

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Inductive Commercial Arithmetic

A PRACTICAL TREATISE ON BUSINESS COMPUTATION

BY

CARL COREN MARSHALL,
AUTHOR OF THE "MARSHALL BUSINESS TRAINING SERIES,"

AND

SAMUEL HORATIO GOODYEAR, A. M.,
AUTHOR OF GOODYEAR'S ACCOUNTANCY COURSES, AND
PROFESSOR OF ACCOUNTANCY IN THE IOWA
STATE UNIVERSITY.

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FOREWORD

In the preparation of this text the authors have departed somewhat from conventional lines. To those who may examine the work, we suggest for their consideration the following special features:

1. A careful plan of inductive development, through which the student is introduced, in one topic, to many of the terms and processes that are brought out more fully in succeeding topics. Thus, in the consideration of simple numbers, there is an elementary presentation of fractions and decimals; and in the topics of fractions and decimals, some of the more common applications of the terms and processes of percentage.

2. A liberal introduction of script and business forms.

3. A consistent correlation of arithmetic and accounting.

4. An unusual exemplification of the more important counting-room methods.

5. A presentation of practical rather than analytic problems.

6. A very complete chapter on the computations required in the trades and arts.

It is believed that these features will particularly appeal to teachers, who are looking for a business arithmetic, that really conforms to modern business.

THE AUTHORS.

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Inductive Commercial Arithmetic

CHAPTER I

NUMBER

1. The idea of **number** is suggested by the question, how many? This idea is natural to every sound human mind. It begins with the idea of units or single things. By a *number* we mean simply a collection of single things or units. It is only by the idea of number that we are able to express our ideas as to the quantity, magnitude, or value of things.

2. The nature or value of a number depends upon its units; the unit of a number may consist of a group or groups of other units. Thus, in speaking of a quantity of eggs, we may say, "there are six 30-dozen cases," the unit of the number six, being *one* 30-dozen case. Each case may then be thought of as 30 units of *one* dozen eggs each. Finally, we may think of each dozen as a number, the unit of which is one egg.

3. After some experience with numbers in their application to the things about us, we are able to conceive of the numbers themselves, and without any thought of their application to objects. Thus, we may think of 6, or 10, or 15, without thinking of any particular unit as the number basis. Numbers thus thought of or expressed are called **abstract numbers.**

4. When we apply numbers directly to definite objects, as in the expressions, seven dollars, five sheep, eight dozen eggs, etc., the numbers are said to be **concrete.**

NOTE.—Travelers report that the Terra del Fuegians, and certain tribes of the Australian Aborigines, are unable to count beyond five, the number of fingers on one hand. It is a curious historical fact that the fingers (digits) were primarily used by all peoples as a means of indicating number. This is why all number systems use five and ten as the basis in forming number groups.

There is much difference among individuals in the ability to recognize numbers at sight, and without stopping to count. Some persons cannot recognize more than five objects without counting, while others can recognize eight or nine. It is an interesting experiment to test this power in the case of a class of students, by arranging irregular groups of from five to ten objects, as beans, bits of chalk, kernels of corn, etc., allowing the students to name at sight, as far as they can, the number in each group. The power to see instantly the number of objects in a group is susceptible of cultivation.

5. Arithmetic, as a science, treats of the principles and relations of numbers and the accepted methods of computing by means of the figures or characters that represent them. As an art, it teaches us to apply these principles and methods to the common affairs of life.

6. The **fundamental processes** of arithmetic are, *addition, subtraction, multiplication,* and *division.*

SYMBOLS FOR EXPRESSING NUMBERS

7. The method of expressing number by means of symbols is called **notation.**

8. The representation of numbers by means of special symbols is of very ancient origin—perhaps as old as language itself. Among most ancient peoples, numbers were represented by letters of the alphabet. In the Greek and the Hebrew notation, each of the nine digits was denoted by a letter, while other letters represented groups of 10's, as 20, 30, 40, 50, 100, etc. In the Roman system the grouping was carried still further and fewer characters were used.

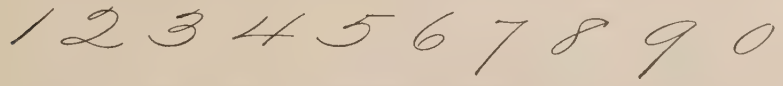
9. The Arabic system. Of the various systems used by the ancients, the one known as the *Arabic* is the simplest as well as the most scientific, and has now come into general use throughout the civilized world. It was not invented by the Arabs, as its name would indicate, but by the mathematicians of India, sometime before the beginning of the Christian Era. The Arabs, however, introduced the system into Europe about 700 years ago, where it gradually superseded the inconvenient Roman system.

10. The distinctive features of the Arabic system are as follows:

1. The use of nine characters or digits (1, 2, 3, 4, 5, 6, 7, 8, 9) to indicate, respectively, the first nine numbers.

2. The use of a tenth character, 0, (called naught, cipher, or zero) to indicate the negation, or absence of number.

The following are the written or script forms of the cipher and the nine digits:



3. The third and most important feature of the Arabic system is the variation of the **unit value** of the several digits according to the place they occupy in the number. Thus, in the number 456 the unit value of each unit in the 6 is 1; in the 5, the unit value is 10, and in the 4, it is 100 and so on. This increase of the unit value from right to left on a scale of 10 is known as the **decimal system.**

11. By the order of a digit is meant the value of its units, as indicated by its position in the number.

The orders for the first eighteen positions are named as follows:

Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
4	3	5,	3	6	7,	4	7	8,	5	2	3,	7	1	6,	4	3	2

For convenience in reading numbers, the orders are grouped into periods of three figures each, and in reading, the unit value of the right hand figure in each period is named, instead of naming the unit value of all the figures. Thus, in reading the above number, 4353674785237164 32, we do not read, "4 hundred-quadrillions, 3 ten-quadrillions, 5 quadrillions, 3 hundred-trillions, 6 ten-trillions," etc., but we separate the number into periods of three figures each, beginning at the right, thus, 435,367,478,523,716,432, and then read 435 quadrillion, 367 trillion, 478 billion, 523 million, 716 thousand, 432, the unit value of the unit's period not being expressed.

12. Numeration is the expression, in words, of numbers that are indicated by figures, letters, or other characters.

To read Arabic numbers readily, one must be able:

1. To read at sight any group of three figures.

2. To name the periods, in their proper order, from highest to lowest.

NOTE.—Strictly speaking, there is no limit to the number of periods; but numbers beyond the sixth period, or quadrillions, are rarely used. The student should learn the periods at least this far, and be able to name them from left to right.

EXERCISE

Read at sight the following number-groups, without reference to their location in periods:

123	101	020	001	905	090	909
328	025	008	075	770	202	060
469	606	206	010	080	003	100
510	011	110	050	709	030	005

In the following numbers, the periods are arranged in columns, the name of the period being placed at the head of the column. Practice in reading these numbers rapidly from left to right until you can name the several periods readily without looking at the names above the columns. If a period place is occupied by ciphers, pass without mention. Notice that the *left hand* period may consist of fewer than three figures. In reading, the name of the first, or unit-period is omitted. Omit, also, the word, *and;* say "five hundred seventy-six," *not* "five hundred *and* seventy-six." The first of the following numbers should be read, "one hundred sixty quadrillion, forty-five trillion, one billion, two hundred six million, five hundred seventy thousand, two."

QUAD.	TRILL.	BILL.	MILL.	THOUS.	.
160	045	001	206	570	002
14	120	000	567	010	206
506	200	014	000	005	709
870	002	569	703	000	504
9	000	000	010	000	530
	53	010	278	005	500
16	000	000	050	005	020
		500	000	030	003
120	601	250	006	310	037

In the following numbers, the periods are separated by commas. Read them.

20,120,160,130,701
150,000,627,100,034
573,020,050,006
5,006,279,130,162
300,000,017,235,837
20,371,500,006,008
40,271,638,798,580,367

Separate the following numbers into periods, beginning at the right, and read them:

127003710025673
800000670001256
127000008670000124
372607165002016000
530802016300000167

FRACTIONAL AND DECIMAL NUMBERS

13. Numbers that express units and combinations of units, as 6, 25, 246, 5167, etc., are called whole numbers or integers.

14. Fractional numbers are those that indicate the division of a unit into equal parts. Thus, the expression, "one-fourth of an apple," indicates that the unit, *one apple*, is assumed to be divided into four equal parts; also, that *one* of these parts is taken or considered. It is written $\frac{1}{4}$. In like manner, the expression, "three-fifths of a dollar," indicates that a dollar is assumed to be divided into five equal parts and three of these parts taken. It is written $\frac{3}{5}$.

In printing, the parts of a fraction are separated by a *horizontal* line, but in writing, the line is usually *oblique*, as in the following script forms:

15. The second number in the fraction which indicates the number of equal parts into which the unit is divided, is called the *denominator* of the fraction; the first number which shows the number of equal parts taken is called the *numerator*. Thus, in the fraction, $\frac{5}{6}$, 6 is the denominator, and 5 is the numerator.

EXERCISE

1. In the fraction $\frac{5}{6}$, the unit is assumed to be divided into how many equal parts? How many of these parts are taken?

2. In the case of each of the following fractional numbers, name the number of equal parts into which the unit is divided and the number of parts that are taken: $\frac{4}{5}$, $\frac{7}{9}$, $\frac{5}{12}$, $\frac{9}{121}$, $\frac{2}{5}$.

3. Read the following fractions: $\frac{21}{25}$, $\frac{125}{217}$, $\frac{361}{478}$, $\frac{19}{100}$, $\frac{217}{1000}$, $\frac{47}{101}$, $\frac{501}{1350}$.

4. Write, in figures, the following expressions: twenty-nine, forty-fifths; one hundred sixty, two hundred forty-firsts; five hundred eighty-five, seven hundred ninety-ninths.

16. When a whole number and a fraction are combined in one expression, the result is called a mixed number. Thus, $5\frac{4}{9}$ (read, five and four-ninths) is a mixed number.

EXERCISE

1. Read the following mixed numbers:

$$29\tfrac{1}{2},\ 15\tfrac{5}{11},\ 101\tfrac{2}{5},\ 207\tfrac{21}{25},\ 100\tfrac{5}{6},\ 2190\tfrac{18}{21}.$$

2. Express in figures the following: one hundred five, and twenty-five twenty-ninths; two hundred seventy-eight, and forty-five fifty-firsts; two hundred, and fourteen eighty-sevenths.

17. A decimal fraction is a fraction whose denominator is 10, 100, 1000, 10000, etc. Thus, $\frac{5}{10}$, $\frac{17}{100}$, $\frac{58}{1000}$, are decimal fractions.

In writing decimal fractions, the denominator is not usually written, but is indicated by a period or "decimal point," so placed in the numerator as to show the number of ciphers in the denominator. Thus, $\frac{65}{100}$ is written .65; and $\frac{13}{1000}$ is written .013. The number of figures to the right of the decimal point indicates the number of ciphers in the denominator of the decimal fraction. Decimal fractions thus written are called **decimals.**

18. In the case of mixed numbers, the whole number is written to the left of the decimal point. Thus, $27\frac{19}{100}$ would be written 27.19, and $128\frac{13}{1000}$ would be written 128.013.

NOTE.—Observe that if there are fewer figures in the numerator than there are ciphers in the denominator, the needed figures are supplied by prefixing ciphers. Thus, $\frac{7}{1000}$ is written .007, and $\frac{9}{10000}$ is written .0009.

EXERCISE

1. Read the following decimal numbers:

.17; .016; .147; .05; .9; .009; .25.6; 121.12; 256.05; 123.116; 29.007; 6.0005.

2. Express in figures the following: twenty-five, and six tenths; one hundred twenty-five, thousandths; sixteen, and seven hundredths; seventeen ten-thousandths.

GENERAL EXERCISE IN NOTATION

Express in figures the following numbers:

NOTE.—In this and all other exercises in arithmetic, take the utmost pains to make your figures plain, uniform in size, and neat in general appearance. Be especially careful that your 6's and 0's, and your 7's, 9's, and 1's, and your 2's and 8's are easily distinguishable. These matters are especially important to those who expect to engage in business pursuits, where legibility of figures is a necessity.

1. Nine hundred, one.
2. One hundred, nine.
3. Two thousand, ten.
4. Eleven hundred, eleven.
5. Sixteen hundred, one.
6. Eighty thousand, ten.
7. Four hundred thousand, eleven.
8. Sixty-two million, twenty-six.
9. Forty-eight hundred, sixty.
10. Three million, forty.
11. Forty million, three.
12. Eleven million, twenty thousand, four.
13. Seven hundred one million, forty thousand, eleven.
14. Ninety million, ten thousand, one.

Write in figures the following fractional numbers:

1. Twenty, and seven fortieths.
2. One hundred five ninths.
3. One hundred, and five ninths.
4. Two hundred nine, and fourteen seventeenths.
5. Seventy-five one hundred firsts.
6. Nineteen, and nineteen twentieths.

Write in figures the following decimals:

7. Seventy-nine thousandths.
8. One and seven hundredths.
9. One hundred six, and six tenths.
10. Twenty one, and fifty-four thousandths.
11. Ten, and one tenth.
12. One hundred, and thirteen hundredths.
13. Six hundred forty eight ten-thousandths.
14. Forty, and seventy-five hundredths.

THE ROMAN NOTATION

19. This system was in general use throughout Europe until about the thirteenth century, and it is still used to indicate the hours on clock-dials, in numbering the chapters and other divisions of books, and for a few other minor purposes.

In the Roman notation, all numbers are represented by means of seven letters of the alphabet, and the combinations formed by them.

These seven letters and the numbers represented by them are as follows:

I = One.
V = Five.
X = Ten.
L = Fifty.
C = One Hundred.
D = Five Hundred.
M = One Thousand

20. The principles governing the combining of these number-letters are as follows:

1. When a letter is followed by the same letter, or a letter denoting a lower number, the number value of the combination is equal to the sum of the letters. Thus, VI = six; XX = twenty; XVIII = eighteen.

2. When a letter is preceded by a letter denoting a smaller number, the value of the combination is equal to the difference between the numbers. Thus, IV = four; XL = forty; XC = ninety; CD = four hundred; CM = nine hundred.

3. A dash (—) placed over a letter or combination of letters increases their value a thousand fold. Thus, $\overline{V} = 5000$. $\overline{XV}CC = 15200$.

Two or more dashes are used to indicate the multiplying of a number by a million, a billion, etc. Thus, $\overline{\overline{X}}$, $\overline{\overline{VI}}$, XL = 10,006,040.

21. Since, at the present time, Roman numerals are rarely if ever used to indicate large numbers, the study of this notation is a matter of historical, rather than of practical importance. Familiarity with the combinations in the following table will enable the student to read any Roman numeral that he is likely to meet with in modern works.

TABLE OF ROMAN NUMERALS

I,	1	XV	15	CC,	200
II,	2	XVI,	16	CCC,	300
III,	3	XVII,	17	CD,	400
IV,	4	XVIII,	18	D,	500
V,	5	XIX,	19	DC,	600
VI,	6	XX,	20	DCC,	700
VII,	7	XXX,	30	DCCC,	800
VIII,	8	XL,	40	CM,	900
IX,	9	L,	50	M,	1000
X,	10	LX,	60	MM,	2000
XI,	11	LXX,	70	$\overline{X}$	10000
XII,	12	LXXX,	80	$\overline{L}$,	50000
XIII,	13	XC,	90	$\overline{C}$,	100000
XIV,	14	C,	100	$\overline{M}$,	1000000

CURRENCY NOTATION OF THE UNITED STATES

22. The United States has a decimal system of currency the unit of which is the *dollar*. The hundredth part of a dollar is called a *cent*, and sums of money are, in business affairs, expressed in dollars and cents, the latter being written decimally as so many hundredths of a dollar.

23. The dollar sign ($) is used as a convenient means of indicating the expression of a sum of money. It is written to the left of the amount. In reading, the number to the left of the decimal point is read "dollars" and the *first* two *figures* of the decimal, "cents."

Thus, $16.05 is read "sixteen dollars and five cents." In order to save time where many amounts are to be read, the reading is often shortened. Thus, $7,945.21 would be read, "seven nine forty-five dollars, twenty-one cents," or $225.06 would be read "two twenty-five dollars, six cents," and $1,005.70, would be read "one double o (or naught) five dollars, seventy cents."

Using this short method, read the following:

$ 561.15	$1156.09	$3478.19	$21547.80
1006.02	1011.90	2087.05	8720.20
109.01	2020.20	2511.25	10521.07

NOTE.—When amounts are arranged in columns, the dollar sign is used only with the first amount in the column.

24. The tenth part of a cent (the thousandth part of a dollar) is called a **mill.** (Latin *mille,* a thousand.) The word does not indicate a monetary unit; it is merely a convenient term by which to designate tenths of a cent. Thus, the amount, $4.768 would be read "four dollars, seventy-six cents, eight mills." If there are more than three figures to the right of the decimal point, those beyond mills are described as the decimal part of a mill; thus, $9.7254 would be read "nine dollars, seventy-two cents, five mills, four tenths." Some accountants read the mills and figures beyond as a decimal of the cents, as "nine dollars, seventy-two cents, fifty-four hundredths," or "seventy-two and fifty-four hundredths cents." In many commercial employments, it is very important to be able to read such amounts readily.

Read the following:

$ 141.2987	$1346.09123	$ 305.09736
1660.429	.1935	2462.2506
1001.0376	11.0057	1021.058

Further facts about the currency and coinage of the United States are given in another part of this book.

CHAPTER II

ADDITION

25. To **add** numbers is to unite them into a number that has the same number of units as the numbers added.

How many units in the numbers, 2, 3, and 4?

Mary spent 3 cents for a pencil, 1 cent for a pen, and 4 cents for a tablet, how many cents did she spend for all?

26. Like numbers are those having the same kind of units.

EXAMPLES: 13 books and 8 books; 9 letters and 5 letters; $15 and $12.

Abstract numbers (See 3) are also like numbers.

27. Principle. *Only like numbers can be added.*

28. The operation of finding a number that is equal to two or more given numbers is called **addition.**

29 The numbers to be added are called **addends.**

30. The result of the addition is called the **sum.**

31. Addition is indicated by the sign called **plus** (+), which, when written between two numbers, implies that they are to be added.

32. The **sign of equality** (=) when written between two numbers, or combinations of numbers, implies that they are equal. Thus, the expression, "9 + 6 = 15," is read, "nine plus six equals fifteen," (or "nine and six are fifteen"), and implies that the two numbers, 9 and 6, are together equal to 15, or, what is the same thing, that the sum of 9 and 6 is 15.

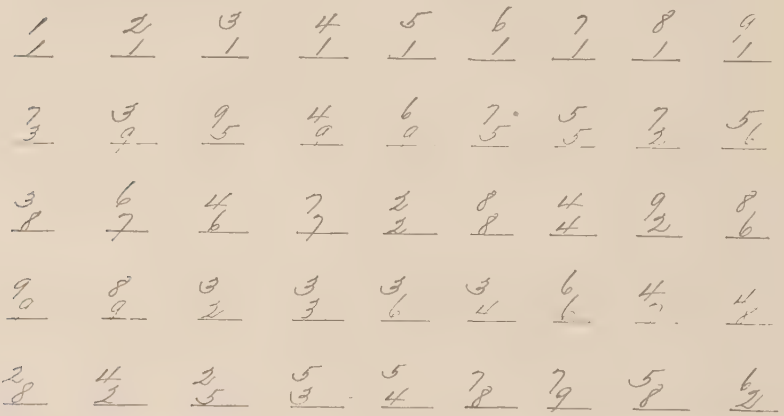
33. In mathematics, the expression of equality between two or more quantities is called an **equation**

METHODS OF ADDING

34. Counting. The result of an addition may be found by the slow process of *counting* the units in the several numbers to be added. Children in badly taught primary schools are sometimes seen to add in this way, using their fingers in the counting process.

35. Grouping. To add with facility, it is necessary to know at sight, and *without an instant's hesitation,* the results of certain fundamental addition combinations. By extending one's knowledge of combinations still further, additions can be performed still more rapidly.

36. The first essential is to know the results of all the groups of two figures each. There are forty-five of these two-figure groups, as given in the following exercise. Practice reading results from left to right, from right to left, and up and down, until you can name the results as rapidly as you can speak. Try to read each combination as *one number,* thus, in the combination $\frac{7}{4}$ do not go through the mental process, "7 and 4 are eleven;" simply *call* the combination "eleven." This ability to group and name *at once* the results of combinations, is the secret of all rapid addition.



37. Constant unit figure. If different groups of addends have the same unit figures, the unit figure of the several sums will be alike. Thus, since 7 added to 9 gives 16, the sum of any two numbers having these unit figures (as 17 and 9; 29 and 7;

87 and 19; 29 and 37; etc.) will have the unit figure 6. In like manner the unit figure for 4 and 9, is 3; for 5 and 7, is 2, for 8 and 9, is 7, and so on.

This principle enables us to add combinations like 29 and 7, 47 and 9, 69 and 7, 157 and 9, as readily as we add 7 and 9.

ORAL EXERCISE

Applying the above principle, add mentally and as rapidly as you can the following combinations:

9	48	25	46	79	27	98	39	35	46	69	49	84
52	6	8	4	6	3	7	9	9	6	7	5	7
5	3	94	3	55	7	76	16	8	49	5	28	57
69	28	7	29	8	95	9	7	76	4	99	8	7
6	8	9	78	36	42	29	22	64	28	43	29	43
95	44	99	7	14	38	16	31	36	17	19	31	36

Here is an excellent addition drill:

1. Begin with 5 and add 7 successively until you get a result greater than 50.

NOTE.—Do not say "5 *and* 7 are 12 *and* 7 are 19, etc." Simply announce the results, 12, 19, 26, etc.

2. Begin with 6 and add 5's until you get a result of 56.

3. Beginning with 9, add 7's until you reach a number whose unit figure is 9.

4. Beginning with 4, add 6's until you have a sum greater than 100.

5. Begin with 2 and add 9's until the sum exceeds 100.

6. Beginning with 5, add 7's until a number is obtained whose unit figure is 5.

7. Begin with 10 and add 8's until you get a sum whose unit figure is 0.

8. Begin with 16 and add 7's until the sum is greater than 125.

9. Begin with 85 and add 8's until the sum is greater than 200.

NOTE.—The student should practice similar exercises until a reasonable rapidity is secured.

38. One should add tens, hundreds, etc., as readily as he adds units. Practice the following until you can announce the results instantly:

27	69	84	90	75	27	50	97	62	29
40	70	30	46	60	90	45	80	50	80
91	83	40	90	88	75	80	59	83	94
60	50	79	96	90	80	97	60	80	70

569	716	200	400	749	609	807	406
600	900	945	860	800	507	508	709

39. Success in adding requires *quickness of eye* as well as familiarity with combinations. Add the following mentally, as rapidly as you can. Add the tens first, then the units, reading the result as it is formed.

23	46	82	25	36	54	21	38	53	34
35	31	16	44	52	31	57	21	45	23
42	26	35	51	27	41	53	15	24	18
34	51	62	38	52	37	26	43	13	41
35	14	61	52	21	58	35	11	74	17
23	73	17	25	14	41	12	28	21	81

Read as rapidly as you can, the results of the following combinations. Do not "add:" simply *read* the results from left to right.

714	115	385	221	516	353	252	324
181	822	411	348	543	335	517	313
254	463	624	531	215	652	342	156
532	126	173	348	544	335	517	313
714	612	931	625	846	536	925	446
635	552	758	761	932	722	734	723

40. In the following combinations, the sum of the units is 10 or more. In adding such combinations mentally, join to one of the numbers the *tens* of the other, then add the units. Thus, in adding 48 and 75, combine 48 and 70 making 118 and 5 making 123. This is simpler than adding the units first and "carrying" the ten.

Announce the results of the following:

19	64	47	68	35	58	56	47	59	47
28	27	36	27	47	29	28	38	38	26
95	57	47	58	96	86	79	28	47	68
78	69	94	63	24	75	54	95	56	79
95	76	48	53	85	37	87	68	39	89
66	59	97	97	46	93	44	75	95	64

41. The two-column method of adding is merely the application of paragraph 40 to columns of figures. In adding up or down the column, the tens of each new number are added first and then the units.

EXAMPLE: Add the following by the two-column method:

39
73
46
28

186

EXPLANATION.—Beginning at the bottom of the column, 28 and 40 make 68 and 6 make 74, and *thinking* the remaining results without naming each process, 144, 147, 177, 186.

ORAL EXERCISE

Add the following in the two-column or double-column method, naming only the results in each combination of tens and units:

43	29	34	43	28	54	83	19	32	29	81	28
21	16	16	28	37	63	31	36	23	19	46	19
86	34	28	62	16	27	29	42	44	36	37	38
27	28	29	37	54	38	16	21	83	48	28	74
44	17	19	25	29	43	47	34	16	37	43	36

42. Where the numbers to be added stand in columns, advantage is taken of the method known as "grouping," or adding the *sums* of certain groups of figures instead of adding the figures

separately. This method is used by nearly all persons who have much adding to do, and skillful practice in it often results in great expertness.

43. The beginner should first learn to use the groups that produce sums less than 11. After acquiring facility in the use of these groups, he may learn to use others.

In adding 9 to any number, the sum, of course, has one less unit than if 10 were added. If, for instance, we begin with 8 and add 9 consecutively, we get as results, 17, 26, 35, 44, 53, 62, etc.

44. In adding 11, the sum has one *more* unit than if 10 were added. Thus, beginning with 2 and adding 11, we obtain 13, 24, 35, 46, 57, 68, etc. Keeping these facts in mind, it will be found almost as easy to add groups whose sum is 9 or 11, as groups whose sum is 10.

ORAL EXERCISE

In the following exercise, the numbers that are enclosed in braces are to be added as a single number. Practice adding these columns from bottom to top, then from top to bottom. Treat each group as one number and add as rapidly as possible.

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*
5	3	4	8	7	9	7	3	2
3	6	6	5	6	7	3	9	9
8	2	8	6	5	5	8	4	3
7	7	3	7	3	3	6	8	8
6	9	8	4	4	2	5	6	4
4	4	5	9	6	4	9	5	7
9	8	7	3	2	6	2	2	6
6	0	2	6	9	8	4	4	5
5	3	6	2	8	3	6	7	8
7	2	4	4	3	5	3	9	3
2	6	7	6	6	9	8	3	9
8	5	7	8	7	7	7	6	2
0	1	4	9	5	6	6	4	7
1	8	3	5	4	2	2	8	3
4	6	5	8	8	6	5	9	5
7	3	9	3	3	5	9	1	9
8	4	2	0	7	8	3	7	6
3	3	8	7	2	9	7	6	8
9	2	7	6	6	3	4	8	4
8	8	9	9	4	8	9	4	7

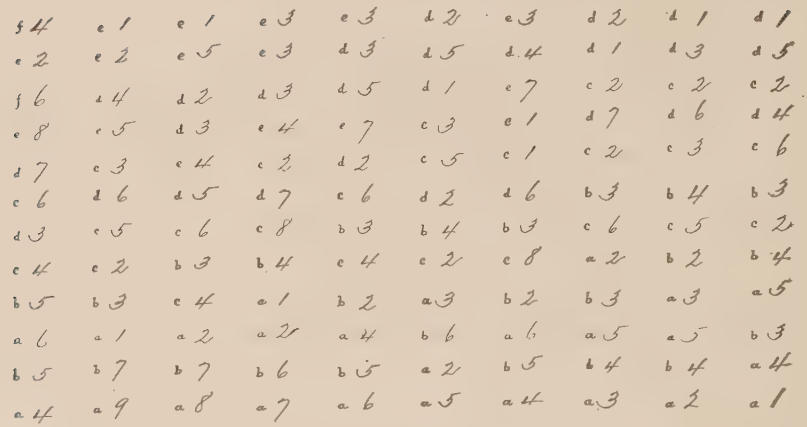
Add the following as rapidly as possible, forming your own groups:

10.	*11.*	*12.*	*13.*	*14.*	*15.*	*16.*	*17.*	*18.*	*19.*	*20.*	*21.*	*22.*	*23.*
5	9	6	4	7	3	7	9	8	4	3	4	3	9
9	2	9	3	4	6	2	2	4	3	9	7	6	5
3	7	5	8	5	2	9	3	2	9	8	6	0	8
6	3	7	6	6	8	8	6	9	7	4	3	7	6
7	5	2	3	3	4	6	8	8	8	6	5	2	7
2	8	8	5	2	9	3	4	7	9	2	8	5	4
4	4	4	4	4	6	8	9	9	7	9	2	6	9
9	1	9	2	9	5	4	7	8	6	8	7	9	3
1	6	3	5	3	4	8	6	3	8	3	3	1	3
6	9	7	7	6	6	9	4	8	7	6	5	6	6
7	7	6	6	5	2	7	8	7	6	5	9	0	7
3	3	2	1	5	7	3	7	2	9	8	6	3	2
8	6	8	3	9	3	5	1	6	2	3	2	7	4
6	7	1	0	1	5	6	9	0	3	5	4	5	3
9	9	5	8	3	0	7	3	5	4	6	3	0	8
2	4	6	2	6	1	3	0	1	8	7	5	2	5
7	1	9	9	2	7	8	5	7	7	4	8	8	1
8	6	8	1	8	2	6	8	2	6	8	7	4	0
6	8	6	6	0	4	7	4	8	3	2	4	3	6
3	5	9	4	1	8	3	6	3	5	6	6	8	6
8	9	6	8	8	3	5	2	5	4	2	5	9	5

45. Another method of grouping consists in combining all groups of from two to four figures that form 10's, 20's, or 30's, and merging into these such other figures as do not enter into these groups. This method involves passing over certain figures, and "looking ahead" for others that may be used in forming the groups. It is more difficult than the grouping illustrated in the previous exercises, and great care must be taken not to overlook figures, but the method, when thoroughly mastered, results in very rapid addition.

In the following exercises, the grouping is indicated by the letters, a, b, c, etc., the groups being formed in alphabetical order upward.

EXERCISE IN GROUPING NUMBERS



46. How to avoid errors. The one matter of prime importance in addition, is *correctness*. There is no advantage in adding rapidly, unless the rapidity is coupled with accuracy. Errors in addition arise from three principal causes:

1. Illegible figures.
2. Irregularity in the position of figures.
3. Inattention, or the inability to keep one's mind concentrated on the work.

The first two of these causes may be avoided simply by a little care; the third can be overcome by practice.

47. Checking, or proving additions. No matter how proficient one may be, errors in adding will occur occasionally, hence no accountant will allow an important addition to go unproved. The simplest and most common method of checking addition results is to add the figures in reverse order, that is, add them first from the bottom upward, and then from the top downward. If the results agree, the work is considered correct.

WRITTEN EXERCISE

Copy the following problems neatly, and then add, grouping wherever you can do so to advantage, and proving your work by adding all columns in reverse order.

1.	2.	3.	4.	5.	6.
364	327	643	842	634	3426
742	645	864	375	784	3548
436	327	963	842	643	7263
743	846	327	963	426	4986
375	364	842	643	962	5432

7.	8.	9.
4	4983649	9
26	837463	84
342	54289	356
5463	7436	8974
79834	842	765
378426	97	98
5897634	8	6

10.	11.	12.	13.	14.
21524	34253	22736	13125	12567
13065	25321	16853	24143	45526
43753	12579	40632	13560	30415
10476	40055	12345	30234	57939
44342	54204	43210	45746	21473
13526	13652	25607	25612	65114
26437	31148	34051	14200	13496
10596	26554	15779	17488	28744

15. The following are the items of cost for Mr. Granger's new residence property: Cost of lot, \$1875; lumber, \$626.06; carpenter work, \$314.25; masonry, \$175.20; plastering, \$85.00; plumbing and wiring, \$93.05; windows and doors, \$45.10; painting, \$76.00; paper hanging, \$41.75. What is the total cost of the property?

16. Frank's yearly allowance for college expenses is as follows: Tuition and fees, \$150; board, \$175; clothes, \$75; books, \$25; laundry, \$30; stationery, \$5; car fare, \$35; lectures and amusements, \$25; incidentals, \$15. What is his total yearly allowance?

17. A dealer delivered ten loads of coal with recorded weights as follows: 2142 lb., 1956 lb., 2047 lb., 2265 lb., 2143 lb., 1989 lb., 2008 lb., 2106 lb., 1989 lb., 2047 lb. What was the total weight?

18. The seven townships of Jefferson county show school enrollments as follows: Merrimac, 567; Clinton, 906; Walnut, 1146; Coldwater, 428; Marion, 1019; Rock, 479; Harding, 1428. Find the total school enrollment of the county.

19. Find the sum of all the odd numbers from 0 to 50.

20. Find the sum of all the even numbers from 0 to 51.

21. Find the sum of:

One hundred twenty-five,
Three hundred thousand, two hundred sixty-four,
One thousand, seven,
Fifty thousand, five hundred twenty-six,
Twenty-seven,
Three hundred ninety-six,
Seven hundred seventy-five,
Three thousand, three hundred thirty-three.

22. Add:

7 hundred 25,
3 thousand, 7 hundred 45,
8 thousand, 9 hundred 64,
5 thousand, 9 hundred 84,
9 thousand, 3 hundred 26,
8 thousand, 17,
4 thousand, 37,
8 hundred 24
3 thousand, 28.

48. Recording partial results. It often happens that a clerk or accountant must add under conditions where he may be interrupted before the result is obtained. To keep a record of his partially completed work, the sum of the several columns may be recorded separately, as illustrated in the accompanying example. By using this method, one may leave his work on completing the addition of any column, and on returning, may resume it where he left off.

29645	
46976	45
58764	49
74256	56
9845	54
47876	27
895	
6438	
274695	

Explanation.—The first number, 45, is the footing of the first column, giving us 5 to write under units' column, and 4 to carry to tens' column. 49 is the total of the second column, including the 4 tens carried. After the footings of all the columns are found and proved, the result is read, beginning with the last number and reading up the units' column (274695). The result is then written as the required footing.

WRITTEN EXERCISE

Copy and add the following, recording the separate results, as illustrated in the foregoing example:

23.	*24.*	*25.*	*26.*
$524.25	$1242.36	56,432,156	$257.32
247.19	1154.22	75,439,965	242.28
325.30	2425.12	28,365,283	420.50
132.75	3204.25	96,543,216	134.36
157.45	1115.38	76,896,543	175.25
284.50	2250.50	76,987,654	200.00
356.15	1425.42	76,345,923	324.14
575.25	1350.24	91,683,468	455.50
142.75	1556.18	23,456,837	255.75
381.54	2345.15	12,764,389	125.80
434.34	2412.20	32,378,425	137.25
150.50	3525.32	21,631,283	143.56

49. Adding horizontal columns. In extending the totals in the items of a bill, in preparing statistical tables, and in many other kinds of accountancy work, it is often necessary to add horizontal columns of figures. With a little practice, this can be done quite as readily as though the figures were in vertical columns.

ORAL EXERCISE

Add the following horizontal columns from left to right, then from right to left. Add by groups as much as possible.

1. 5, 6, 3, 0, 7, 4, 8, 1, 2, 9, 3, 8, 2, 5, 7, 4, 3, 2, 6, 5, 1
2. 7, 4, 2, 9, 0, 6, 5, 8, 3, 7, 4, 6, 5, 5, 6, 4, 3, 2, 1, 8, 7
3. 7, 8, 9, 6, 1, 7, 4, 8, 9, 7, 6, 5, 7, 1, 1, 6, 7, 4, 9, 5, 6
4. 6, 9, 5, 8, 6, 7, 3, 4, 6, 9, 8, 1, 7, 6, 3, 2, 5, 8, 7, 2, 9
5. 9, 1, 7, 5, 4, 8, 7, 6, 5, 4, 6, 7, 8, 9, 3, 5, 7, 4, 6, 8, 5

Copy and add the exercises on page 22. Prove all your results. You should drill in exercises of this kind until you can add at the rate of one hundred figures per minute.

27.

496281
763859
429153
783692
475789
596291
783756
954983
478821
296247

28.

519472
472896
597483
967983
479354
264395
447857
598154
519783
429671

29.

547291
738692
579456
397287
462986
739649
578492
497387
964289
472697
598734
329156
578797
647231
547897

30.

59432651
7493478
569173
21497837
4579649
578497
4261910
3874396
497837
5761434
39764875
5967381
42669178
9756410
428977
5174192
56493875
2143964
78374951

31.

4182179
57492682
72965473
9783495
429675
9781349
42998164
5793786
4959437
54789753
7837964
517839
54259281
7349561
47578357
5495648
734954
47578976

32.

4567893471
4796548347
5427964278
5739714683
7296548737
5129647347
4296548178
7895478136
5473916246
7849548342
9756713847
4298768239
5747965173
4983729647
2467539746
7839542644
7328975642
5120974030
5428374961
4156947839
7682477642
9751596147
5429764321
4837592673
8179442795
3426593788
4564397647
7394876194
2849678796
5472958476
3305425431
1959116428
7896578645
2158749362
5947347165

WRITTEN EXERCISE

In copying amounts in dollars and cents for horizontal addition, write the cents as numerators of common fractions, with the denominator, 100, understood. Thus, $4\frac{75}{}$, $5\frac{15}{}$, $\frac{45}{}$, $6\frac{03}{}$, $9\frac{75}{}$, $\frac{70}{}$, $\frac{19}{}$, $2\frac{80}{}$, $7\frac{45}{}$.

Copy, add horizontally, and prove the following:

33.	$4.75,	$5.15,	$ .45,	$6.93,	$9.75,	$ .70,	$ .19,	$2.80,	$7.45
34.	4.30,	8.75,	9.15,	5.25,	.38,	1.56,	2.95,	.48,	1.58
35.	2.87,	3.91,	5.75,	4.18,	7.29,	.87,	.54,	7.39,	8.64
36.	9.26,	.49,	6.28,	.86,	4.48,	3.74,	6.91,	.07,	2.94
37.	.40,	3.75,	1.80,	9.57,	.68,	.20,	8.40,	2.74,	3.59
38.	1.25,	4.25,	1.72,	3.65,	1.22,	1.15,	3.20,	2.15,	3.23
39.	2.29,	1.63,	2.15,	1.25,	2.17,	3.23,	1.28,	2.15,	1.32
40.	4.16,	2.28,	1.75,	1.56,	3.21,	1.34,	2.12,	3.13,	4.41
41.	1.25,	2.30,	.70,	1.54,	2.19,	3.12,	1.48,	2.15,	1.33
42.	1.85,	.93,	2.20,	1.36,	2.15,	1.33,	6.15,	4.34,	.92

Copy the following table, then complete it by adding both the horizontal and vertical columns. If the work is correct, the sum of the footings of the vertical columns will equal the sum of the several totals of the horizontal columns. This final total, if correct, is entered in the lower space at the right.

43.

													Totals	
	279	65	64	46	109	50	279	07	305	76	895	62		
	62	07	285	94	5	19	301	59	638		85	27		
	895	72	596	34	960	06	95	08	800	07	96			
	35	87	207	56	79	07	109	30	876	29	521	15		
	19	56	389	45	892	54	78	46	309	07	862	79		
	428	57	680		943	87	264	12	829	69	328	54		
	127	63	458	27	64	49	956	14	800	01	29			
	275		647	30	808	29	365	16	297	50	406	04		
	94	65	165	37	38	52	27		706	07	956	50		
	173	48	85		37	60	95	86	879	98	347	30		
Totals														

44. The following table represents the total daily attendance of the public schools in Hamilton township for the ten school months of 1909-1910. Copy carefully this tabulation on a sheet

of blank paper, add the columns and fill in the total for each month and for each school. The total attendance for the year is entered in the last space to the right, and it should be proved by adding the totals of the different months, also for the several schools.

SCHOOL	Sept	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	TOTALS
Lincoln	1126	1019	987	928	1097	1065	989	1074	994	917	
Washington	1049	984	946	898	1036	978	943	960	917	889	
Longfellow	876	807	758	716	847	827	810	864	793	719	
Douglas	948	892	868	807	849	808	796	757	714	658	
Clay	1075	1043	985	924	1019	981	967	1007	947	939	
Fulton	829	757	749	717	837	806	795	801	824	783	
Franklin	743	704	685	612	728	694	709	716	697	651	
Whittier	843	807	784	757	838	816	824	775	786	727	
TOTALS											

45. The Holland Cement Co. operates five plants at as many different towns. The several Superintendents' reports of output in barrels for the week ending June 19 are as follows:

Milltown: Mon. 765; Tues. 873; Wed. (shut down); Thurs. 563; Fri. 759; Sat. 942.

Fairview: Mon. 426; Tues. 512; Wed. 387; Thurs. 462; Fri. 486; Sat. 347.

Peidmont: Mon. 865; Tues. 916; Wed. 925; Thurs. 879; Fri. and Sat. shut down for repairs.

Pine Valley: Mon. 429; Tues. 396; Wed. 419; Thurs. 384; Fri. 476; Sat. 358.

Springville: Mon. 827; Tues. 795; Wed. 858; Thurs. 846; Fri. shut down; Sat. 759.

Prepare a table of these data showing (a) total output of each mill; (b) total output for each day of the week, and (c) total output of all the mills for the week.

CHAPTER III

SUBTRACTION

50. The word *subtraction* comes from two Latin words, *sub*, under, and *trahere*, to draw, meaning, "to draw away underneath." In arithmetic it means finding the difference between two numbers by taking the lesser from the greater.

51. There are, therefore, three terms to consider in every subtraction (1) the greater number or *minuend*, (2) the lesser number, or *subtrahend*, (3) the *difference* or *remainder*.

PRINCIPLES

52. In any subtraction it is obvious:

I. *That the minuend, subtrahend, and difference must be like numbers.* (See 26.)

II. *That the sum of the subtrahend and difference equals the minuend.*

III. *That if the difference be subtracted from the minuend, the result will equal the subtrahend.*

53. The **sign of subtraction** is the dash (—); it is read "minus," and when written between two numbers implies that the number to the right of the sign is to be subtracted from the number to the left. Thus, $12 - 4 = 8$.

54. Two views of subtraction. From Principle 2 as above, it is evident that if either of two numbers be subtracted from their sum, the result or difference will be the other number. Instead of thinking of the subtraction process as the taking of a less number from a greater, we think of it as *the adding of such a number to the less number as will make it equal to the greater.* Thus, in finding the difference between 7 and 10, we may think "7 and 3 more make 10," instead of "7 from 10 leaves 3." This method is almost universally used by clerks and shop-keepers in making change, and has therefore been called the "making change" method of subtraction. Both of these processes depend, however, upon a knowledge of addition combinations, and to be able to subtract with facility one must be able to add with equal facility.

ORAL EXERCISE

Subtract 8 from each of the following numbers:

13, 19, 12, 16, 11, 20, 15, 10, 14.

What number must be added to 9 in order that the sum may be equal to 13? 19? 15? 17? 16? 11? 14? 20?

Begin at 50 and subtract 7 until a remainder is obtained that is less than 7. In the same way begin at 60 and subtract 9; begin at 75 and subtract 8; begin at 100 and subtract 6; begin at 80 and subtract 11; begin at 65 and subtract 7; begin at 95 and subtract 12; begin at 250 and subtract 20; begin at 370 and subtract 40.

Read as rapidly as you can the difference between the larger and the smaller number in each of the following subtractions:

12	16	18	17	9	8	11	13	15	18	11	9	7	12
6	9	9	8	6	5	4	4	7	9	6	4	3	7
17	13	16	16	14	9	13	14	18	17	10	12	10	14
9	5	7	8	6	5	8	9	9	8	8	4	6	7
12	13	16	14	13	11	13	10	17	12	14	13	15	11
8	9	8	8	6	7	7	3	9	5	5	4	6	5

Without copying the numbers, write as rapidly as you can the results of the following subtractions:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*	*10.*
67	96	26	43	85	28	64	79	87	58
24	32	14	21	52	17	43	54	35	26
11.	*12.*	*13.*	*14.*	*15.*	*16.*	*17.*	*18.*	*19.*	*20.*
84	96	37	48	56	27	75	59	37	65
41	52	25	13	25	13	52	24	16	23

What is the value of X in each of the following equations?

$61 - 12 = X$	$29 - 16 = X$	$X = 27 - 16$
$42 - 24 = X$	$58 - 29 = X$	$X = 91 - 68$
$83 - 75 = X$	$50 - 28 = X$	$X = 72 - 29$
$64 - 29 = X$	$71 - 34 = X$	$X = 51 - 36$
$43 - 31 = X$	$62 - 38 = X$	$X = 43 - 28$

55. *Borrowing and carrying.* When all the figures of the minuend represent greater numbers than the corresponding figures of the subtrahend, as in the subtraction, 9786 — 6523, the process is

simple and the result, 3263, may be read at a glance. But it more often happens that some of the figures in the subtrahend represent larger numbers than the corresponding figures in the minuend, as in the subtraction, 8643 — 2725. In such cases, 10 is added to the smaller figure in the minuend, (it being assumed that the 10 was "borrowed" from the next higher order in the minuend). Then, when the lower figure is subtracted, the next order of the minuend is considered to be lessened by 1, or else 1 is added or "carried" to the next order of the subtrahend. (Both methods are taught in the schools.) It is assumed that students of this book are familiar with this method of transferring the order units of the minuend as required in ordinary subtraction.

56. Proof of subtraction. Since the remainder is the difference between the subtrahend and the minuend, the minuend is equal to the sum of the subtrahend and remainder; hence,

To prove subtraction.—*Add the subtrahend to the remainder; if the sum is equal to the minuend, the subtraction is correct.*

WRITTEN EXERCISE

Solve the following and prove your results:

1. 9124 — 857 = ?
2. 2147 — 1429 = ?
3. 24582 — 9385 = ?
4. 82430 — 29572 = ?
5. 940367 — 47038 = ?
6. 102012 — 87965 = ?
7. 720164 — 385726 = ?
8. 80000127 — 5000864 = ?
9. $17965.42 — $12867.92 = ?
10. $100046.21 — $74506.38 = ?
11. $1006.20 — $95.87 = ?
12. 2964301 — 780067 = ?
13. 42716301 — 743986 = ?

Find the difference between

14. 4275 and 958.
15. 1256 and 8794.
16. $2796.42 and $37601.15.
17. $1215627 and $862479.
18. 3874216 and 982475.
19. 10201670 and 34876216.
20. 937624 and 1000000.

57. Complementary numbers. Complementary numbers are any two numbers whose sum is equal to a unit of the next higher order. Thus, 6 and 4, 7 and 3, 8 and 2, etc., are complementary numbers, as the sum of each of these pairs is 10; and 36 and 64, 27 and 73, 12 and 88, are complementary numbers, as the sum of each pair is 100.

A number is said to be the *complement* of another when the sum of the two is a unit of the next higher order; thus, 7 is the complement of 3, 26 is the complement of 74, and 364 is the complement of 636.

In all complementary numbers of more than one figure, the sum of the unit figures is 10, while that of the other corresponding orders is 9.

EXAMPLE:	4632	247
	5368	753
	$999_{10} = 10000$	$99_{10} = 1000$

By applying the foregoing principle, a little practice will enable one to name, at sight, the complement of any number. The ability to do this is very useful in the business office in making change, etc.

ORAL EXERCISE

Name as rapidly as you can the complements of the following numbers: 36, 26¢, 64, 73, 28¢, 83, 42, 34, 65¢, 48, $26, 36, 57, $21, 32, 63, 76, 19, 83, 55, 22, 31, 43, 82, 53.

Give the complements of the following: 125, 236, 328, 475, 643, 764, 238, 753, 146, 321, 816, 458, 735, 593, 637.

In case a ten-dollar bill is offered in payment, name the amount of change required for each of the following amounts: $2.75, $1.45, $3.56, $8.27, $3.42, $6.29, $1.53, $2.35, $5.15, $8.21, $6.15, $4.38, $7.23, $2.91, $3.54, $2.74, $8.75, $4.18, $3.37, $2.85.

58. Making change. The most important application of subtraction is that of "making change," as this is required in so many of the common affairs of life. Salesmen, ticket sellers, car conductors, express and telegraph messengers and others *must* learn to make change rapidly and accurately, and all the rest of us *ought* to do so. As previously observed, (54), the salesman "subtracts" by adding to the less number such an amount as will make it equal to the greater number. For instance, if a lady buys a pair of gloves for $2.35, and tenders in payment a five-dollar bill, the salesman may return to her a two-dollar bill, a half dollar, a dime, and a nickel, saying as he does so, "two-thirty-five, four-thirty-five, eighty-

five, ninety-five, five dollars; thank you!" The salesman of experience will use as few bills or coins as possible in making change, since the more coins he handles, the more of his time is taken, and the greater the possibility of mistakes.

ORAL EXERCISE

1. Supposing that \$1 is tendered in payment for each of the following purchases, name the coins that would be given in return, also the amount of the change:

9¢, 65¢, 30¢, 19¢, 49¢, 67¢, 60¢, 42¢, 24¢, 58¢, 26¢, 38¢, 72¢, 29¢, 63¢, 41¢, 79¢, 61¢, 39¢, 54¢.

2. Name the coins and amount of change where \$5 is tendered in payment for sales as follows:

\$2.35	\$3.79	\$1.98	\$3.19	\$1.54
3.15	2.28	2.66	1.26	2.13
1.28	3.54	3.74	2.58	2.48
2.56	2.72	1.52	3.10	3.05
4.19	3.25	2.25	2.97	2.04

3. In each of the following sales, \$10 is tendered by the customer. Name denomination of bills or coins, and amount of change required. Use the "complement" method in finding amount of change:

\$6.45	\$7.25	\$1.98	\$2.54	\$4.68	\$3.18
4.22	6.34	2.62	5.18	3.91	5.67
8.06	2.86	3.71	3.27	5.13	4.81
1.56	3.29	4.19	2.11	3.92	6.55
5.24	5.36	7.48	3.79	4.33	3.15

59. *Combination problems.* In business, problems often occur in which the sum of several numbers must be subtracted from another number. It is convenient to make such computations in one operation.

ORAL EXERCISE

In each of the following exercises, find the change required by subtracting the sum of the items from the sum paid. Where possible, find the result mentally:

	Items	Paid		Items	Paid
1.	15¢, 12¢, 50¢,	\$1	*8.*	75¢, 24¢, 47¢,	\$2
2.	24¢, 36¢, 18¢,	\$1	*9.*	\$1.25, 60¢, 22¢,	\$5
3.	45¢, 15¢, 36¢,	\$2	*10.*	\$3.25, 50¢, \$1.25	\$5
4.	21¢, 18¢, 12¢,	\$1	*11.*	\$4.60, \$2.25, \$1.20	\$10
5.	45¢, 55¢, 48¢,	\$2	*12.*	\$1.20, \$1.50, \$2,	\$5
6.	23¢, 34¢, 25¢,	\$1	*13.*	\$3.60, \$4, 55¢,	\$10
7.	63¢, 8¢, 4¢,	\$1	*14.*	\$4.25, \$5.50, \$8.20,	\$20

ILLUSTRATIVE PROBLEM

From a car-load of coal weighing 24562 lb. net, three loads have been taken, weighing respectively, 2646 lb., 2138 lb., and 2387 lb. How many pounds remain in the car?

OPERATION

24562
2646
2138
2387
17391

SOLUTION.—Using the "Making change" method, $7+8+6=21$ and *1* are 22; write 1, carry $2+8+3+4=17$ and *9* are 26; write 9; carry $2+3+1+6=12$ and *3* are 15; write 3, carry $1+2+2+2=7$ and 7 are 14; write 7, 1 and 1 are 2; write 1. If preferred, subtract the unit figure of the sum of each column mentally in the ordinary way. The "making change" method however, will be found simplest when it is once learned.

Use the combination method in solving the following problems:

WRITTEN EXERCISE

21. On Jan. 1, Harper & Stone's bank balance was $3251.90. On Jan. 5, they drew a check for $641.75; on Jan. 10, another for $354.29, and on Jan. 17, a check for $1154.26. What was their bank balance on Jan. 18?

22. From a tract of 2634 acres, the following allotments have been sold: 647 acres, 224 acres, and 548 acres. How many acres remain unsold?

23. On a debt of $6534.15, the following payments have been made: May 5, $356.25; May 15, $528; May 20, $2754.45; May 31, $427.50. What is the balance due on June 1?

24. On Jan. 1, the several members of the firm of Wilson, Davis & Brown, had investment credits as follows: Wilson, $7648.96; Davis, $6524.90; Brown, $1812.55. During January the partners made withdrawals from the firm's capital as follows: Wilson, $255.75, $341.82, and $456.90; Davis, $249.15, $359.54, and $135.75; Brown, $429.70, $176.85, $96.38, and $276.42. What is each partner's investment credit on Feb. 1?

25. Copy the following form, filling in all the balances and totals as indicated. If the work is correct, the footing of each debit column should agree with the footing of the corresponding credit column:

Account Titles	Debits	Credits	Dr. Bal.	Cr. Bal.
C. R. Walker & Co.	1259 60	947 19	....	
A. C. McGuire	2435	1758 65	....	
Benson & Harland	3864 15	2589 76	....	
C. B. Esley & Sons	6528 35	9586 54		....
R. B. Bryan & Co.	2139 46	3754 95		....
Miller & Day	3836 14	4567 28		....
Cash Balance	3141 67		....	
Totals	....	....	....	....

60. Accounts and account balances. In bookkeeping the various charges and credits relating to one person or firm, or to one class of transactions, are assembled in groups called *accounts.*

The accompanying form represents an account with Hall & Morton. That part of the account at the left of the central dividing line is called the *debit side* of the account, and that part to the right is called the *credit side.* The various sums or items on the left are called *debits,* and those on the right are called *credits.*

61. The difference between the sums of the debits and credits is called the *balance* of the account.

62. To *close* or *balance* an account, is to enter the balance on the lesser side and rule up the account, as shown in the accompanying form.

63. When an account is properly closed, the totals or footings of the two sides should be equal.

Hall & Morton

19—				19—			
Jan.	2	Mdse	126 50	Jan.	6	Cash	50
	7	"	254 25		11	Mdse	162 25
	21	Cash	75		29	"	64 90
Feb.	4	Mdse	115 75	Apr.	30	Balance (RED)	512 90
	16	"	45 10				
	21	Cash	50				
	23	Mdse	123 45				
			790 05				790 05
Mar.	1	Balance	512 90				

64. An account with a person or firm is called a **personal account.**

The debit side of a personal account shows the charges, or what the person or firm has received from the business. The credit side shows the credits, or what the business has received from the person or firm. If the debit side is the larger, the account shows that the person owes the business; if the credit side is the larger, it shows that the business owes the person.

WRITTEN EXERCISE

Copy the following nine forms, supplying the missing amounts.

In number 26, find what Cameron owed us on Dec. 31, and how much he owes us or we owe him at the close. In numbers 27 and 28, find whether the firm owes us, or whether we owe the firm and how much.

26.

J. R. Cameron

19-				19-			
May	1	Mdse 30 days	254 73	May	20	Cash	200
	16	" 60 "	587 98	June	19	"	400
June	24	" 10 "	357 99	July	25	Note 60 days	357 99
Aug.	20	" 30 "	348 97	Aug.	25	10 days check	250
Sep.	5	" Net	566 70	Dec	15	Cash	500
Dec	24	" 60 days	529 89		31	Balance	?
			?				?
19-				19-			
Jan	1	Balance	?	Jan	10	Note 1 mo.	938 27
	5	Mdse 30 days	621 70	Feb	20	Cash	400
	18	" 10 "	542 98				
Feb	12	" Net	371 90				
	16	" "	421 62				

27.

W. O. Gorman & Co.

19-				19-			
Apr	4	Material & Labor	13 50	Apr	10	Cash	20
	11	" "	2 50	May	12	"	20
	14	" "	6	June	9	"	20
	23	" "	5 40				
May	3	" "	2 69				
	15	" "	40 61				
	28	" "	6 57				
June	5	" "	1 50				
	8	" "	6 43				
	21	" "	4 20				
	28	" "	5 90				

28.

John Erickson

19–					19–				
June	5	Mdse	426	90	June	15	cash	300	
	8	"	57	80		18	Note	500	
	17	"	30	29		20	draft	228	99
	21	"	1567	94		27	Cash	1000	
	26	"	2132	16		29	"	500	
	27	"	400	90		30	Note	267	94

65. In a **cash account,** the debits show the amounts received and the credits the amounts paid out; the balance shows the amount of cash on hand. In numbers 29, 30 and 31, find the balance of cash.

29.

Cash

19–					19–				
Sep.	1	On hand	1957	46	Sep.	3	Rent Ellz	100	
	2	H. Adams 2/c	127	87		4	P. Preston 2/c	50	
		Geo. Benson 2/c	607	9		5	Part Dues Note	150	
	8	Mdse Sales	327	60			Int " "	7	50
	9	" "	142	29		12	J.B. Cunningham	659	74
	10	" "	131	67		15	W.L. Sauls 2/c	179	59
	12	E.B. Mason 2/c	200	90					

30.

Cash

19–			Dr		Cr	
Mar	1	Amount on hand	400			
	1	Rent to April 1			25	
	1	Feb. 2/c Acme Grocery Co			31	80
	1	" " Black Dry Co			21	60
	1	" " City Market			24	31
	1	" " City Light & Power Co			8	34
	5	Bill Chas Peterson for service			5	10
	7	James Madison Part Note	61	80		
	10	Henry Blackman account			7	49
	15	Salary to date	62	50		
	24	Personal Expenses			16	59
	29	Subscription to Herald			4	
	31	Salary to Date	62	50		

31.

Cash

19-			Dr.	Cr.
May	1	Balance on hand	1474 59	
	3	Paid for Safe		275
	3	" " Mdse.		1075
	4	Received from Note discounted	996 50	
	6	" " G. B. Gumble on a/c	400	
	8	Loaned to S. K. Gorman		300
	8	Sold to A. S. Bashor for cash	140 28	
	9	Loaned to R. B. Gulliam		420 78
	12	Recd. from S. K. Gorman on loan	100	
	15	Sold to A. B. Trainman	60 27	
		Paid Gas Bill		12 60
	18	Recd. of C. A. Barber on acct.	130	
	21	Paid Note & Int. Citizens Bank		506 40
	26	Recd. of G. Benson on acct.	600 50	
	28	Mdse. Sales to date	600 20	
	30	Furniture Bill, Gladman & Co.		290 11
	31	Recd. from C. B. Gumble on acct.	45 61	

66. In **accounts with property,** the debits show what the property has cost us, and the credits show the sums the property has yielded us through sales or otherwise. The balance shows the gain or loss.

In numbers 32, 33, and 34, find the gain or loss.

32.

Merchandise

19-				19-			
Jan.	1	Inventory	7391 46	Jan.	31	Sales	5987 20
	31	Purchases	4629 30	Feb.	28	"	13792 43
Feb.	28	"	6291 87	Mar.	31	"	8691 77
Mar.	31	"	1391 62	Apr.	30	"	3921 78
Apr.	30	"	4697 83	May	31	"	6897 52
May	31	"	6173 49	June	30	"	8479 69
June	30	"	5781 62	July	31	"	9621 82
July	31	"	166 49	Aug.	31	"	6791 88
Aug.	31	"	12579 48	Sep.	30	"	1564 73
Sep.	30	"	4291 69	Oct.	31	"	16399 25
Oct.	31	"	1379 69	Nov.	30	"	12587 92
Nov.	30	"	873 19	Dec.	31	"	13421 87
Dec.	31	"	261 69		31	Inventory	14396 43

33.

Gladbrook Farm

19–			Dr	Cr
Feb	16	Cost of Farm	12000	
	20	Cord Wood Sold		48
	27	Costs "		9870
Mar	5	Cost of Milk House	650 90	
	7	Costs Sold		16070
	19	Cost of Well	78 40	
	26	" " Fencing	120 90	
	31	" " Labor	260 50	
	31	" " Taxes	69 79	
Apr	20	" " Repairs Building	310 60	
	23	" " Advertising Sale	30	
May	19	Proceeds Sale of Farm		14500

34.

House & Lot, #137 Main St

19–				19–			
Aug	5	Purchase Price	2200	Oct	1	Rent 1 Month	25
	6	Recording Deed	1	Nov	1	" "	25
Sep	7	Repairs Basmt	60 10	Dec	1	" "	25
	15	Painting Bldg	164 74		18	Proceeds Sale	2500
	15	Adv for Financ	3 90				
Oct	1	Taxes	21 10				
Nov	19	Two Elm Trees	40				
Dec	13	Gas Fixtures	12 80				
	17	Coal House	41 70				

67. A Statement is any written exhibit of the facts of a business or of a business transaction. In business, many forms of statements are used, and these usually require the footing of columns and the entry of balances.

A statement of account is a written exhibit of the debits and credits of a person or firm as shown by the books of the firm rendering the statement. It is usually drawn from the ledger accounts and may give the balance due, or a list of the debits with their dates, followed by a list of credits with the dates, and the resulting balance.

Business men usually send out statements of account to their debtors on or near the first of each month, or at any other time when the account is to be paid.

On succeeding pages are given three model forms of statements of account with memoranda for preparing statements in similar form.

The following form of statement is used when itemized bills have been sent to the customer.

CHICAGO, May 30, 19—

Messrs. Gladden & Collins,
147 Harriott St,

IN ACCOUNT WITH

H. C. JURGENSON & CO.

19—						
May	5	Mdse per bill	49	26		
	7	" "	5	67		
	10	" "	14	93		
	18	" "	16	78		
	27	" "	31	10		
	30	" "	42	19	159	93
		Cr.				
May	8	Mdse. returned	4	91		
	15	Cash	50			
	26	"	50		104	91
					55	02

35. Prepare a statement of account in above form from (the Student) & Co. to J. C. Farmer, 216 Main street, your city, for the following Mdse. per bills rendered: Oct. 1, $129.50; Oct. 5, $172.20; Oct. 12, $232.15; Oct. 25, $179.75; Oct. 30, $215.20—with the following credits: Oct. 10, Cash, $190, Merchandise returned, $12.50; Oct. 20, Cash, $225.

The following form of statement is used when itemized bills have not been sent to the customer.

St. Louis, Mo., Sept. 1, 19-

The Acme Mills Co,
Minneapolis, Minn.

IN ACCOUNT WITH

SMYTHE FURNITURE CO.

Aug	5	56 ft. Counter with eight Wickets 6.40	358	40		
	12	80 ft. Partitions 3-	240			
	12	1 extra Wicket	6	50		
	19	8 Office Chairs 5-	40			
		2 Type W. Desks 21-	42			
	21	3 Tables 30"-72"-30" 7.25	21	75		
	26	1 Roll Top Desk	24	50		
	31	36 hr. Labor 60	21	60		
					754	75
		Cr.				
Aug	15	Cash	250			
	25	2 Off. Chairs Returned	10			
	28	30 day note	294	75		
					554	75
					200	
		Received Payment Sep 9, 19-				
		John Garver & Co				
		by Lane				

36. Prepare a statement of account in above form from (the Student) & Co. to Chambers & Lee, Clinton, for the following items: Nov. 1, 1 M No. 10 Envelopes, printed, $3.75; Nov. 6, 5 M 4-page Circulars, $24.75; Nov. 10, 3 M Letter Heads, $8.75; Nov. 20, 2 M Statements of Account, $17.50—with the following credits: Nov. 15, Cash $20; Nov. 21, overcharge on Statements, $10; Nov. 25, Cash $15.

The following form of statement is used by firms who number their invoices.

Cincinnati, Jan 1, 19—

The J. K. Black Dry Goods Co,
Waterloo, Iowa

IN ACCOUNT WITH

Hibbard, Bartlett, Spencer & Co.

Dec	4	Inv# 15973 60 days	429	69	
	6	16148 60 "	583	97	
	9	16315 30 "	24	96	
	18	16872 60 "	240	08	
	21	16959 30 "	860	19	
	26	17040 30 "	144		
	28	17112 30 "	2007		
					2160 40
		Cr.			
Dec	8	Cash 421 10 Dis 8 59	429	69	
	12	" 572 29 " 11 68	583	97	
					1013 66
					1146 74

37. Prepare a statement of account in above form from (the Student) & Co. to Johnson & Stevens, Washington, for the following numbered invoices and credits: Dec. 5, Inv. No. 2929, 30 ds., $212.50; Dec. 10, Inv. No. 3015, 60 ds., $179.50; Dec. 15, Inv. No. 3120, 90 ds., $372.29; Dec. 20, Inv. No. 3219, 30 ds., $275.50—with the following credits: Dec 15, Cash $208.25, Discount $4.25; Dec. 20, Cash, $175.91, Discount $3.59.

CHAPTER IV

MULTIPLICATION

68. Multiplication is a short method of adding equal numbers.

At 3 cents each, what is the cost of 7 pears?

SOLUTION.—If the pears were bought one at a time, we should have to pay out 3 cents *seven times*, and the cost would equal $3+3+3+3+3+3+3=21$ cents. If, when we add the seven 3's, we remember the result, the next time we have a similar problem we would not need to add, but would simply say, "The pears will cost 7 times 3 cents, or 21 cents."

69. The results of a certain number of continuous additions, are shown in a table known as "The Multiplication Table."

70. In multiplication, the several numbers to be added are thought of and spoken of as one number, which is called the **multiplicand.** The number that indicates the number of times the multiplicand is taken, is called the **multiplier.**

The result of the multiplication is called the **product.**

Thus, in the expression, "5 times 9 are 45," 9 is the *multiplicand*, 5 is the *multiplier*, and 45 is the *product.*

The multiplier and multiplicand are also called *factors* of the product.

71. The sign of multiplication is $(\times)$, and when written between two numbers indicates that either one is to be taken as an addend as many times as there are units in the other. Thus, the expression "$4 \times 6 = 24$" may be read "four times six are twenty-four," or "four multiplied by six equals 24."

72. Continued multiplication is the process indicated by joining a series of numbers with the multiplication sign, as $5 \times 2 \times 3 \times 4$. The result of a continued multiplication is obtained by multiplying any of the numbers by any other, and this product by the third, and so on; the result being the same, regardless of the order in which the numbers are taken.

PRINCIPLES

I. *The multiplier is always an abstract number.*

II. *The multiplicand and product are like numbers.*

III. *Either term of the multiplication is contained in the product as many times as there are units in the other term.*

ORAL EXERCISE

1. Prove by addition that 8 times 7 is equal to 7 times 8.

2. What is the product of $2 \times 3 \times 5 \times 7$?

In how many ways can you get the result?

3. Find, by addition, the cost of 5 acres of land at $25 per acre.

4. Name the multipler, the multiplicand, and the product in the following problem: What is the total length of seven boards, if each board is 9 ft. long?

5. Find, by subtraction, the number that must be multiplied by 7, to give 91 as a product.

6. Find, by addition, the number that 14 must be multiplied by to give 84 as a product.

7. Find, by addition, the product of $3 \times 4 \times 5$.

8. How many distinct multiplications are necessary to find the product of $6 \times 7 \times 8 \times 4 \times 12$?

9. Prove that the product of $6 \times 3 \times 5 \times 2$ is the same regardless of the order in which the several multiplications are performed.

10. If $2 \times 4 \times (x) = 72$, find x by subtraction.

73. The multiplication table. To apply the operation of multiplication to numbers generally, it is indispensable to know the multiplication table as far as 9×9, but in the application of multiplication to business affairs, it is necessary to know combinations much beyond this. In some texts in arithmetic the table is extended to 25×25 but the mastery of such a table is a burdensome and unnecessary task.

Perhaps the most practical form of a multiplication table is that which includes *all* combinations up to a given product, say 100. Such a table includes combinations like 2×17, 3×26, 4×15, etc., which are constantly used in business, and are therefore of much more importance than combinations like 11×12, 17×19, 13×23, which occur much less frequently. Multiplications involving the *small* numbers, 2, 3, 4, and 5, are much more common in ordinary business affairs than are combinations involving larger numbers.

It is upon the foregoing considerations that the mastery of the following table is advised. The student will note that the "2 line"

extends to 2×50, the "3 line" to 3×33, the "4 line" to 4×25, etc, the *product limit* being 100. The table, therefore, includes *all* the factors of *all* the products up to and including 100. The *thorough mastery* of this table will prove a great "time saver" to the student, and greatly aid him in his future work.

THE BUSINESS MULTIPLICATION TABLE

(Including the factors of all products up to 100.)

NOTE.—The table is shortened by omitting in each group the combinations given in the previous groups. Thus, the "six line" begins with 6×6, since the combinations 2×6, 3×6, 4×6, and 5×6 have been given in previous groups.

$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 4 = 8$
$2 \times 5 = 10$
$2 \times 6 = 12$
$2 \times 7 = 14$
$2 \times 8 = 16$
$2 \times 9 = 18$
$2 \times 10 = 20$
$2 \times 11 = 22$
$2 \times 12 = 24$
$2 \times 13 = 26$
$2 \times 14 = 28$
$2 \times 15 = 30$
$2 \times 16 = 32$
$2 \times 17 = 34$
$2 \times 18 = 36$
$2 \times 19 = 38$
$2 \times 20 = 40$
$2 \times 21 = 42$
$2 \times 22 = 44$
$2 \times 23 = 46$
$2 \times 24 = 48$
$2 \times 25 = 50$
$2 \times 26 = 52$
$2 \times 27 = 54$
$2 \times 28 = 56$
$2 \times 29 = 58$
$2 \times 30 = 60$

$2 \times 31 = 62$
$2 \times 32 = 64$
$2 \times 33 = 66$
$2 \times 34 = 68$
$2 \times 35 = 70$
$2 \times 36 = 72$
$2 \times 37 = 74$
$2 \times 38 = 76$
$2 \times 39 = 78$
$2 \times 40 = 80$
$2 \times 41 = 82$
$2 \times 42 = 84$
$2 \times 43 = 86$
$2 \times 44 = 88$
$2 \times 45 = 90$
$2 \times 46 = 92$
$2 \times 47 = 94$
$2 \times 48 = 96$
$2 \times 49 = 98$
$2 \times 50 = 100$

$3 \times 3 = 9$
$3 \times 4 = 12$
$3 \times 5 = 15$
$3 \times 6 = 18$
$3 \times 7 = 21$
$3 \times 8 = 24$
$3 \times 9 = 27$

$3 \times 10 = 30$
$3 \times 11 = 33$
$3 \times 12 = 36$
$3 \times 13 = 39$
$3 \times 14 = 42$
$3 \times 15 = 45$
$3 \times 16 = 48$
$3 \times 17 = 51$
$3 \times 18 = 54$
$3 \times 19 = 57$
$3 \times 20 = 60$
$3 \times 21 = 63$
$3 \times 22 = 66$
$3 \times 23 = 69$
$3 \times 24 = 72$
$3 \times 25 = 75$
$3 \times 26 = 78$
$3 \times 27 = 81$
$3 \times 28 = 84$
$3 \times 29 = 87$
$3 \times 30 = 90$
$3 \times 31 = 93$
$3 \times 32 = 96$
$3 \times 33 = 99$

$4 \times 4 = 16$
$4 \times 5 = 20$
$4 \times 6 = 24$

$4 \times 7 = 28$
$4 \times 8 = 32$
$4 \times 9 = 36$
$4 \times 10 = 40$
$4 \times 11 = 44$
$4 \times 12 = 48$
$4 \times 13 = 52$
$4 \times 14 = 56$
$4 \times 15 = 60$
$4 \times 16 = 64$
$4 \times 17 = 68$
$4 \times 18 = 72$
$4 \times 19 = 76$
$4 \times 20 = 80$
$4 \times 21 = 84$
$4 \times 22 = 88$
$4 \times 23 = 92$
$4 \times 24 = 96$
$4 \times 25 = 100$

$5 \times 5 = 25$
$5 \times 6 = 30$
$5 \times 7 = 35$
$5 \times 8 = 40$
$5 \times 9 = 45$
$5 \times 10 = 50$
$5 \times 11 = 55$
$5 \times 12 = 60$

$5 \times 13 = 65$	$6 \times 7 = 42$	$7 \times 7 = 49$	$8 \times 9 = 72$
$5 \times 14 = 70$	$6 \times 8 = 48$	$7 \times 8 = 56$	$8 \times 10 = 80$
$5 \times 15 = 75$	$6 \times 9 = 54$	$7 \times 9 = 63$	$8 \times 11 = 88$
$5 \times 16 = 80$	$6 \times 10 = 60$	$7 \times 10 = 70$	$8 \times 12 = 96$
$5 \times 17 = 85$	$6 \times 11 = 66$	$7 \times 11 = 77$	
$5 \times 18 = 90$	$6 \times 12 = 72$	$7 \times 12 = 84$	$9 \times 9 = 81$
$5 \times 19 = 95$	$6 \times 13 = 78$	$7 \times 13 = 91$	$9 \times 10 = 90$
$5 \times 20 = 100$	$6 \times 14 = 84$	$7 \times 14 = 98$	$9 \times 11 = 99$
	$6 \times 15 = 90$		
$6 \times 6 = 36$	$6 \times 16 = 96$	$8 \times 8 = 64$	$10 \times 10 = 100$

ORAL PROBLEMS FOR ANALYSIS

1. What will 17 lemons cost at 5 cents each?

ANALYSIS.—They will cost 17 times 5 cents, or 85 cents.

NOTE.—In giving the analysis of problems in multiplication, do not reverse the terms in the expression. Take for the multiplicand the number that is of the same denomination as the required product. Thus, in the foregoing problem it would be illogical to say "five times seventeen," instead of "seventeen times five," notwithstanding that both expressions involve the same numerical result.

2. Allowing 8 rails to the panel, how many rails will be required for 12 panels of fence?

3. If 15 quires of paper are used in one copy of a certain book, how many quires will be used in an edition of 3000 copies?

4. How many days are there in 13 weeks?

5. Find the value of 5 acres of land at $17 an acre.

6. How many eggs in 2 cases containing 30 dozen each?

7. If there are 16 ounces in a pound, find the weight, in ounces, of 6 pounds of nails.

8. What is the value of 13 cases of berries at $3 each?

9. Find the cost of 5000 ft. of lumber at $19 per thousand.

10. What is the value of 11 yards of ribbon at 12 cents a yard?

11. How far will a man travel in 9 hours at the rate of 60 miles an hour?

12. At 25 cents an hour, what can a man earn in 3 days of 8 hours each?

13. What is the distance around a square field that measures 17 rods on each side?

14. If a boy runs 12 feet per second, how far can he run in a minute?

15. In an orchard there are 12 rows of trees with 9 trees in each row; how many trees in the orchard?

16. A room is 60 feet long and 40 feet wide; how many square feet of floor space in the room?

NOTE.—The square feet in any surface is equal to the product of the length by the width. The analysis is as follows:

If the surface were 60 feet long and 1 foot wide, the surface would equal 60 square feet; therefore, a surface 60 feet long and 40 feet wide will contain 40 times 60 square feet, or 2400 square feet.

17. Find the cost of 8 quarts of berries at 5 cents a pint.

18. What is the length of 5 panels of fence, the length of a panel being 16 ft?

19. Mary's flower bed is 14 feet long and 6 feet wide, how many square feet in its surface?

20. What is the value of 5 dozen eggs, each egg being worth 2 cents?

21. A certain room is 16 feet long and 10 feet wide, what is the entire distance around the room? How many square feet in the floor?

22. Find the cost of 15 tons of coal at $5 per ton.

74. Factors. The factors of any number are any group of two or more numbers which, being multiplied together, will produce the number. Thus, 3 and 4, or 6 and 2, are factors of 12; while the groups of 2, 3, and 5, or 5 and 6, or 3 and 10, or 2 and 15, are factors of 30.

75. The **multiples** of a number are the products obtained by multiplying it by other numbers. Thus, 24, 36, 48, 60, etc., are multiples of 12.

ORAL EXERCISE

1. What are the factors of 51? 27? 65? 91?

2. Name three different groups of factors that will produce 60.

3. Think of as many pairs of factors as you can that will produce 72, 96, 100.

4. Name the missing factor in the following equations:

$$5 \times 3 \times \text{—} = 90$$
$$2 \times 3 \times 4 \times \text{—} = 96$$
$$7 \times 2 \times \text{—} = 70$$
$$3 \times 2 \times \text{—} = 75$$
$$2 \times 8 \times \text{—} = 80$$

5. Name, in order, all of the multiples of 12 less than 100. Of 13; of 18; of 16.

6. Of what numbers is 60 a multiple? 72? 96? 85? 68?

7. Name a number that is a multiple of 5 and 12. Of 9 and 15; of 18 and 4; of 7 and 13.

8. Name all multiples, less than 100, of the following numbers: 7, 14, 15, 17, 19, 21, 29.

WRITTEN EXERCISE

Multiply 5679 by 6.

(a)

$$\begin{array}{r} 5679 \\ \times\ 6 \\ \hline 54 \\ 42 \\ 36 \\ 30 \\ \hline 34074 \end{array}$$

EXPLANATION.—If all results are recorded in full, the operation would appear as in (a); but in practice, the additions are performed mentally, the tens of each product being carried to the units of the next, only the general products being recorded, as in (b).

(b)

$$\begin{array}{r} 5679 \\ \times\ 6 \\ \hline 34074 \end{array}$$

1. Multiply each of the following numbers by 4: 964; 7446; 9241; 24563; 9768.

2. Multiply the following by 5: 2046; 7487; 92507; 76906; 19867.

3. Multiply the following by 6: 7438; 20906; 38720; 84769; 372067.

4. Multiply the following by 7: 42876; 89735; 48738; 90764; 58678.

5. Multiply the following by 8: 9675; 6958; $97.65; $189.48; $9476.28.

6. Multiply the following by 9: 87¢; $9.85; $597.62; $9839.46; 46758.

ORAL EXERCISE

Announce, as rapidly as you can, the results of the following multiplications:

1. $13 \times 3 = ?$
2. $15 \times 6 = ?$
3. $19 \times 2 = ?$
4. $16 \times 6 = ?$
5. $14 \times 5 = ?$
6. $13 \times 6 = ?$
7. $29 \times 2 = ?$
8. $24 \times 3 = ?$
9. $17 \times 4 = ?$
10. $46 \times 2 = ?$
11. $18 \times 4 = ?$
12. $23 \times 3 = ?$
13. $19 \times 5 = ?$
14. $15 \times 5 = ?$
15. $36 \times 2 = ?$
16. $28 \times 3 = ?$
17. $14 \times 6 = ?$
18. $18 \times 5 = ?$
19. $13 \times 7 = ?$
20. $23 \times 4 = ?$
21. $350 \times 2 = ?$
22. $1900 \times 4 = ?$
23. $18 \times 30 = ?$
24. $260 \times 2 = ?$
25. $1700 \times 5 = ?$
26. $160 \times 30 = ?$
27. $300 \times 48 = ?$
28. $130 \times 60 = ?$
29. $2800 \times 3 = ?$
30. $470 \times 2 = ?$
31. $1300 \times 40 = ?$
32. $380 \times 20 = ?$
33. $1700 \times 30 = ?$
34. $140 \times 300 = ?$
35. $180 \times 40 = ?$
36. $120 \times 12 = ?$
37. $160 \times 50 = ?$
38. $500 \times 19 = ?$
39. $700 \times 130 = ?$
40. $29 \times 30 = ?$
41. $1314 \times 3 = ?$
42. $1912 \times 5 = ?$
43. $1713 \times 4 = ?$
44. $2648 \times 2 = ?$
45. $1815 \times 5 = ?$
46. $918 \times 3 = ?$
47. $514 \times 6 = ?$
48. $1918 \times 4 = ?$
49. $2946 \times 2 = ?$
50. $1525 \times 3 = ?$
51. $1327 \times 3 = ?$
52. $2319 \times 4 = ?$
53. $1131 \times 3 = ?$
54. $1618 \times 5 = ?$
55. $2114 \times 3 = ?$
56. $1729 \times 30 = ?$
57. $3213 \times 3 = ?$
58. $1319 \times 50 = ?$
59. $4728 \times 20 = ?$
60. $16140 \times 6 = ?$
61. $\$1.90 \times 5 = ?$
62. $\$18.25 \times 3 = ?$
63. $\$16.40 \times 2 = ?$
64. $\$13.18 \times 4 = ?$
65. $\$12.29 \times 3 = ?$
66. $\$6.48 \times 2 = ?$
67. $\$7.15 \times 5 = ?$
68. $\$28.17 \times 3 = ?$
69. $\$47.25 \times 2 = ?$
70. $\$15.17 \times 5 = ?$
71. $\$14.18 \times 4 = ?$
72. $\$26.45 \times 2 = ?$
73. $\$18.32 \times 3 = ?$
74. $\$12.16 \times 6 = ?$
75. $\$8.19 \times 5 = ?$
76. $\$27.16 \times 3 = ?$
77. $\$19.25 \times 2 = ?$
78. $\$17.28 \times 2 = ?$
79. $\$13.25 \times 3 = ?$
80. $\$15.18 \times 5 = ?$
81. $504 \times 19 = ?$
82. $1700 \times 40 = ?$
83. $\$1316 \times 50 = ?$
84. $204 \times 23 = ?$
85. $320 \times 30 = ?$
86. $17019 \times 30 = ?$
87. $111517 \times 5 = ?$
88. $12018 \times 4 = ?$
89. $\$190.29 \times 3 = ?$
90. $\$240.15 \times 2 = ?$
91. $\$180.15 \times 5 = ?$
92. $\$305 \times 16 = ?$
93. $\$507.03 \times 13 = ?$
94. $\$306.04 \times 14 = ?$
95. $\$1825 \times 30 = ?$
96. $\$13.15 \times 60 = ?$
97. $\$25016 \times 30 = ?$
98. $\$140.15 \times 5 = ?$
99. $\$1700 \times 40 = ?$

WRITTEN EXERCISE

Multiply 4678×537.

4678
537

32746
14034
23390

2512086

EXPLANATION.—Beginning with units, multiply successively by the figures of the multiplier, writing the several products so that figures of the same order may stand in the same column. The several products are then added, the result being the final product.

7. Multiply 7464 by 29.
8. Multiply 9348 by 58.
9. Multiply 7468 by 89.
10. Multiply 567 by 246.
11. Multiply 946 by 708.
12. Multiply 1234 by 567.
13. Multiply $87.46 by 987.
14. Multiply $679.43 by 369.

Find the required products for the following continued multiplications:

15. $64 \times 87 \times 9 = ?$
16. $9 \times 8 \times 6 \times 7 \times 15 = ?$
17. $47 \times 102 \times 64 = ?$
18. $3 \times 9 \times 6 \times 54 \times 42 = ?$
19. $35 \times 127 \times 86 = ?$
20. $53 \times 12 \times 43 \times 124 = ?$

CONTRACTIONS IN MULTIPLICATION

76. A contraction is a shorter method or process for obtaining a result than the one ordinarily used.

A number of contractions are employed in multiplication, the following being among the most useful:

77. *Where there are ciphers at the right of the multiplier or the multiplicand.* If either factor is 10, 100, 1000, 10000, etc., the product is obtained by writing the ciphers at the right of the other factor. Thus, $10 \times 56 = 560$; $157 \times 1000 = 157000$.

562
2400

2248
1124

1348800

234000
16

1404
234

3744000

If either factor consists of other figures than 1, with ciphers to the right, multiply regardless of the ciphers, and annex them to the product.

243000
2500

1215
486

607500000

If both factors have ciphers at the right, multiply together the remaining digits, and annex as many ciphers as there are at the right of both factors.

WRITTEN EXERCISE

21. Multiply 768 by 1000.
22. Multiply 346 by 4600.
23. Multiply 1,234,000 by 75.
24. Multiply 346,700 by 9000.
25. Multiply 106,000 by 30,400.
26. Multiply 740,000 by 28,000.

78. *Where one part of the multiplier is a multiple of another part.*

Multiply 1254 by 637.

1254
637
8778
79002
798798

EXPLANATION.—Since 63 is 9 times 7, we multiply the first partial product by 9 instead of continuing the multiplication in the usual way.

Multiply 2351 by 856.

2351
856
18808
131656
2012456

EXPLANATION.—Since 56 is 7 times 8,, we first multiply 8, then multiply this partial product by 7.

Multiply 120304 by 48246.

120304
48246
721824
2887296
5774592
5804186784

EXPLANATION.—Since 24 is 4 times 6, we first multiply by 6, and this product by 4, for the second partial product. Since 48 is 2 times 24, we multiply the second partial product by 2. The sum of the three partial products is the complete product. We could also find the third partial product by multiplying the first one by 8.

79. *When the multiplicand is slightly less or slightly greater than 10, 100, 1000, etc.*

Multiply 857 by 99.

85700
857
84843

EXPLANATION.—As 99 is 1 less than 100, first multiply by 100, then subtract 857.

Multiply 948 by 11.

9480
948
10428

EXPLANATION.—As 11 is 1 more than 10, first multiply by 10, then add 948. Such multiplications may usually be performed mentally.

Multiply 436 by 995.

436000
2180
433820

EXPLANATION.—First multiply by 1000, then subtract 5×436 from the result.

Multiply $26.48 by 102.

$2648
52.96
$2700.96

EXPLANATION.—Multiplying $26.48 by 100 gives $2648; add to this amount 2 × $26.48, or $52.96.

80. Cross multiplication. The process called cross multiplication may be classed as a short method, in that time is gained by doing much of the work mentally, the final results only being written. The method may be used with any number of figures in multiplier and multiplicand, but it is hardly practical as a short method except with multiplier and multiplicand of two figures each. It is especially valuable in billing.

The method consists in computing all partial products of a given denomination and adding them without recording the partial products.

WRITTEN PRACTICE

Find the product of 28×23.

28
23
644

EXPLANATION.—$3 \times 8 = 24$. Write 4 in units place and carry 2 to tens place. 3 times 2 tens (6 tens) + 2 tens times 8 (16 tens) + 2 tens = 24 tens. Write 4 in tens place and carry 2 to hundreds. 2 tens times 2 tens = 4 hundreds + 2 hundreds = 6 hundreds, which should be written in hundreds place in the product.

Copy the following problems, perform the multiplications by the method just illustrated, and record the products after the sign of equality.

27.	$22 \times 26 =$	*32.*	$33 \times 18 =$	*37.*	$36 \times 24 =$
28.	$41 \times 33 =$	*33.*	$27 \times 14 =$	*38.*	$22 \times 27 =$
29.	$16 \times 18 =$	*34.*	$18 \times 22 =$	*39.*	$36 \times 28 =$
30.	$24 \times 31 =$	*35.*	$43 \times 26 =$	*40.*	$42 \times 19 =$
31.	$27 \times 13 =$	*36.*	$17 \times 24 =$	*41.*	$61 \times 34 =$

81. Another short method of multiplication, especially practical in billing, can be used with two or three figures in multiplicand and multiplier, when the sum of the units in the two factors equal 10 and when the remaining figures in the two factors are alike. In such case we write the product of the units as units and tens, supplying the cipher in ten's place if necessary. We add 1 to the tens multiplier, and write the product of the remaining figures in multiplier and multiplicand as hundreds in the product.

Multiply 33 by 37.

33
37
1221

EXPLANATION.—$3 \times 7 = 21$ written as units and tens. 7 units times 3 and 3 units times 3 equals 10 units or 1 ten times 3. 3 tens plus 1 ten equals 4 tens times 3 tens equals 12 hundreds.

Multiply 71 by 79.

71
79
5609

EXPLANATION.—$9 \times 1 = 9$ written as units, placing a cipher in tens place. 8 times 7 tens equals 56 hundreds.

ORAL EXERCISE

Apply the short method just illustrated and announce the following products without using a pencil.

1.	28×22	*6.*	46×44	*11.*	92×98
2.	27×23	*7.*	37×33	*12.*	83×87
3.	81×89	*8.*	62×68	*13.*	74×76
4.	72×78	*9.*	55×55	*14.*	69×61
5.	63×67	*10.*	25×25	*15.*	53×57

In case of ciphers in one or both factors, perform the multiplication regardless of ciphers, annexing the required ciphers after completing the multiplication.

16.	270×23	*19.*	25×250	*22.*	304×306
17.	540×560	*20.*	240×260	*23.*	208×202
18.	190×110	*21.*	710×790	*24.*	507×503

WRITTEN EXERCISE

Perform the following mutiplications, shortening the work, wherever practical, by means of any of the foregoing contractions:

42.	$3564 \times 287 = ?$	*48.*	$796500 \times 24000 = ?$
43.	$1356 \times 4900 = ?$	*49.*	$324610 \times 9072 = ?$
44.	$2456 \times 2408 = ?$	*50.*	$349 \times 98 = ?$
45.	$27800 \times 540 = ?$	*51.*	$656 \times 11 = ?$
46.	$124760 \times 56287 = ?$	*52.*	$\$13.47 \times 105 = ?$
47.	$345200 \times 7208 = ?$	*53.*	$\$9.58 \times 97 = ?$

54. Find the value of 267 tons of coal at $4.75 per ton.

55. If there are 16 lots to the block, how many lots are there in the Adams Addition to the town of Mayfair, which extends 7 blocks in one direction and 5 blocks in the other. What is the total value of the Addition, if the average value of the lots is $175 each?

56. On a Dakota ranch there are 2180 sheep. If the fleeces average 8 pounds each, what is the value of the wool clip when the price of the wool is 18 cents a pound?

57. In a certain cornfield there are 469 rows, and 528 hills in each row; how many hills of corn in the field?

58. In the town of Belleville, each of the 6 street cars makes 32 trips per day, and the receipts of each car per trip average $1.45. What are the total receipts for one year of 365 days?

59. When the price of lumber is $18.75 per thousand feet, what is the value of 187 thousand feet?

60. There are 320 rods in one mile; how many rods in 129 miles?

61. A pile of iron rails weighing 65 pounds to the yard contains 148 rails, each 8 yards long; what is the weight of the pile?

62. The net cost of making a certain book was found to be 59 cents per copy. What was the cost of an edition of 7500 copies?

63. Scott & Co., contractors, employ 75 men at wages averaging $1.75 per day. What is the amount of the pay-roll for one month of 26 working days?

PROMISCUOUS WRITTEN PROBLEMS

64. Mr. Bennet bought three tracts of land, including respectively, 325, 79, and 146 acres, at $17.50 per acre. He afterwards sold the largest tract at $20 per acre, and the other tracts at $16.25 per acre. Did he lose or gain on the speculation, and how much?

65. John Clark sold to Carter Bros. 79 bushels of potatoes at 45 cents per bushel and 125 pounds of butter at 17 cents per pound. He received in payment groceries amounting to $17.25, and $20 in cash. How much do Carter Bros. still owe him?

66. Clarence earns $9.50 a week. He estimates his weekly expenses as follows: Board, $3.50; laundry, 40 cents; books and

newspapers, 50 cents; incidentals, 75 cents. Allowing $50 per year for clothing, what should he save in one year (52 weeks)?

67. At 35 cents a rod, what will it cost to plant a hedge around a field 65 rods long and 48 rods wide?

68. A lumber dealer sold to a farmer 34 thousand feet of lumber at $16.25 per thousand, receiving in payment 7 tons of hay at $9.50 a ton, 16 cords of wood at $3.75 a cord, and a check for the balance. What was the amount of the check?

69. A grocer bought from a huckster 357 pounds of butter @ 13 cents, and 175 dozen eggs @ 9 cents. He sold the butter @ 16 cents per pound and the eggs @ 12 cents per dozen. What was his gain on the purchase?

NOTE.—The character (@) signifies "each" or "apiece," and is used to indicate the market price per pound, per bushel, per dozen, etc.

70. A dealer gained $75.90 on 55 tons of baled hay, which he bought at $7.75 per ton. For what sum did he sell it?

71. Miller & Brown, real estate dealers, bought a farm containing 375 acres for $9500. They expended on the property for repairs and building, $758.25, and for taxes $102.15. They sold wood from the land to the amount of $95.25, and then sold the entire property for cash at $31.50 per acre. What did they gain through the speculation?

72. A contractor engaged to build a cement walk 155 feet long and 6 feet wide, for $95. He then sublet the work at 9¢ per square foot. How much did he gain on the contract?

NOTE.—The product of the length multiplied by the width will give the number of square feet.

CHAPTER V

DIVISION

82. Division may be defined as a short method of making several successive subtractions of the same number.

Walter having 12 plums gives 2 to Henry; how many remain? He then gives 2 to Frank, and 2 to Chester; how many are left? How many times can he give 2 away? In the same way successively take 3 out of 15 until nothing is left. How many subtractions are required? How many times can 4 be taken out of 20? Out of 12? Out of 28? How many 5's in 35? In 45?

How many times can you take 3 out of 14? What number will remain?

If Ethel has $25, how many books can she buy at $5 each, and what sum will she have remaining? How often can 9 be taken out of 50, and what number will remain?

By the process of *successive subtractions*, we may find how many times any number is contained in any other larger number. By using our knowledge of the multiplication table, we solve such problems without resorting to the slow process of successive subtractions. For instance, we know by the multiplication table that 7 times 9, or 9 times 7 equals 63. We may therefore conclude that 7 is *contained* in 63, (or may be *taken from* 63) 9 times, also that 9 is *contained* in 63, (or may be *taken from* 63) 7 times.

83. The process of finding how many times one number is contained in another is called **division.**

Thus, when we say, "20 divided by 5 equals 4," we mean that 5 is contained 4 times in 20.

84. The **divisor** is the number that we divide by.

85. The **dividend** is the number that is divided.

86. The **quotient** is the result of the division.

Thus, in the expression, "20 divided by 5 equals 4," the *dividend* is 20, the *divisor* is 5, and the *quotient* is 4.

87. When the division is not exact, the remaining or undivided part of the dividend is called the **remainder.** Thus, when we divide 38 by 5, we find that the divisor is contained in the dividend 7 times, with a remainder of 3.

88. The general sign of division is (÷); when written between two numbers it indicates that the number on the left (dividend) is to be divided by the number on the right (divisor).

Thus, the equation $42 \div 7 = 6$, is read, "42 divided by 7 equals 6."

There are other methods of indicating division which will be considered later in this chapter.

89. Division is also defined, *"the process of separating a number into equal parts."*

NOTE.—The word *divide* comes from two Latin words, *dis*, apart, and *videre*, to pierce. Hence, literally, "to pierce (or cut) into two parts." We still use the word in this sense when we say, "the room is divided by a partition," or, "the people are divided in their opinions."

The two division processes, however, are fundamentally the same, and both are based on subtraction, as may be seen by the following problem and analysis:

If 5 pencils cost 30 cents, what is the cost of one pencil?

ANALYSIS: At one cent each, the pencils would cost 5 cents; to cost 30 cents, each pencil must cost as many cents as 5 cents is contained times in 30 cents.

The more common, as well as more convenient, formula for problems of this kind is to say, "one pencil will cost *one-fifth* of 30 cents," but it should be noted that to find *one-fifth of 30 cents*, we must *find how many times 5 cents is contained in 30 cents.*

Although these two division processes are fundamentally alike, being based on successive subtractions of one number from another, they are *in purpose, logically distinct,* and will be so considered here.

90. *Measurement division or ratio.* This is the *measurement* of one number by another of the same kind. There are various ways of expressing this idea. Thus, we say:

1. 12 apples is 4 times 3 apples.
2. 4 apples is contained in 12 apples 3 times.
3. 4 apples is one-third ($\frac{1}{3}$) of 12 apples.
4. The ratio of 12 apples to 4 apples is 3.
5. The ratio of 4 apples to 12 apples is $\frac{1}{3}$.

The foregoing expressions all mean the measurement of one number by another, or what is the same thing, the finding of the ratio of one number to another.

91. Ratio is indicated in arithmetic by the sign (:). When this sign is written between two numbers, the meaning is that the number on the left is to be measured, or divided by the one on the right. Thus, $12:4=3$, the expression being read: "The ratio of 12 to 4 is 3."

92. Separative division or partition. This process of division may be defined as the separation of one number into as may equal parts as there are units in another.

Thus, in the problem: "If 7 acres of land cost \$91, what will one acre cost? The *purpose* is to separate \$91 into as many equal parts as there are units in 7, that is, into 7 equal parts.

93. Fractions. Fractional numbers (See 14) illustrate the idea of separative division, or that something is to be considered as being divided into equal parts.

94. In mathematics generally, the fractional form is used as a convenient means of expressing any unexecuted division. Thus, $\frac{a}{b}$ means, in algebra, that a number represented by a is to be divided by a number represented by b.

95. The ratio of one quantity to another is often expressed by a fraction, as $3:5=\frac{3}{5}$, $X:Y=\frac{X}{Y}$, or $20:4=\frac{20}{4}$.

Also, a fraction may be considered to mean simply the division of its numerator by its denominator. Thus, $\frac{3}{4}$ means $3 \div 4$.

In this view, the numerator of every fraction may be considered as the dividend and its denominator as the divisor of an *unexecuted* division. Likewise, any ordinary division may, if desired, be ex-

pressed in the fractional form. Thus, the division, $24 \div 8$, may be expressed as $\frac{24}{8}$, or $17 \div 5$ as $\frac{17}{5}$.

96. The fractional form is also used to indicate the division of the remainder when the division is inexact. Thus the division of 58 by 9 may be expressed $58 \div 9 = 6\frac{4}{9}$, or $\frac{47}{8}$ may be written $5\frac{7}{8}$.

97. It is very important for the student to become familiar with these various methods of indicating division, as division is by far the most important relation of numbers. The subjects of *fractions, decimals, percentage,* and *proportion,* which include much the larger part of practical arithmetic, are merely so many applications of this foundation process. The student who thoroughly learns the forms and principles of division, will have little difficulty with their further application as required in the above named subjects.

GENERAL PRINCIPLES OF DIVISION

98. All processes and applications of division are governed by the following General Principles which should be thoroughly understood:

I. *The divisor and quotient are factors of the dividend.*

II. *In measurement division, or ratio, the dividend and divisor are like numbers, and the quotient is an abstract number.*

III. *In partition, or separative division, the quotient and dividend are like numbers, and the divisor is an abstract number.*

IV. *A change in the dividend produces a proportionate and similar change in the quotient; a change in the divisor, produces an opposite and proportionate change in the quotient.*

That is, if we increase the dividend (retaining the same divisor) we increase the quotient in the same proportion, or if we diminish the dividend, we diminish the quotient. But if we increase the divisor (retaining the same dividend) we *diminish* the quotient in the same proportion, and if we diminish the divisor, we *increase* the quotient in the same proportion.

V. *If both dividend and divisor be either increased or diminished, in the same proportion, the quotient remains the same.*

ORAL DRILLS

Announce as rapidly as you can, the results of the following divisions:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*
2)26	2)52	2)72	4(84	13)26	13)39	31)62	15)75
3)45	2)34	5)75	7)91	14)42	13)78	14)70	15)90
2)38	3)54	2)74	5)85	29)58	17)68	24)72	31)93
3)39	2)56	2)78	3)87	32)64	23)69	29)87	19)95
2)54	4)60	4)76	6)84	18)36	18)90	27)81	14)98
2)28	3)57	5)85	5)90	17)51	18)54	16)64	47)94
3)42	5)65	3)81	2)92	13)65	19)57	6)90	16)80
2)46	2)64	2)82	3)93	17)34	18)72	23)92	22)88
4)52	4)68	2)84	2)94	16)48	26)52	14)42	36)72
2)32	3)66	4)72	4)96	14)56	21)63	13)91	32)96
2)58	3)63	5)90	6)96	19(38	17)85	26)78	39)78
3)51	5)70	6)78	7)98	21)84	15)60	24)96	13)52

Give the quotients and remainders of the following:

9.	*10.*	*11.*	*12.*	*13.*	*14.*	*15.*	*16.*
7)24	8)44	8)41	6)71	7)55	26)53	13)15	15)100
5)36	7)62	7)60	12)59	12)82	15)76	14)79	21)100
8)42	6)43	6)51	11)60	19)97	13)68	18)91	12)1000
9)56	5)52	12)62	9)89	3)53	14)58	14)100	18)100
5)38	9)49	8)20	6)70	18)59	24)100	16)100	17)100

Name rapidly the quotients in the following indicated divisions:

$\frac{92}{46}$, $\frac{96}{12}$, $\frac{121}{11}$, $\frac{144}{12}$, $\frac{68}{17}$, $\frac{91}{13}$, $\frac{54}{27}$, $\frac{85}{17}$, $\frac{64}{16}$, $\frac{78}{26}$, $\frac{81}{27}$, $\frac{57}{19}$, $\frac{63}{21}$, $\frac{90}{18}$, $\frac{45}{15}$, $\frac{96}{16}$, $\frac{72}{24}$, $\frac{92}{23}$, $\frac{74}{37}$, $\frac{48}{16}$, $\frac{98}{14}$, $\frac{75}{15}$, $\frac{80}{16}$, $\frac{76}{19}$, $\frac{84}{21}$, $\frac{78}{39}$, $\frac{51}{17}$, $\frac{98}{49}$, $\frac{36}{18}$, $\frac{70}{14}$, $\frac{90}{15}$.

Name the whole number and fraction that result from each of the following divisions:

$98 \div 8$	$36 \div 7$	$55 \div 17$	$87 \div 16$	$60 \div 19$	$95 \div 13$
$87 \div 5$	$95 \div 6$	$81 \div 13$	$59 \div 18$	$71 \div 11$	$53 \div 16$
$83 \div 15$	$100 \div 12$	$50 \div 13$	$90 \div 17$	$68 \div 13$	$70 \div 19$

Express in simplest form the value of the following ratios:

91 : 7	87 : 3	17 : 5	5 : 17	26 : 6	40 : 13	13 : 40
50 : 14	30 : 12	12 : 30	29 : 14	95 : 13	100 : 14	19 : 15
64 : 21	39 : 8	5 : $2\frac{1}{2}$	19 : 5	19 : $3\frac{4}{5}$	51 : 17	51 : 3

Find three-fourths of each of the following numbers:

12, 24, 32, 44, 52, 56, 64, 72, 76, 28, 36.

Find two-thirds of each of the following numbers:

57, 87, 96, 63, 99, 45, 75, 81, 69, 78, 93, 42.

Find five-eighths of each of the following numbers:

64, 96, 48, 32, 72, 80, 40, 56, 24, 32.

What is the ratio of:

29 to 5? 60 to 12? 12 to 60? 91 to 7? 10 to 4? 10 to $2\frac{1}{2}$? 20 to 6? 87 to 16? 50 to 13?

ORAL TEST PROBLEMS

At 15 cents, how many pineapples may be bought for 75 cents?

ANALYSIS.—As many pineapples may be bought as 15 cents is contained times in 75 cents, or 5; hence, 5 pineapples may be bought.

Analyze the following:

1. If there are 8 pints in one gallon, how may gallons in 56 pints?

2. How many weeks are there in 84 days?

3. At $3 a case, how many cases of eggs can be bought for $81?

4. If a train is running at the rate of 18 miles an hour, in what time will it run 72 miles?

Frank sold 6 bushels of apples for $9; what was the price per bushel?

ANALYSIS.—Each bushel was sold for one-sixth of $9, or $1.50.

Analyze the following:

5. Henry earned $45 in 15 days; what did he earn in a day?

6. A train ran 87 miles in 3 hours; what was the rate of speed?

7. Fred planted 85 hills of potatoes. If there were 5 rows, there were how many hills in each row?

8. If seven hogs weighed 840 pounds, what was their average weight?

9. Carl raised 150 bushels of potatoes on a six-acre lot; what was the yield per acre?

10. At 30 cents each, how many music books can be bought for $6.30.

11. If 20 tons of coal may be loaded in one car, how many cars will be required for 780 tons?

12. To fence a lot 20 rods long and 15 rods wide, costs $84; what is the cost of the fence per rod?

13. Of a field containing 80 acres, Henry plowed 16 acres; what part of the field did he plow?

ANALYSIS.—Since 16 acres is contained in 80 acres 5 times, he plowed one-fifth of the field.

14. 8 days is what part of 8 weeks?

15. From a bin of wheat containing 560 bushels, 140 bushels were sold; what part of the wheat was sold?

16. Fred planted 85 hills of potatoes. If there are 17 hills in each row, there were how many rows?

17. If there are 18 rows in an orchard containing 720 trees, there are how many trees in each row?

18. If 40 arithmetics can be packed in a box of given size, how many similar boxes will be required for an edition of 6000 books?

19. Henry had $84 and has spent $14; what part of the whole amount has he spent?

20. How many feet in 96 inches?

21. $108 was paid for a cement walk 2 yards wide and 60 yards long; what was the price per square yard?

22. Chester wishes to invest \$96 in chickens at \$3 per dozen; how many dozen can he buy?

23. A carload of coal containing 13 tons was sold for \$52; what was the price per ton?

24. A company of men packed 750 tons of ice in 15 days; what was the average quantity packed per day?

25. Arthur has 96 cents; how many pencils can he buy at 4 cents each? After he has purchased 6 pencils, what part of the remainder of his money will be required to purchase two more?

26. A train moving at the rate of 17 miles per hour will require how long to run 85 miles?

27. A lady received 3 pounds of coffee for 7 dozen eggs *@* 9¢. What was the coffee worth per pound?

28. Four men built 60 rods of fence in 3 days. What was the average number of rods per day for each man?

29. If cranberries are worth 15¢ a quart, how many quarts should be given in exchange for 5 pounds of butter @ 18¢?

30. A torpedo boat running at the rate of 24 knots an hour, will steam how far in 15 minutes?

31. A grocer sells lemons at 20¢ a dozen and oranges at 30¢. At these prices, how many oranges are worth the same as 36 lemons?

32. Henry and his 5 brothers share equally a legacy of \$18,000. Henry invests his share in land at \$15 an acre. How many acres does he purchase?

33. Five men united in a speculation, buying 5000 bushels of grain *@* 65¢ and selling it at 75¢. If the expenses were \$100 and they shared the profits equally, what sum did each gain?

34. In a certain town the water tax is 13 cents per thousand gallons, with 15 cents per quarter for meter rent. If Mr. A's quarterly water bill is \$2.75, how much water was used?

35. Five heirs inherit an estate valued at \$60,000, which they are to share equally. Afterwards another heir is found: to what amount will this discovery diminish the share of each of the other five?

99. Division processes. When the dividend is a number so large that the quotient can not be determined by inspection, a written process of division must be used. There are two forms of the process called respectively, *long division* and *short division.*

100. In **long division,** all the steps in the division process are shown, as illustrated in the following example:

Divide 25521 by 7.

```
7)25521(3645
  21
  ---
  45
  42
  ---
   32
   28
   --
    41
    35
    --
     6
```

EXPLANATION.—Taking enough of the left hand digits of the dividend to contain the divisor, we find that 25 (thousands) contains 7, 3 (thousands) times. 3 (thousands) times 7 gives 21 (thousands), which, subtracted from 25 (thousands), leaves 4 (thousands). Annexing the next figure and dividing, we find that 7 is contained in 45 (hundreds) 6 (hundreds) times. Multiplying and subtracting as before, we have left the remainder, 3 (hundreds). Bringing down the next figure of the dividend, we have 32 (tens), which, divided by 7, gives us 4 (tens). Multiplying, subtracting, and bringing down the next figure, as before, we have 41, which divided by 7, equals 5. Again multiplying and subtracting, we have a final remainder of 6.

101. Short division is merely an abbreviated form in which the various multiplications and subtractions are performed mentally, thus:

```
7)25521
  ------
  3645 + 6 rem.
```

EXPLANATION.—Ascertaining mentally that 7 is contained in 25 3 times, with 4 remainder, we write the 3 in the first figure of the quotient, and *mentally prefix* the 4 to the next figure, which gives 45 as the dividend in the next step, and so on.

Short division can be employed only when the divisor is a number small enough for us to perform the multiplications and subtractions of the process mentally. The student should never employ long division unless the divisor exceeds 12.

WRITTEN EXERCISE

1.	Divide 1978 by 7.	*11.*	$21786 \div 19 = ?$
2.	Divide 8976 by 6.	*12.*	$87463 \div 28 = ?$
3.	Divide 10274 by 8.	*13.*	$10271086 \div 146 = ?$
4.	Divide 7861247 by 4.	*14.*	$94207658 \div 67 = ?$
5.	Divide 20761201 by 5.	*15.*	$560217563 \div 496 = ?$
6.	Divide 12172946 by 9.	*16.*	$85205617 \div 649 = ?$
7.	Divide \$8673.57 by 3.	*17.*	$417601924 \div 4567 = ?$
8.	Divide \$407232.45 by 9.	*18.*	$93681596 \div 837 = ?$
9.	Divide 9706421 by 11.	*19.*	$170215862 \div 299 = ?$
10.	Divide 201634596 by 12.	*20.*	$37021675 \div 7019 = ?$

102. When the divisor is a multiple of ten. If the divisor is 10, 100, 1000, 10000, etc., the division is indicated by pointing off from the right of the dividend as many figures as there are

ciphers in the divisor. Thus, $25267 \div 100 = 252.67$ and $31496 \div 1000 = 31.496$. The quotient is that part of the result to the left of the period, or "decimal point" and the part to the right is the remainder. Results of this form of division are called *decimals*. (See 17.)

103. If the divisor consists of other figures than 1, with ciphers to their right, the division process is shortened, as illustrated in the following example:

Divide 246713 by 1700.

```
17|00)2467|13(145
      17
      --
       76
       68
       --
        87
        85
        ---
         213
```

EXPLANATION.—Cut off the ciphers of the divisor, and from the right of the dividend, an equal number of figures. Divide the remaining figures of the dividend by the divisor, exclusive of its ciphers. To the first remainder, if there be one, annex the figures cut off from the right of the dividend, for a complete remainder.

WRITTEN EXERCISE

21. $24678 \div 1400 = ?$

22. $18276201 \div 9000 = ?$

23. $74216730 \div 870000 = ?$

24. $8726013 \div 52000 = ?$

25. $674120167 \div 24600 = ?$

WRITTEN PROBLEMS

26. It requires about 8 minutes for light to pass from the sun to the earth, a distance of 92,500,000 miles. What is the approximate velocity of light per minute? Per second?

27. During 18 days in the month of May, the total attendance at the Fairview school amounted to 576 days. What was the average daily attendance? If there are 36 pupils enrolled, what was the average number of days each pupil attended during the month?

28. A tract of land containing 2425 acres was sold for $39600; what was the price per acre?

29. By a drop of 7 cents in the market price of wheat, a speculator lost $1330; how many bushels did he buy?

30. The product of two numbers is 40548. One of the numbers is 327; what is the other?

31. Mr. Howland sold his spring wool clip at 17 cents a pound, receiving $400.86. How many pounds did he sell?

32. If a steam pump has a capacity of 575 gallons a minute, how long will it require to fill a tank holding 32775 gallons?

33. By a rise of 25 cents per bushel in the market price, the value of a farmer's potato crop is increased $198.75. How many bushels did he raise?

34. If a cruiser can sustain an average speed of 19 knots an hour, how many days and hours will she require to cover a distance of 5662 knots?

35. Mr. L. finds that he spends for tobacco an average of $2 per month. In how many years and months would this expenditure amount to enough to purchase a $500 piano?

CHAPTER VI

GENERAL PROPERTIES OF NUMBERS

104. Arithmetic is a practical application of the fundamental number processes—*addition, subtraction, multiplication* and *division*. There are, however, some general principles and properties of numbers, as well as certain special processes that have not been considered in the previous chapters, and to which we shall now give attention.

DIVISIBILITY

105. A number is said to be **divisible** by another number when the former will contain the latter a whole number of times. Thus, 12 is divisible by 2, or by 3, or by 4, but it is not divisible by 5 or by 7.

106. An **exact divisor** of a number is any number by which it is divisible. The exact divisors of a number are usually spoken of as "factors," but, strictly speaking, the factors are the numbers which multiplied together will produce the number. Thus, the groups, 2, 2, 2, and 3; or 2, 4, 3; or 3, 8; or 2, 12, are factors of 24. But the exact divisors of 24, viz.: 2, 3, 4, 6, 8, and 12, are not properly spoken of as "factors of 24," since the product is not 24, but 13824.

107. In considering the divisibilty of a number, the unit 1 and the number itself are not regarded as either factors or divisors of the number.

108. A **prime number** is a number that is not divisible by any integer, or whole number. Thus, 11, 13, 17, 61, 73, and 97 are prime numbers.

109. A **composite number** is a number that has integral factors.

EXAMPLES: 12, 15, 9, 51, 91.

110. The **prime factors** of a number are those prime numbers which multiplied together will produce the number. Thus, 2×2

$\times 3 \times 5$ are the prime factors of 60. The method of finding the prime factors of a number is here illustrated.

What are the prime factors of 72?

2|72	EXPLANATION.—The number is divided by any of its prime factors. The quotient obtained, is divided by one of its prime factors, this quotient by another prime factor, and so on until a quotient is obtained that is a prime number. The several divisors and the last quotient, 2, 2, 2, 3, 3, are the required prime factors of the number.
2|36	
2|18	
3|9	
3	

111. Factoring is the process of finding the integral factors or divisors of numbers.

ORAL EXERCISE

1. Name all of the exact divisors of 60.

2. Name five pairs of factors, each of which will produce 96.

3. Find the prime factors of 100, of 40, of 72.

4. What prime numbers are exact divisors of 90?

5. Name the series of prime numbers which multiplied together will produce 108.

6. What divisors of 72 are composite?

7. Name the missing factor in each of the following equations:

$2 \times \_\_ \times 5 \times 7 = 210$ $\qquad$ $2 \times 2 \times 6 \times 2 \times \_\_ = 96$

$3 \times 2 \times \_\_ \times 5 = 270$ $\qquad$ $2 \times 8 \times 3 \times \_\_ = 144$

8. Name the prime numbers from 1 to 101.

9. Factor the following numbers: 39, 51, 58, 69, 78, 87.

10. Determine by inspection which of the following numbers are prime and which are composite.

(Read the numbers as rapidly as you can.)

61, 45, 9, 39, 79, 41, 27, 37, 91, 57, 69, 31, 93, 87, 59, 51, 43, 83, 77, 85, 29, 63, 71, 89, 97.

112. Tests of divisibility. It is possible to tell by inspection whether a number is divisible by *2, 3, 4, 5, 6, 8, 9, 10,* or *11.*

A number is divisible:

By 2, when its unit figure is 0, 2, 4, 6, or 8.

NOTE.—*Even numbers* are those divisible by 2, *odd numbers* are those not divisible by 2.

By 3, when the sum of its digits is divisible by 3. Thus, 2343 is divisible by 3 because the sum of its digits ($2 + 3 + 4 + 3 = 12$) is 12, which is divisible by 3.

By 4, when the number expressed by the two right-hand figures is divisible by 4, or when these two figures are ciphers. *Examples,* 1116, 2508, 1396, 1900.

By 5, when the right-hand figure is 0 or 5.

By 6, when it is an even number and the sum of its digits is divisible by 3, (that is, when the number is divisible by both 2 and 3).

By 8, when the number expressed by the three right-hand figures is divisible by 8, or when these three figures are ciphers. *Examples,* 5128, 6008, 7000, 5240.

By 9, when the sum of its digits is divisible by 9.

By 10, when its right-hand figure is 0.

By 11, when the sums of its alternate digits are equal. Thus, the number 5236 is divisible by 11, since the sum of the alternates, $6 + 2$ equals the sum of the remaining alternates, $3 + 5$.

For the same reason the following numbers are divisible by 11: 77, 121, 561, 2222, 9108, 3575, 6246361.

This principle is the basis of the so-called "Check Eleven System," a method used by acountants for safe-guarding postings and additions.

ORAL TEST EXERCISE

1. Apply the foregoing tests and determine what numbers are divisors of the following numbers:

2524	3600	50505	78100
3165	4005	26739	39611
3522	1101	47502	55550
6128	7902	30330	44550

2. Form a number having 5 figures that will be divisible by 9 and by 5.

3. What years between 1907 and 1930 will be leap years?

NOTE.—Only years divisible by 4 are leap years; centennial years are leap years only when divisible by 400.

4. What centennial years since the year 1000, by the calendar now used, would not have been leap years?

5. Form a number having 6 figures that is divisible by 11.

6. Form a number having 5 figures that will be divisible by 2, 5, 9, and 8.

POWERS AND ROOTS

113. A **power** of a number is the product arising from using that number two or more times as a factor.

EXAMPLES: $4(=2\times2)$; $27(=3\times3\times3)$; $49(=7\times7)$; $32(=2\times2\times2\times2\times2)$.

Using a number twice as a factor produces its *second power*, or *square;* using it three times as a factor produces its *third power*, or *cube;* four times, its *fourth power* and so on. Thus the second power or square of 2 is 4; its third power or cube is 8; its fourth power is 16; and its fifth power is 32.

NOTE.—The first power of a number is the number itself.

114. A **root** is the factor used in producing a given power. Thus, 2 is the second, or *square root* of 4, while 3 is the third, or *cube root* of 27.

ORAL EXERCISE

1. What is the third power or cube of 5? The second power or square of 12? The cube of 3? The square of 20? The fourth power of 3?

2. What is the square root of 49? of 64? of 100? What is the cube root of 27? of 125? of 8?

3. Square the following numbers: 6, 8, 10, 11, 12, 15, 20, 30, 50, 100.

4. Cube the folowing: 2, 3, 4, 5, 6, 7, 8, 9, 10.

NOTE.—The second power of a number is called its *square* because of the fact that if the side of a square be multipled by itself, the result will be the area of the square. Thus, a square that is 12 inches on each side, contains 12×12, or 144 square inches.

In like manner, the third power of a number is called its cube, because the three dimensions of a cube multiplied together produce the volume of a cube in cubic units. Thus, a cubical block that is 12 inches along each edge, contains $12\times12\times12$, or 1728 cubic inches.

5. How many square feet in a floor 16 feet long and 16 feet wide?

6. If a square lot of land contains 81 square rods, what is the length of one side?

7. A cubical block of marble contains 125 cubic feet, what is the length of one edge?

8. What is the square root of 16? of 100? of 1600? of 25? of 2500? of 900?

9. What number multiplied by itself will give 4900 as a product?

10. A man paid $400 for a piece of land; the number of acres he bought was equal to the price per acre. How many acres did he buy?

11. Find the fourth power of 5. The cube of 40.

12. Arrange the following factors into two groups, each composed of the same factors: $2 \times 2 \times 3 \times 3 \times 5 \times 5$. What is the product of each group? What is the product of all the factors? What is the square root of this product?

13. What are the prime factors of 144? Separate these factors into two equal groups. What is the product of each group? What is the square root of 144?

14. What numbers between 1 and 100 are second powers? Give the root of each.

15. What numbers between 1 and 100 are third powers? Give the root of each.

16. The square of 2 multiplied by the square of 5 produces what number? What is its square root?

COMMON FACTORS

115. One of the definitions of the word *common* is, "belonging to all." Thus, we speak of persons owning property "in common," that is, jointly; or we may say, "anger is a passion *common* to man and beast."

116. A **common factor** or **common divisor,** of two or more numbers, is a number that will exactly divide each of them. Thus, 4 is a common factor of 24, 60, and 84.

117. The **greatest common factor** or **divisor** of two or more numbers, is the largest number that will exactly divide each of them. Thus, 12 is the greatest common divisor of 24, 60, and 84.

118. When numbers have no common factor they are said to be *prime to each other.* Thus, 16, and 27, and 25, are prime to each other.

ORAL EXERCISE

1. What is the largest number that will exactly divide 96 and 72?

2. What is the greatest common factor of 90 and 108?

3. What is the largest number that is a common divisor of 30, 60, 54, and 48?

4. Name a common divisor of 34, 51, and 85.

5. Name all the numbers less than 100 of which 19 is a common divisor.

6. Name five numbers that have 12 for their greatest common divisor.

7. Name 3 numbers each of which is divisible by 2, 3, and 5. What is their greatest common divisor?

8. Name 3 composite numbers that are prime to each other.

9. What is the highest power of 2 that will exactly divide 48, 64, and 80?

10. What is the highest power of 3 that will exactly divide 27, 54, 81, 90?

11. Name three numbers each of which is divisible by the first power of 2 and the square of 3.

12. What is the greatest common divisor of 48, 72, 96, and 120?

119. A common factor of two numbers is also a factor of their sum and of their difference and of the remainder after division. Thus, 15 is a common factor of 75 and 180; it is also a factor of $75 + 180$, or 255, and of $180 - 75$, or 105, and of 30, or the remainder which results from dividing 180 by 75.

This principle is applied in finding the greatest common divisor of numbers that are too large to be easily factored.

Find the greatest common divisor of 1159, 703, and 1387.

```
703)1159(1
    703
    ---
    456)703(1
        456
        ---
        247)456(1
            247
            ---
            209)247(1
                209
                ---
                 38)209(5
                    190
                    ---
                     19)38(2
                        38
                        --
```

EXPLANATION. — Selecting the two lesser numbers, 703 and 1159, we divide the greater by the less, obtaining for a remainder, 456. Since the common divisor, if there be one, is either this remainder or a divisor of it (See 119), we divide the first divisor by the remainder, and continue the process until we have a remainder of 19, which exactly divides the previous divisor. This last divisor, 19, is the greatest common divisor of 703 and 1159, and as it also exactly divides 1387, it is the greatest common divisor of the three numbers. If two numbers are prime to each other, that is, have no common divisor, the final remainder will be 1.

Although this method of finding the greatest common divisor is seldom applied in practical arithmetic, the student should not be unfamiliar with it.

Find the greatest common divisor of 168, 216, 240, 264.

4	168	216	240	264
2	42	54	60	66
3	21	27	30	33
	7	9	10	11

$4 \times 2 \times 3 = 24$, g. c. d.

EXPLANATION.—Divide the numbers by any number that will divide *all* of them. Divide the resulting quotients in the same way until the quotients obtained have no common divisor. The product of the several divisors is the greatest common divisor.

WRITTEN EXERCISE

1. Find the greatest common divisor of 1037 and 629.

2. What is the largest number that will exactly divide 455, 364, 286?

3. Find whether the numbers 151 and 175 are prime to each other.

4. What is the greatest common divisor of 360, 252, 396, 180?

5. A carpenter is directed to make the widest possible side walk, no wastage, out of the following planks each 1 foot in width and 2 inches in thickness: 9 planks 20 feet long, 8 planks 16 feet long, and 6 planks 12 feet long. What will be the length and width of the walk?

COMMON MULTIPLES

120. A **common multiple** of two or more numbers is any number that is exactly divisible by each of them. Thus, 60 or 90 is a common multiple of 2, 3, 5.

121. The **least common multiple** of two or more numbers is the *least* number that is exactly divisible by each of them.

EXAMPLE: 30 is the least common multiple of 2, 3, 5.

PRINCIPLES

122. I. *A multiple of a number is a multiple of any factor of that number.*

II. *The least common multiple of two or more numbers is the product of the highest powers of the factors in these numbers.*

What is the l. c. m. (least common multiple) of 24, 80, 75, 60, 108?

$24 = 2^3 \times 3$
$80 = 2^4 \times 5$
$75 = 3 \times 5^2$
$60 = 2^2 \times 3 \times 5$
$108 = 3^3 \times 2^2$
l. c. m. $= 16 \times 25 \times 27 = 10800$

EXPLANATION.—Applying Prin. II., we multiply together the several "highest powers." These are, $2^4=16$, $3^3=27$, and $5^2=25$. The product, 10800, is therefore the least number that will contain all the factors of the numbers, hence the l. c. m. of the numbers themselves.

123. In the ordinary operations of arithmetic, it is rarely necessary to find the l. c. m. of numbers greater than 100. In such numbers the factors of the l. c. m. should easily be found by inspection. To do this readily, it is only necessary to be familiar with the lower powers of the smaller prime numbers, 2, 3, 5, and 7, and to recognize these when they are factors in numbers less than 100. To this end the following table should be learned.

TABLE OF POWERS

1st	2nd	3rd	4th	5th	6th	7th
2	4	8	16	32	64	128
3	9	27	81	243		
5	25	125	625			
7	49	343				

ORAL EXERCISE

1. What is the highest power of 2 in 96? in 80? in 68? in 100? in 72?

2. What is the highest power of 3 in 108? in 72? in 45? in 54? in 81? in 90?

3. What is the highest power of 5 in 75? in 250? in 100?

4. What is the highest power of 7 in 98? in 490? in 84?

WRITTEN EXERCISE

Find the l. c. m. of

6. 16, 30, 55, 33.
7. 50, 75, 36, 28.
8. 8, 9, 14, 18.
9. 5, 25, 35, 48.
10. 4, 12, 20, 30.
11. 20, 25, 30, 40.
12. 60, 120, 240, 360.
13. 21, 24, 72, 30.
14. 25, 75, 125, 225.
15. 9, 21, 35, 105.

REDUCTION AND CANCELLATION

124. We have learned (98 Prin. V) that in any case of division, the divisor and dividend may be proportionally increased or

diminished without affecting the quotient. Thus, if in the example $91 \div 65$, we divide both the dividend and divisor by 13, we shall have the simpler and equivalent division $7 \div 5$. In like manner, the division, $144 \div 36$, equals $12 \div 3$, (obtained by dividing both dividend and divisor by 12.)

125. Many practical operations of arithmetic involve division in which either the dividend or the divisor is *compound*, that is, it consists of several factors instead of a single number. Thus, the expression, $(12 \times 8 \times 5) \div (4 \times 6 \times 2)$ means that the product of 12, 8, and 5, is to be divided by the product of 4, 6, and 2. This expression is more conveniently written $\frac{12 \times 8 \times 5}{4 \times 6 \times 2}$

126. Now, applying Prin. V, we may greatly simplify such compound divisions by striking out, or "cancelling," equal factors from both dividend and divisor, as in the accompanying operation.

$$\frac{\overset{2}{\cancel{12}} \times \cancel{8} \times 5}{\underset{2}{\cancel{4}} \times \cancel{6} \times \cancel{2}} = 10$$

EXPLANATION.—We cancel the 2 and the 4 in the divisor and the 8 in the dividend. This *divides both dividend and divisor by* 8. We then cancel the 6 in the divisor and cancel the 12 in the dividend reserving the resultant factor 2. As all the terms of the divisor are canceled, the quotient is the product of the remaining terms of the dividend $(2 \times 5 = 10)$.

127. Cancellation is the process of simplifying compound divisions by striking out, or canceling equal factors in dividend and divisor. By this process we divide factors in the dividend and divisor by the same number; canceling the numbers divided and writing the quotient of each, if more than unity.

Divide $(12 \times 31 \times 40 \times 36)$ by $(14 \times 48 \times 9 \times 5)$.

$$\frac{\overset{4}{\cancel{12}} \times 31 \times \overset{8}{\cancel{40}} \times \cancel{36}}{\underset{7}{\cancel{14}} \times \underset{4}{\cancel{48}} \times \cancel{9} \times \cancel{5}} = \frac{124}{7} = 17\tfrac{5}{7}$$

EXPLANATION.—We cancel 12 in dividend and 48 in divisor, dividing each by 12, and write the quotient 4 below 48. 4 and 9, factors in the divisor (product 36), will cancel 36 in the dividend. 5 in the divisor will cancel 40 in dividend, leaving the factor 8. 2, a factor of 14, will cancel 2, a factor of 8, leaving the factor 7 in the divisor, and 4 in the dividend. Two factors, 31 and 4, remain in the dividend making a product of 124. Dividing this product by 7, the remaining factor in the divisor, gives as a quotient $17\frac{5}{7}$.

ORAL TEST EXERCISE

1. Reduce the following divisions to simpler forms by applying (98) Prin. V.

$96 \div 64$; $120 \div 36$; $91 \div 39$; $68 \div 51$; $65 \div 25$; $110 \div 35$; $190 \div 57$; $108 \div 48$.

2. Reduce to lower terms each of the following ratio pairs:

85 : 34; 220 : 66; 114 : 84; 260 : 91; 270 : 63; 180 : 99; 480 : 72; 570 : 76.

3. Name 3 multiples of 19.

4. Name all the exact divisors of 96.

5. Name all the common divisors of 36, 54, and 90.

6. What is the smallest number that may be exactly divided by both 16 and 12? By 7, 6, and 3? By 4, 5, and 6? By 24, 18, and 36?

7. What number has for its factors the cube of 2 and the square of 3? The cube of 3 and the square of 2? The square of 2 and the square of 5? The cube of 2 and the square of 5?

8. Name three numbers that have 18 for their greatest common divisor.

9. Name four numbers whose least common multiple is 60.

WRITTEN EXERCISE

16. What is the quotient of $24 \times 36 \times 6 \times 7 \times 5$ divided by $18 \times 4 \times 8 \times 14$?

17. Divide $72 \times 210 \times 95 \times 60 \times 42$ by $21 \times 19 \times 12 \times 10 \times 15 \times 7$.

18. Divide the product of $84 \times 1728 \times 48 \times 75$ by the product of $144 \times 21 \times 15 \times 36$.

19. $(16 \times 15 \times 17 \times 36) \div (8 \times 5 \times 24 \times 6) = ?$

Solve the following, shortening the operations by cancellation:

20. A farmer brought to market 15 turkeys averaging 12 pounds each, which he sold at 9 cents per pound. He took his pay in sugar at 4 cents per pound. How many pounds did he get?

21. A man sold 32 bushels of oats at 33 cents per bushel and took his pay in beef at 11 cents per pound. How many pounds did he receive?

22. How many bushels of potatoes, at 75 cents per bushel, can be had for 24 sacks of corn, each containing 2 bushels, at 50 cents per bushel?

23. How many pieces of cloth, each piece containing 48 yards, at 10 cents per yard, should be received for 2 loads of oats of 60 bushels each at 25 cents a bushel?

24. A farmer brought to market 5 loads of rye, averaging 16 bags of 2 bushels each. The merchant paid for the rye with 16

sacks of sugar of 100 pounds each at 5 cents per pound. What price per bushel was allowed for the rye?

PROBLEMS AND THEIR ANALYSIS

128. A *problem* in arithmetic is any question requiring for its answer the application of arithmetical principles and processes.

129. A *solution* is an orderly statement of the arithmetical processes employed in solving a problem.

130. An *operation* is an exhibit of the computations required by a solution.

131. An *analysis* is a logical explanation of the steps taken in the solution of a problem.

132. There is much confusion in the use of the foregoing terms, not only by teachers and students, but by authors of text books. The following illustration will help to make clear the meaning of these terms.

PROBLEM

If a farm of 75 acres is valued at \$1875, what would be the valuation of 53 acres?

SOLUTION

$\$1875 \div 75 = \$25.$
$\$25 \times 53 = \$1325.$

OPERATION

```
75)1875(25        25
   150            53
   ---            --
   375            75
   375          125
   ---          ----
                1325
```

ANALYSIS

One acre is valued at $\frac{1}{75}$ of \$1875, or \$25; and 53 acres should be valued at 53 times \$25, or \$1325.

133. In solving problems, the solution is ordinarily considered sufficient, but the student should be prepared to submit the analysis or operation if called upon. The analysis of a problem should consist of accurate, concise statements free from needless repetition. The operation should be neat and the figures uniform and legible. There should be none of the undue haste that results in careless or slovenly work.

WRITTEN EXERCISE

Write solutions of the following problems, and be prepared to give a clear analysis of each. Keep the work or operation for

inspection by the teacher. The analysis may be either oral or written as the teacher may require.

25. A mill owner sold 250,000 shingles at $3.00 a thousand and with the money bought 15 acres of land. What did the land cost an acre?

26. A man paid $85 for a cement walk 5 feet wide and 153 feet long. If there are 9 square feet in a square yard, what did the walk cost per square yard?

27. An orchard contains 25 rows of trees, with 40 trees in each row. The apple crop from the orchard was sold for $4500. What was the value of the yield per tree?

28. A real estate dealer sold a tract of 278 acres of land for $12510, receiving $13 per acre more than it cost him. What did he pay per acre for the tract?

29. A farmer sold his wheat crop at 85¢ per bushel, and with the money bought 124 stock hogs at $4.25 each. How many bushels of wheat did he sell?

30. A locomotive runs at the rate of 4 rods per second. If there are 320 rods in a mile, how many miles and rods will the locomotive run in an hour (3600 seconds)?

31. A man bought 16480 pounds of wool @ 19¢, and sold it @ 26¢. He then invested the gains of this speculation in pork at 8¢ a pound, which he afterwards sold at 11¢ a pound. What did he gain on the pork?

32. Walter's expense account while attending Marion College for five months was as follows: Board and room, $75.50; tuition and books, $62.85; clothing and laundry, $41.15; incidentals, $7.95. What were his average monthly expenses?

33. The items of cost for an edition of 6000 books are as follows: Paper, $618; printing and binding, $1545; packing, $15. If the books are sold at 50¢ each, what is the gain on each book?

34. A speculator bought a piece of city property for $2650, borrowing money to pay for same. Before selling the property he paid for taxes, $53, repairs, $27, and for interest on the loan $159. He received for rent of the property $300, and at the end of the year sold the property for $3000. How much did he gain?

COMBINATIONS AND FORMULAS

134. It is often desirable to indicate two or more successive arithmetical processes by means of one equation. Such an equation

is called a *formula,* and the *order* of the several processes is indicated by means of the *vinculum* ————————, the *parenthesis* ( ), and sometimes by the *comma.*

135. The vinculum and the parenthesis are used to enclose one or more indicated operations, the results of which are considered as one quantity. Thus, $(12 + 8) \times 2$, or $\overline{12 + 8} \times 2$, means that 12 and 8 are to be added and the sum multiplied by 2, the result being 40. But $12 + (8 \times 2)$, or $12 + \overline{8 \times 2}$, means that the product of 8×2 is to be added to 12, the result being 28.

136. Both the vinculum and the parenthesis may be used in an arithmetical formula. Thus, $27 - \overline{24 \div (12 - 8)}$ indicates that 24 is to be divided by the quantity $12 - 8$, and the quotient subtracted from 27.

ORAL EXERCISE

Verify the following:

1. $(17 - 8) \div 3 = 3.$
2. $\overline{51 \div 17} + 9, - (2 \times 4) = 4.$
3. $34 \div (15 + \overline{(3 \times 2) \div 3}) = 2.$
4. $\dfrac{(75 \div 15) + 9}{2} = 7.$

Find the result of the following:

5. $54 \div (12 + \overline{78 \div 13}) + 15 - (8 - 4) = ?$
6. $\dfrac{(25 - 6) \times (52 \div 13)}{(3 \times 13) - 37} = ?$
7. $(7 + 8 + 9, \div 6) \times (56 \div \overline{19 - 5}) \times 6 = ?$
8. $\dfrac{48}{17 + 7} \times \overline{(52 \div 2,} \ - (4 + 8, \times 2) \ ) = ?$
9. $((3 \times 12) - 24) \times (76 \div 19) + 15 = ?$

137. It is sometimes desirable to formulate a problem instead of writing the complete solution, or analysis. It may be assumed that a student can solve any problem for the solution of which he is able to write the correct formula. The method of formulation is illustrated by the accompanying problem and its formula.

WRITTEN PROBLEMS

Mr. A. sold some cattle for \$950 and some sheep for \$375. With a part of the money he bought 3 cows at \$35 each and with the remainder bought 8 horses. What was the average cost of the horses?

$\overline{(\$950 + \$375) - (3 \times \$35)} \div 8 =$ Average cost of horses.

FORMULA FOR SOLUTION

Formulate each of the following problems and find the result of each formula.

35. A man bought a house and lot for $1500, expended $250 for repairs, and then sold the property for $2000, investing his gains in wheat at $1 per bushel. How many bushels did he buy?

36. A certain tank holds 40,000 gallons. It has a supply pipe that brings in 150 gallons per minute, and a discharge pipe that carries away 175 gallons per minute. If the tank is full, and both pipes are left open, how many hours will be required to empty the tank?

37. A man bought a farm for $5000, 60 head of cattle at $25 each, and 15 horses at $75 each. He then sold the farm and stock for $8000. What sum did he gain?

38. If there are 160 square rods in an acre, what is the value at $25 an acre of a field 96 rods long and 70 rods wide?

39. Mr. Markham earns $3 per day and works an average of 5 days each week. His weekly expenses are as follows: Board, $4; clothing, $3, other expenses, $2. The remainder of his earnings he deposits in bank. What sum should he have to his credit at the end of one year, or 52 weeks?

40. Mr. L. sold 965 bushels of wheat at $1, receiving in payment 5 milch cows, a town lot valued at $125, 140 stock hogs worth $4 each, and cash $80. What was the average cost of the cows?

41. Ellerton is one of five heirs who inherits an estate consisting of 640 acres of land valued at $45 per acre. If Ellerton has other property valued at $2250, cash $500, and owes debts to the amount of $3000, what is his present worth after his debts are paid?

42. A residence property rents for $35 per month. The annual taxes average $75 and the repairs $50. The owner pays the water rate of $1 per month; what is the net income from the property during a period of five years?

43. A committee has a certain sum with which to purchase a carpet for a lodge room. They find that if they purchase carpeting at $2 per yard, they will lack $80 of having enough money to pay for it, but if they purchase a grade at $1 per yard, they will have $80 of their fund unexpended. How many yards are required?

44. Walter Bright bought a 30-acre wood lot at $26 per acre. He paid $275 for fencing the lot, sold from it 160 cords of wood at $2 per yard, and then sold the lot for $870. What did he gain by the speculation?

Prepare written solutions for the following problems:

45. Lillian finds that if she buys a certain number of yards of dress goods at $1.25 a yard, she will have $1.50 remaining. But if she buys the same number of yards of a quality that costs $1.44 a yard, she will lack 78 cents of having enough money to pay for it. How much money has she?

46. A buyer for a department store is allowed a certain sum with which to purchase bicycles. He bought 60 bicycles at $25 each. How many could he have purchased of a higher grade quoted at $30 each?

47. Benton & Sons sold 525 barrels of flour at $4 a barrel, and invested the proceeds in pickled pork at $20 a barrel. How many barrels of pork did they buy?

48. A contractor engages to build a house for $2875, provided he uses lumber costing $18 per thousand feet; but if lumber worth $20 per thousand is used, he will ask $2945. If the cheaper grade is selected, what will be the amount of the lumber bill?

49. On a field of wheat containing 75 acres, a farmer harvested 1397 bushels; on a second field of 64 acres, he harvested 1123 bushels; and on a third field of 57 acres, he harvested 1008 bushels. What was the average yield per acre for the whole crop?

50. Five men bought a paper mill for $47,500, investing equally. They operated the mill for one year at a total outlay of $48,559. During the time they sold paper to the amount of $69,744. They then sold the mill for $50,000. What was the amount of each man's share of the gain?

51. A cyclist rode 4 hours at the rate of 15 miles an hour. He then rode back over the same road in just 5 hours. At what average rate per hour did he ride on his return trip?

52. Mrs. B. finds that if she selects for a new gown material that costs $1.25 per yard, the whole cost of the garment will be $27.85; but if she selects material worth $1.48, the cost will be $30.61. What is the cost of the gown aside from the cloth?

53. By the terms of Mr. Beverly's will, his estate, which is valued at $375,000, is to be divided as follows: his wife is to receive $25,000; the eldest son, $20,000; each of the three remaining children, $10,000. After paying these and the other special bequests to the amount of $5,750, the remainder of the estate is to be divided equally among the five heirs. What is the value of each one's interest in the undivided portion of the estate?

CHAPTER VII

COMMERCIAL METHODS AND APPLICATIONS

BUYING AND SELLING

138. In commercial transactions, values are measured by means of money. Money in America consists of the various coins of copper, nickel, silver, or gold, also of government and bank notes and certificates.

139. By the term **coin** is meant the *metallic* money manufactured by the Government.

140. **Paper money** consists of the various promises to pay money, which are issued by the government or by banks, and which are allowed by law to circulate as money.

141. **Currency** is a term including both coin and paper money.

142. **Cash** is a term which includes not only money, but checks, drafts, and other commercial paper that is convertible into money on the demand of the holder.

143. **Cost** or **price** is the amount paid, or promised to be paid, for an article. In business, the term price usually implies the quoted market value of one unit of a commodity, as, "the price of wheat is 90 cents a bushel," while *cost* refers to the *whole amount* that is paid for an article or commodity, as, "the property cost $5000," or "the goods, including freight, cost $75.15."

144. A **sale** is a transfer for value of ownership in property from the seller to the buyer.

145. **Gains** or **profits** are the increase that comes from selling goods for an amount greater than their cost.

146. **Losses** are the diminishment of capital that results from selling goods for a less sum than their cost.

147. **Gross gain** is the excess of sales over cost.

148. **Net gain** is what remains after subtracting expenses from the gross gain.

149. The **capital** or **investment** is the total value of the business.

150. The **net capital** or **net investment** is the total capital or investment, less the total amount of debts for which the business is liable.

ALIQUOT PARTS

151. An **aliquot part** of a number is any quantity that is contained in it a whole number of times.

EXAMPLES: 3, 4, and 6 are aliquot parts of 12, while $7\frac{1}{2}$ and $3\frac{3}{4}$ are aliquot parts of 15. The aliquot parts of 100 are $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, and 50.

152. The term, **factor** or **divisor**, applies only to *whole* numbers; an aliquot part of a number, may be either a whole or a fractional number. Thus, $2\frac{1}{2}$ is an aliquot part of 5, $7\frac{1}{2}$, 10, $12\frac{1}{2}$, 15, etc., because it is contained in each of these numbers a *whole number* of times. But $2\frac{1}{2}$ is not an aliquot part of 6, or 8, or 9, since it is not contained in these numbers a whole number of times. When any number is divided by an integer or whole number, the result or quotient is an aliquot part of the dividend.

Through the use of aliquot parts, many arithmetical operations may be greatly simplified. Suppose for instance, we want to know the value of 168 yards of cloth at $12\frac{1}{2}¢$ per yard. Instead of performing the multiplication we may reason in this way: at a dollar a yard the cloth would cost \$168. Since $12\frac{1}{2}$ is $\frac{1}{8}$ of a dollar, the cost will be $\frac{1}{8}$ of \$168 or \$21. Again, suppose we want to know how many pounds of wool at 25¢ per lb. can be bought for \$147. Here we may reason: since 25¢ is $\frac{1}{4}$ of a dollar, we can buy 4 lb. for each dollar expended; hence, we can buy 147×4 lb. or 588 lb.

The student should commit the following table thoroughly:

ALIQUOT PARTS OF \$1

$\frac{1}{2}$ of \$1 = 50¢
$\frac{1}{3}$ of \$1 = $33\frac{1}{3}$¢
$\frac{1}{4}$ of \$1 = 25¢
$\frac{1}{5}$ of \$1 = 20¢
$\frac{1}{6}$ of \$1 = $16\frac{2}{3}$¢
$\frac{1}{8}$ of \$1 = $12\frac{1}{2}$¢
$\frac{1}{12}$ of \$1 = $8\frac{1}{3}$¢
$\frac{1}{16}$ of \$1 = $6\frac{1}{4}$¢
$\frac{1}{20}$ of \$1 = 5¢
$\frac{1}{25}$ of \$1 = 4¢
$\frac{1}{40}$ of \$1 = $2\frac{1}{2}$¢

ORAL EXERCISE

1. Find the cost of 128 yd. of cloth @ 50¢.

NOTE.—The "price sign" @ means "at," "each," or "apiece."

2. How can you find the cost when the number of articles or quantity is given and the price is 25¢? when the price is $12\frac{1}{2}$¢? $6\frac{1}{4}$¢?

3. At $12\frac{1}{2}$¢ per lb., how many pounds of meat can be bought for $12? for $7? for $30.

4. When the total cost is given, how can you find the quantity when the individual cost is 20¢? 25¢? $8\frac{1}{3}$¢? $33\frac{1}{3}$¢? $12\frac{1}{2}$¢?

5. Find the cost of 25 yd. of cloth @ $1.40.

6. What is the cost of $12\frac{1}{2}$ lb. @ 24¢? $6\frac{1}{4}$ yd. @ 48¢? $12\frac{1}{2}$ gal. @ 56¢? 25 doz. @ 28¢?

Find the cost of

7. 360 yd. cloth at $12\frac{1}{2}$¢; 25¢; 20¢; 50¢.

8. 800 lb. at $\frac{1}{2}$¢; 5¢; 4¢; $12\frac{1}{2}$¢; 25¢.

9. 144 gal. at 50¢; 25¢; $12\frac{1}{2}$¢.

10. $12\frac{1}{2}$ yd. at 96¢; 48¢; 56¢.

11. 25 bu. at 28¢; 40¢; 64¢; 76¢.

12. At 3 lb. for a dollar how much coffee may be had for $1.50? for $7? for $5.50? for 25¢? for $2.25?

13. If 48 yd. cost $15, what is the cost of 16 yd.? 24 yd.? 12 yd.?

14. If 12 lb. cost $5, what is the cost of 36 lb.? 60 lb.? 96 lb.?

15. At $12.50 per ton, how much coal can be bought for $600?

SUGGESTION.—$12.50 is $\frac{1}{8}$ of $100.

16. At $25 a ton, how many tons of steel rails can be bought for $1000? for $5000? for $10000? for $100000?

17. At $2\frac{1}{2}$¢ per lb., what is the cost of 48 lb. of plaster?

SUGGESTION.—The cost is $\frac{1}{4}$ the cost at 10¢ per lb.

18. At $2.50 per bu., what is the cost of 36 bu. of flax seed? $12\frac{1}{2}$ bu.? 25 bu.? 64 bu.?

19. What short method can you suggest for multiplying any number by 25? by $12\frac{1}{2}$? by $33\frac{1}{3}$?

20. Announce the result of the following:

24×25	25×68	96×25	52×25
$96 \times 12\frac{1}{2}$	$48 \times 12\frac{1}{2}$	$12\frac{1}{2} \times 16$	$12\frac{1}{2} \times 56$
$33\frac{1}{3} \times 51$	$33\frac{1}{3} \times 75$	$54 \times 33\frac{1}{3}$	$57 \times 33\frac{1}{3}$

21. How many times is 25 contained in 300? in 500? in 900?

22. How many times is $12\frac{1}{2}$ contained in 200? in 400? in 700?

23. Give a short rule for dividing any number by 25.

24. Divide each of the following by 25: 600, 150, 350, 800, 900.

25. State a short method of dividing any number by $12\frac{1}{2}$, by $33\frac{1}{3}$, by 50.

26. At 25¢ per pound, how many pounds can be bought for \$3? for \$4.50? for \$17? for \$19.75? for \$21.25? for \$16.75?

27. \$5 will pay for how many yards at $6\frac{1}{4}$¢? at $12\frac{1}{2}$¢? at 25¢? at 50¢?

28. If a certain grade of tea sells at the rate of 3 lb. for \$1, how much can be bought for \$1.50? for \$2.50? for \$1.25? for \$5.75?

29. Multiply \$2.40 by 50; by $33\frac{1}{3}$; by 25; by $12\frac{1}{2}$.

30. Divide \$19 by 25; by $33\frac{1}{3}$; by 50; by $12\frac{1}{2}$.

31. Find the value of 250 acres at \$48 per acre.

SUGGESTION.—250 acres is $\frac{1}{4}$ of 1000 acres.

32. What is the cost of 125 tons of steel at \$24 per ton? at \$32? at \$40?

33. What is the cost of 125 lb. coal at \$4.80 per ton?

SUGGESTION.—1 ton = 2000 lb. The cost of 1000 lb. is $\frac{1}{2}$ the cost of a ton.

34. At \$12 a ton, what is the cost of 250 lb. of hay? 125 lb.? 500 lb.? 75 lb.? 900 lb?

35. What is the cost of 19 lb. pork at 15¢?

SUGGESTION.—Find the cost of 20 lb., then subtract 15c.

36. What is the cost of 29 yds. of cloth at 18¢? at 25¢? at 30¢?

37. What is the value of 50 tons of coal at \$4.50? 51 tons? 49 tons? 52 tons? 48 tons?

38. What is the cost of 60 bu. potatoes at 25¢? 63 bu.? 57 bu.? 62 bu.?

39. Find the value of 48 yd. cloth at \$1.75.

SUGGESTION.—\$1.75 = $\frac{7}{4}$ of a dollar.

40. What is the cost of 16 bx. soap at \$3.75? 24 bx.? 12 bx.?

41. What is the cost of a 75 lb. mat of coffee at the rate of 3 lb. for \$1? at 4 lb. for \$1? at 5 lb.? at 6 lb.?

42. What is the cost of 76 pr. shoes at $2.50?

SUGGESTION.—First find the cost at $10.

43. What is the cost of 2 doz. pr. skates at $1.25 per pair? 16 pr.? 48 pr.? 56 pr.?

44. What is the cost of 49 books at $1.50? 25 books? 98 books?

BILLS AND EXTENSIONS

153. A **bill** or **invoice** is a list of property sold, with prices and total cost. It is made out by the seller and delivered to the buyer.

Following is a form of a bill showing items, prices, extensions, and total cost:

Minneapolis, Aug. 15, 1920.

Mr. George Madison
Rochester, Minn.

BOUGHT OF

Franklin Macintosh & Company

Shipped Via

TERMS: Cash

40	bx. D. Peaches	5.40	216	
20	" A.A. Prunes	4.25	85	
30	cs. #1 Raisins	2.70	81	
80	bx. Evap. Apples	4.60	368	750

154. Fractions of a cent. Market prices are often expressed in cents and fractions of a cent, but in accounting and in business transactions generally, cost amounts are expressed in dollars and cents only. If, in any computation, the exact amount includes a fraction or decimal that is *less than one-half cent,* the fraction is disregarded, or "dropped." If the fraction is equal to or greater than one-half, it is counted as a cent. For example, $1.75\frac{1}{4}$, or $1.75\frac{1}{3}$, or $1.75\frac{3}{8}$, or 1.754, would each be entered, $1.75. But $1.75\frac{3}{4}$, or $1.75\frac{5}{8}$, or $1.75\frac{1}{2}$, or 1.756, would be entered $1.76. Fractions of a cent other than halves, fourths, and eighths, are rarely used in market prices.

155. Extensions are the cost amounts of the several items of a bill which are entered in, or "extended," to the first money column. The work of preparing bills and computing and entering the required extensions and totals, comprises much the most important

application of commercial arithmetic. Every competent salesman must be able to do this work rapidly and accurately and to attain this expertness he must be familiar with and able to apply, *instantly*, the various devices for saving time and effort in making his computations. Extensions should be computed mentally whenever possible, and advantage should be taken of every short method that is applicable.

WRITTEN EXERCISE

The following forms are intended to afford the student practice in preparing bills and in making extensions. The items should first be copied neatly on properly ruled paper, then the correct extensions and totals should be entered. Most of the extensions may be computed mentally if the student has made himself familiar with the preceding exercises.

1.

Your City, Sept. 1, 19—

R. E. GRANT,
City

Bought of JONES & SPENCER

Terms: *Cash*

320	lb. Gran. Sugar	.05				
124	lb. Java Coffee	.25				
40	gal. Syrup	.75				
50	lb. E. B. Tea	.65				
84	lb. Starch	.06½				
96	lb. Soap	.02½				
	Received Payment JONES & SPENCER per Student					

2.

Your City, Sept. 2, 19—

M. I. MASON,
232 No. First Street

Bought of THE STUDENT

Terms: *on Account*

280	lb. Dried Peaches	.22				
48	cans Cocoa	.25				
140	lb. N. Y. Cheese	.16				
32	lb. Y. H. Tea	.37½				
5	brl. Flour	4.20				
120	lb. Rice	.07½				
130	lb. Butter	.17				

3.

Your City, Sept. 3, 19—

THOMAS J. HARTMAN,
City

Bought of THE CARTER LUMBER COMPANY

Terms: *30 days*

3400	ft. No. 1 Flooring,	per M.	36.00	122	40	
48	pcs. 2x12—12, pine, 1152 ft.	" "	27.00			
8	" 2x12—14, Pine, 224 ft.	" "	27.00			
2300	ft. No. 1 Sheathing	" "	27.00			
900	ft. No. 1 Y. P. Ceiling	" "	32.00			
1500	ft. Drop Siding	" "	33.00			
8	M. Shingles	" "	3.50			
5	pcs. 1½x6—12 Cypress, 45 ft.	" "	50.00			

4.

Your City, Sept. 4, 19—

LANNING PRINTING COMPANY,
Blairstown, Your State

Bought of THE WESTERN PAPER CO.

Terms: *on Account*

26	Rm. 16x21—20 St. Charles Writing, 520 lb.	.08½			
25	Rm. 24x36—50 Envelope Manila, 1250 lb.	.04½			
18	Rm. 22x34—36 Aurora Writing, 648 lb.	.06			
12	Rm. 24x36—70 Enamel Printing, 840 lb.	.07½			
24	Rm. 24x36—40 R. R. Manila, 960 lb.	.05¼			
5 1/100	Rm. 40x48—200 Pure Fibre, 1040 lb.	.06			
25	M. No. 2 Drug Envelopes, per M.	.42			

5.

Chicago, Sept. 5, 19—

THOMAS & TRENTON,
Galesburg, Ill.

Bought of J. M. SMYTHE & CO.
Wholesalers in Furniture and Office Supplies

Shipped by *C. & N. W. Ry.*
Terms: *60 days*

2	doz. Kitchen Tables, 24	each	$ 4.20			
⅔	" Common Lounges, 9	each	9.00			
½	" Oak Extension Tables, 6	each	14.00			
4	" Dining Chairs	per doz.	18.00			
½	" Brass Bedsteads, 6	each	22.50			
1	" Antique Oak Bed Room Sets, 12	each	27.50			

6.

E. R. Reynolds, St. Louis, Sept. 8, 19—
Blue Rapids, Kans.
Bought of Browne & Browne
Shipped per *Mo. Pac. R. R.*
Terms: *90 days*

24	pr. Congress Shoes	$1.75				
14	" Men's heavy Shoes	1.60				
36	" Ladies' French Kid Shoes	2.40				
12	" Boys' School Shoes	1.80				
24	" Infants' Shoes	.90				
18	" Misses' Gondola Button Shoes	1.75				

7.

S. B. *152* Folio *133* Chicago, Sept. 19, 19—
T. E. Tripp,
Moline, Ill.
Bought of Marshall Field Co.
Terms: *90 days*
Shipped per *C., R. I. & P.*

Case			No. Yds.	Price	Extension		Total	
116	10	pcs. Delaines 29^2 28 27^2 31 32^3 28^2 30 33^1 29^3 28^2	298^3	.16				
	21	pcs. American Prints 36^1 42^2 43 41^1 39^3 38 39^2 44 41^1 39 39^1 41^2 38^3 42 43^2 40 41^1 38 39 39^2 41^3		.07½				

The small figures are quarters of a yard. The total numbers of yards of a given variety should be found by horizontal addition.

8.

Union Fuel Co., Pittsburg, Sept. 10, 19—
College City, Your State.
Bought of Pennsylvania Coal Co.
Terms: *90 days*

Car	Net Wt.		Tons	Price	Extension		Total	
Pa. 31343	28750	Egg Coal	14^{750}	4.25	61	09		
L. V. 47263	38125	Nut Coal		4.50				
D. L. & W. 36195	34220	Grate Coal		4.75				
Pa. 32241	27275	Cannel Coal		7.50				

Enter up the following sales in the form of bills using current datings, and supply addresses of seller and buyer:

9. Bert Clemens bought of F. S. McDaniel for cash, 25 lb. C. sugar @ 5¾¢; ½ bu. pears @ $2.40; 20 lb. butter @ 21¢; 2 sk. P. P. flour @ $1.75; 8 lb. Japan tea @ 75¢; 1 doz. eggs @ 18¢; 5 baskets quinces @ 70¢; 14 lb. graham flour @ 4¢; 5 lb. cheese @ 18¢.

10. A. A. Brewer bought of H. Hulman on acct. 30 days, 10 brl. kerosene, 440 gallons, @ 11¢; 6 brl. M. syrup, 43 gallons each, @ 40¢; 8 sk. rice, 800 lb., @ 7½¢; 4 boxes ground coffee, each 50 lb., @ 28¢; 10 sk. Rio coffee, 1000 lb., @ 15¢; 4 ch. Oolong tea, each 60 lb., @ 45¢.

11. J. G. Weidner bought of J. V. Farwell, on account 60 days,

Case No. 120, 24 pieces Challie, @ 75¢:

46^1, 43^1, 41^1, 40^2, 43^2, 39^3 38^1, 37^1, 42, 43^2, 40^1, 38, 41, 47^1, 43, 39, 43^1, 42, 41^1, 41^3, 42^3, 42^1, 44, 43^3.

Case No. 63, 24 pieces Henrietta cloth, @ 62½¢:

34^2, 30^1, 33^2, 34^1, 36^3, 34, 32^3, 34, 36^2, 33^3, 31, 30^2, 31^3, 34^1, 33^2, 37, 31^1, 31^3, 32^3, 30^1, 32, 31^2, 33^1, 30.

Case No. 143, 20 pieces serge, @ 37½¢:

32^1, 31^2, 33, 21^3, 29, 28^1, 26^3, 31^2, 32^1, 30, 29^1, 27, 32^1, 34^3, 32^1, 33, 31^2, 32^1, 31^2, 33^3.

12. J. G. Foster bought of Ryder & Stuart, on account 90 days,

1 doz. cherry bedroom sets	(each)	$30.00
½ doz. bird's eye maple bedroom sets	(each)	75.00
7 office desks, oak	(each)	16.00
¾ doz. rockers, upholstered	(per doz.)	32.00
2 doz. fancy baby cabs	(each)	12.00

13. Sharp & Strong bought of the Carter Lumber Co., on note at 90 days, 100 pcs. hemlock 2x6—16 @ $26 per M; 120 pcs. pine 4x4—12 @ $32 per M; 80 pcs. 2x6—16 @ $28 per M; 60 pcs. hemlock 2x8—16 @ $30 per M; 24 pcs. pine 8x8—16 @ $34 per M; 4000 ft. No. 1 Y. P. flooring @ $37.50 per M; 24 M lath @ $2.50 per M.

14. Parsons Livery Co. bought of the Acme Feed Co. on account, 12600 lb. timothy hay @ $9 per T.; 14780 lb. clover hay @ $11.50 per T.; 8½ T. prairie hay, baled, @ $8.50 per T.; 2 loads oats, net weight 3740 and 4260 lb. (32 lb. per bu.) @ 37½¢;

2 loads ear corn, net weight 4280 and 4120 lb. (70 lb. per bu.) @ 45¢.

15. Harvey & Son bought of the Union Shoe Company, on account, 28 pr. misses, Gondola button shoes @ $2.29; 36 pr. ladies' kid slippers @ $1.50; 25 pr. calf boots @ $3.20; 18 pr. childs' shoes @ $1.20; 32 pr. men's arctics @ $1.62½; 24 pr. congress shoes @ $1.50.

16. Canfield & Co. bought of the American Carpet Co., on account 90 days, 60 yd. body brussels @ $1.37½; 60 yd. wilton @ $2.25; 125 yd. tapestry brussels @ 90¢; 120 yd. ingrain @ 87½¢; 140 yd. moquette @ $1.60; 80 yd. oil cloth @ 62½¢.

17. N. R. Parvin bought of the Haskell Coal Co., on account, 1 load grate coal (6220-1920) @ $7.25; 2 loads egg coal (5870-1950 and 6125-1975) @ $7.50; 1 load lump coal (5250-1950) @ $5.25; 1 load nut coal (5780-1980) @ $7.75.

18. Trenton & Lee bought of J. V. Farwell & Co., on account 90 days,

8 pcs. cambric @ 22¢:

41^1, 42, 40^3, 43^2, 44, 38^3, 41^2, 44^1.

10 pcs. gingham @ 12½¢:

40, 41^1, 40^3, 42, 42^2, 40^1, 46^3, 39^2, 38, 41^1.

9 pcs. colony flannel @ 37½¢:

46, 45^1, 40^2, 41^1, 43, 40, 38^3, 39^1, 40.

19. Central Transfer Co. bought of the Sherman Feed Co., on account, 21400 lb. red top hay, baled, @ $12.60 per T.; 15260 lb. upland hay @ $9.50 per T.; 2 loads oats, net weight 3125 and 2980 lb. (32 lb. per bu.) @ 38¢; 3 loads ear corn, net weight 4120, 4260 and 4430 lb. (70 lb. per bu.) @ 43¢.

20. Star Printing Co. bought of the Baker Paper Company, on account, 12 reams 17x22—24 lb. London Mills writing @ 9¢; 24 reams 22x32—90 lb. book @ 4½¢; 12 reams 17x28—32 lb. linen ledger @ 16¢; 12 reams 24x36—32 lb. enamel book @ 6½¢.

ORAL TEST PROBLEMS

The student should try to give a clear, concise analysis of each problem, taking advantage of every device for simplifying the solutions.

1. A bolt of silk containing 29 yards was sold at auction for $16.25. At the same rate, what would be the cost of an 87-yard bolt?

SUGGESTION.—29 is ⅓ of 87.

2. If it requires 3 weeks for 15 men to complete a certain contract, in what time could it be completed by 45 men?

3. Frank's yearly salary is $1500. He spends $\frac{1}{3}$ of it for board and $\frac{1}{4}$ of it for other expenses; what sum does he save?

4. A's property investment on beginning business was $6000 and his debts amounted to $1500. At the end of the year his property was valued at $7500 and his debts amounted to $2000. Required his gain for the year.

5. By buying a lot of books at 65¢ each and selling them at 80¢ each, a stationer gained $4.50; how many books in the lot?

6. A grocer gains 60¢ by selling a case of 12 dozen eggs at 25¢ a dozen; what did the case of eggs cost?

7. Two men are 37 miles apart and approach each other, one at the rate of 3 miles an hour, the other, 4 miles an hour; how far apart will they be at the end of 4 hours?

8. If 7 men do a piece of work in 13 days, how long will it take 1 man to do it? 2 men? 13 men?

9. If the paper in a book costs 28¢, the binding 12¢, and the printing 4¢, what profit will be gained by selling the books for 60¢?

10. If 3 oranges are worth as much as 4 lemons, how many lemons should be given for 51 oranges?

11. Bought 19 acres of land at $45 per acre, and sold it at $49 per acre; what was the gain?

12. A square floor contains 81 square feet; what is the length of the four sides?

13. For how much must I sell 16 yd. cloth which cost $1.50 a yard, in order to gain $16?

14. By selling a bolt of cloth at 15¢ a yd. I shall lose 60¢, but by selling it at 18¢ a yd. I shall gain 30¢. What did the cloth cost, and how many yards are in the bolt?

15. What is the value of 98 lb. butter at 18¢ per lb.?

16. Henry worked as many days as he received cents per day and earned $16; how many days did he work?

17. A farmer owes a grocer $15.75. What is the state of the account after the farmer sells the grocer 40 doz. eggs @ $12\frac{1}{2}$¢ and 50 lb. butter at 26¢?

18. What is gained by buying 1600 lb. wool at 20¢ and selling it at $22\frac{1}{2}$¢?

19. What is gained by buying 60 lb. coffee at the rate of 4 lb. for $1 and selling it at 3 lb. for $1?

20. A farm hand receives \$1.50 a day and his board for every day that he works, but is to pay his employer 50¢ a day for his board for every day that he is idle. At the end of 12 days he received \$12 in full of account; how many days did he work?

WRITTEN TEST PROBLEMS

21. Mr. H. harvested 1728 bu. wheat. What is the crop worth at \$1.12½ per bu.?

22. Corn weighs 56 lb. to the bushel; what is the value of 13314 lb. at 40¢ a bushel?

SUGGESTION.— $\frac{13314 \times .40}{56} =$ value.

23. What is gained by buying 96 shares of mining stock at \$62.50 a share and selling it at \$75 a share?

24. If the use, or interest, of a dollar for 1 yr. is 6¢, what should be the interest charged for the use of \$175 for 4 months?

25. In a certain transaction a firm gained \$256.25; which was ⅛ of the investment; what was the investment?

26. A business concern has the following resources: Stock on hand valued at \$7562.15; real estate, \$12500; cash in bank \$1546.75; notes and other debts owed to the firm, \$956.70. Their liabilities (that is, the debts owed to others) amount to \$15564.25. What is the net capital invested?

27. If the interest on a certain sum for one year (360 interest days) amounts to \$25.92, what should be paid for the use of half the sum for 40 days?

28. At \$8.50 per doz., what would be paid for 17 pairs of gloves?

SUGGESTION.—17 *pairs* of gloves will cost $\frac{1}{12}$ of what 17 *dozen* cost.

29. Wheat weighs 60 lb. to the bushel. Find the value of 1256 lb. at 96¢ per bu.

30. Find the value of 3827 lb. corn (56 lb. to the bu.) at 48¢ per bu.

31. A grocer bought 3 mats of Java coffee, weight 67, 64, and 69 lb. at 18¢. After roasting, the coffee is retailed at 3 lb. for \$1. What is the gain on the lot, if the coffee loses ¼ of its weight in roasting?

32. At 6¼¢ per pound, how many pounds of sugar can be bought for 18¾ doz. eggs at 28¢?

SUGGESTION.—6¼¢ is $\frac{1}{16}$ of a dollar.

33. If the freight rate on canned goods from Chicago to Springville is 38¢ per 100 lb., what will be the charge on a shipment of tinned tomatoes weighing 978 lb.? (See 154.)

34. An oil company was fined $5000 for receiving freight rebates of 11½¢ a barrel on 75000 barrels of oil. How much did the rebate exceed the fine?

35. At 12¢ a pound, what is the value of 5 cheeses, wt. 118½, 116¼, 120¾, 119¼, 121½ lb.?

36. On Jan. 1, a real estate dealer bought a three-flat building for $12500, borrowing the money on his note. During the year he rented the first flat for 8 mo. at $27.50 per mo.; the second for 12 mo. at $25, and the third for 10 mo. at $22.50. He paid out for repairs, $68.50; for water supply, $27.25; and for taxes, $63.80. At the end of the year he sold the property for $13150, paying the note with accrued interest amounting to $625. What was his net gain on the transaction?

37. Walter worked at 30¢ an hour from 8:45 A. M. till noon, and from 1:15 P. M. till 5:30 P. M. How much does his employer owe him?

38. A farmer sold to a liveryman three loads of hay, weight 1849, 1856, and 1795 lb. at $16 per ton, and invested the money in fertilizer at $1.10 a hundred. How much fertilizer did he buy?

39. A seamstress receives $1.50 per doz. for finishing vests. At the end of the week she has finished 104 garments; what sum is due her?

40. Through an error in making their estimate, a publishing firm lost $420 by selling an edition of 6000 books at 48¢ each. For what price should they have been sold to gain 10¢ on each book?

CHAPTER VIII

COMMON FRACTIONS

156. Fractional numbers have already been defined and illustrated (14). We have also learned (95) that a fraction is the expression of a division in which the numerator of the fraction is the dividend and the denominator is the divisor. We are now to consider some further facts and processes relating to fractions.

157. The **unit** or **base,** of a fraction, is the thing or quantity to which the fraction applies. In the expression, "$\frac{3}{4}$ of an appple," the *unit* of the fraction is the apple. In the expression, "$\frac{1}{5}$ of \$20," the unit of the fraction is \$20. In the case of unapplied fractions, as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{12}$, etc., the unit, or base, is the unit 1.

Name the unit, or base, in each of the following: \$7; $\frac{2}{3}$ of an acre; $\frac{4}{5}$ of Mr. A's income; $\frac{3}{4}$ of 60¢; $\frac{9}{10}$; $\frac{1}{5}$¢; $\frac{2}{3}$ ft.

158. A **fractional unit** is one of the equal parts into which the unit, or base, of the fraction is divided. Thus, $\frac{1}{4}$ is the fractional unit of $\frac{3}{4}$, and $\frac{1}{12}$ is the fractional unit of $\frac{7}{12}$.

FRACTIONAL FORMS

159. An **improper fraction** is one whose numerator is equal to or greater than its denominator, and whose value is therefore equal to or greater than 1.

EXAMPLES: $\frac{6}{5}$, $\frac{7}{7}$, $\frac{13}{9}$, $\frac{4}{4}$, $\frac{19}{5}$, $\frac{24}{2}$.

160. A **mixed number** is an integer and a fraction united in one expression.

EXAMPLES: $3\frac{1}{5}$, $27\frac{4}{7}$, $1\frac{1}{8}$.

161. A **decimal fraction** is a fraction whose denominator is some power of 10.

EXAMPLES: $.7 = \frac{7}{10}$, $.03 = \frac{3}{100}$, $.013 = \frac{13}{1000}$.

PRINCIPLES

162. Since a fraction is the expression of a division in which the numerator is the dividend and the denominator the divisor, relations of numerator and denominator are governed by Principles

IV and V, (98). As applied to fractions, these principles may be stated as follows:

I. *A change in the numerator (dividend) produces a proportionate and like change in the value of the fraction.* That is, multiplying the numerator, multiplies the fraction, and dividing the numerator, divides the fraction.

II. *A change in the denominator (divisor) produces a proportionate and opposite change in the fraction.* That is, *multiplying* the denominator *divides* the fraction, and *dividing* the denominator, *multiplies* the fraction.

III. *If both numerator and denominator be either increased or decreased in the same proportion, the value of the fraction remains the same.* That is, if we either multiply or divide both terms of a fraction by the same number, the value of the fraction remains the same. This process is called *reduction.*

ORAL EXERCISE

1. (Prin. I.) Multiply the following fractions by 3:

$\frac{5}{29}, \frac{6}{19}, \frac{4}{25}, \frac{17}{100}, \frac{29}{90}, \frac{13}{85}, \frac{18}{97}.$

Divide the following by 5:

$\frac{35}{64}, \frac{85}{27}, \frac{55}{61}, \frac{95}{48}, \frac{70}{59}, \frac{35}{19}, \frac{75}{41}.$

2. (Prin. II.) Divide the following by 7:

$\frac{2}{5}, \frac{10}{13}, \frac{5}{6}, \frac{11}{14}, \frac{8}{9}, \frac{6}{7}, \frac{11}{12}, \frac{9}{11}.$

Multiply the following by 6:

$\frac{29}{54}, \frac{11}{12}, \frac{7}{18}, \frac{13}{24}, \frac{17}{48}, \frac{25}{36}, \frac{29}{60}.$

3. Illustrate two methods of multiplying each of the following by 4:

$\frac{7}{16}, \frac{5}{28}, \frac{13}{64}, \frac{12}{32}, \frac{15}{28}, \frac{19}{72}, \frac{17}{84}.$

4. Illustrate two methods of dividing each of the following by 6:

$\frac{24}{17}, \frac{30}{13}, \frac{18}{14}, \frac{12}{15}, \frac{48}{11}, \frac{54}{60}, \frac{36}{20}.$

5. Of the two methods used in the last two problems, which do you consider preferable? Why?

6. (Prin. III.) Reduce the following fractions to lower terms:

$\frac{12}{20}, \frac{15}{35}, \frac{65}{91}, \frac{34}{85}, \frac{84}{96}, \frac{75}{95}, \frac{57}{76}, \frac{72}{96}.$

7. (Prin. III.) Reduce the following to higher terms:

$\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{8}{17}, \frac{21}{32}, \frac{9}{19}.$

8. (Prin. III.) Change each of the following to lower and to higher terms:

$\frac{10}{12}, \frac{24}{30}, \frac{18}{20}, \frac{19}{38}, \frac{15}{35}, \frac{26}{52}, \frac{12}{18}, \frac{15}{24}.$

REDUCTION—PROBLEMS AND ANALYSIS

163. It is indispensable that the student of arithmetic be familiar with the analysis of problems involving fractions. Those who do not get a clear knowledge of fractions invariably fail when they come to percentage and other practical applications of numbers. The fundamental problems here presented should be completely mastered.

164. *Problem I. To reduce improper fractions to whole or mixed numbers.*

How many ones in $\frac{24}{6}$?

Analysis: In one there are six sixths; hence, in $\frac{24}{6}$ there are as many ones as 6 is contained times in 24, or 4.

How many ones in $\frac{45}{8}$?

Analysis: $\frac{1}{8}$ of the number of eighths equals the number of ones. $\frac{1}{8}$ of $45 = 5\frac{5}{8}$.

ORAL EXERCISE

Using either of the foregoing analyses, solve and analyze the following:

1. How many ones in $\frac{72}{12}$? in $\frac{52}{13}$? in $\frac{91}{7}$? in $\frac{54}{18}$?
2. Change to whole numbers, $\frac{85}{17}$, $\frac{95}{19}$, $\frac{78}{13}$, $\frac{75}{15}$, $\frac{65}{5}$, $\frac{87}{29}$.
3. Change to mixed numbers, $\frac{52}{25}$, $\frac{19}{4}$, $\frac{47}{13}$, $\frac{88}{16}$, $\frac{97}{13}$, $\frac{75}{12}$, $\frac{90}{16}$, $\frac{72}{17}$, $\frac{68}{12}$.

Note.—In the answers returned, the fraction in the mixed number should be in its lowest terms.

165. *Problem II. To reduce whole or mixed numbers to improper fractions.*

How many sixths in 9?

Analysis: In 1 there are six sixths, and in 9 there are nine times six sixths, or $\frac{54}{6}$.

How many fifths in $13\frac{4}{5}$?

Analysis: In 1 there are five fifths, and in $13\frac{4}{5}$ there are $13 \times \frac{5}{5}$, or $\frac{65}{5} + \frac{4}{5} = \frac{69}{5}$.

ORAL EXERCISE

Solve and analyze the following:

1. How many fourths in 17? in 19? in $3\frac{3}{4}$? in $5\frac{1}{4}$? in $13\frac{3}{4}$?
2. How many eighths in $11\frac{5}{8}$? in $5\frac{3}{8}$? in $12\frac{7}{8}$? in $9\frac{3}{8}$? in $10\frac{1}{8}$?
3. Reduce to improper fractions:
   $8\frac{1}{6}$, $5\frac{3}{13}$, $7\frac{8}{9}$, $2\frac{4}{19}$, $11\frac{7}{9}$, $15\frac{5}{6}$, $14\frac{3}{4}$.

166. *Problem III. To reduce a fraction to higher terms.*

Change $\frac{3}{4}$ to twelfths.

Analysis: In one there are 12 twelfths; in $\frac{1}{4}$ there are $\frac{1}{4}$ of 12 twelfths or $\frac{3}{12}$, and in $\frac{3}{4}$ there are 3 times $\frac{3}{12}$ or $\frac{9}{12}$.

2nd Analysis: A fraction is reduced to higher terms by multiplying both terms by the same number. (Prin. III.) To change 4ths to 12ths, both terms of the fraction must be multiplied by 3. Hence $\frac{3}{4} = \frac{9}{12}$.

Note.—In using this analysis, omit the reference to Prin. III; use the last part of the analysis only.

ORAL EXERCISE

Solve and analyze the following:

1. How many 24ths in $\frac{1}{2}$? in $\frac{2}{3}$? in $\frac{5}{6}$? in $\frac{7}{12}$? in $\frac{5}{8}$?

2. Change $\frac{5}{9}$ to 27ths, $\frac{4}{5}$ to 40ths, $\frac{7}{12}$ to 84ths, $\frac{9}{16}$ to 64ths, $\frac{11}{21}$ to 63rds, $\frac{12}{13}$ to 91sts.

3. Reduce each of the following fractions to a fraction whose denominator is 96:

$\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{5}{16}$, $\frac{13}{24}$, $\frac{17}{32}$, $\frac{29}{48}$.

4. Change each of the following fractions to seventy-seconds:

$\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{7}{9}$, $\frac{5}{12}$, $\frac{11}{18}$, $\frac{19}{24}$, $\frac{31}{36}$.

167. *Problem IV. To reduce fractions to equivalent fractions having the least common denominator.*

Fractions are said to have a "common denominator" when they are composed of the same fractional units (See 158). Thus, $\frac{3}{16}$, $\frac{7}{16}$, $\frac{9}{16}$ have the common denominator 16. A series of fractions are said to be "reduced to a least common denominator" when they are changed to other fractions having a common denominator, which is the *least common multiple* of the denominators of the fractions which are to be reduced. Thus, when the series, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, is changed to the series, $\frac{6}{12}$, $\frac{10}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, the common denominator, 12, is the l. c. m. of the denominators, 2, 6, 3, 4, and is the smallest denominator to which the fractional denominators of the first series can be reduced and preserve integral numerators.

Note.—It is, of course, possible to reduce a fraction to *any* given denominator, provided, the numerator is not required to be an integer. Thus, $\frac{3}{4} = \frac{7\frac{1}{2}}{10}$, or $\frac{4\frac{1}{2}}{6}$, but such operations are not required in practical arithmetic.

Change $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{8}$ and $\frac{5}{16}$ to fractions having the least common denominator.

Analysis: The least common denominator is the least common multiple of the several denominators. The l. c. m. of the

denominators, 3, 6, 12, 8, and 16, is 48. Reducing the several fractions to 48ths, proceeding as under Problem III, we have $\frac{32}{48}$, $\frac{40}{48}$, $\frac{28}{48}$, $\frac{18}{48}$ and $\frac{15}{48}$.

ORAL EXERCISE

Reduce to least common denominators:

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{9}$
2. $\frac{3}{8}$, $\frac{3}{4}$, $\frac{7}{12}$
3. $\frac{5}{7}$, $\frac{1}{4}$, $\frac{11}{14}$
4. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{9}$
5. $\frac{3}{11}$, $\frac{1}{4}$, $\frac{3}{22}$
6. $\frac{4}{5}$, $\frac{7}{15}$
7. $\frac{5}{6}$, $\frac{9}{14}$
8. $\frac{7}{16}$, $\frac{5}{12}$
9. $\frac{3}{4}$, $\frac{5}{13}$
10. $\frac{6}{11}$, $\frac{4}{7}$
11. $\frac{7}{15}$, $\frac{3}{25}$
12. $\frac{11}{18}$, $\frac{7}{24}$
13. $\frac{3}{4}$, $\frac{8}{19}$
14. $\frac{11}{32}$, $\frac{5}{24}$
15. $\frac{4}{25}$, $\frac{3}{20}$

ORAL TEST PROBLEMS

1. Express at sight in their lowest terms, the following:

$\frac{12}{16}$, $\frac{16}{24}$, $\frac{52}{65}$, $\frac{36}{48}$, $\frac{45}{60}$, $\frac{72}{96}$, $\frac{24}{50}$, $\frac{75}{100}$, $\frac{32}{48}$, $\frac{42}{56}$, $\frac{60}{84}$, $\frac{85}{100}$, $\frac{125}{1000}$, $\frac{30}{45}$, $\frac{75}{90}$, $\frac{30}{75}$, $\frac{65}{100}$, $\frac{48}{80}$, $\frac{54}{76}$, $\frac{28}{84}$, $\frac{57}{95}$.

2. How many 8ths in $\frac{3}{4}$? in $\frac{4}{16}$? in 9? in $2\frac{1}{2}$? in $5\frac{1}{4}$? in $7\frac{1}{2}$? in $3\frac{5}{8}$? in 12?

3. Change $\frac{2}{3}$ ft., $\frac{1}{6}$ ft., and $\frac{3}{4}$ ft. to 12ths of a foot.

Change at sight:

4. $\frac{25}{60}$ of an hour to 12ths of an hour.
5. $\frac{120}{144}$ of a square foot to 6ths of a square foot.
6. $\frac{96}{128}$ of a cord to 4ths of a cord.
7. $\frac{720}{1728}$ of a cubic foot to 12ths of a cubic foot.
8. $\frac{84}{60}$ of a minute to 5ths of a minute.
9. $\frac{18}{27}$ of a cubic yard to its lowest terms.
10. $\frac{54}{60}$ of an hour to its lowest terms.
11. $\frac{1}{2}$ equals how many fourths? sixths? eighths?
12. $\frac{1}{3}$ equals how many sixths? ninths? twelfths?
13. $\frac{2}{3}$ equals how many sixths? ninths? twelfths?
14. $\frac{5}{3}$ equals how many sixths? ninths? twelfths?
15. $\frac{1}{4}$ equals how many twelfths? twenty-fourths?
16. $\frac{3}{4}$ equals how many twelfths? twenty-fourths?
17. How many fourths in 1? 2? $2\frac{1}{4}$? $3\frac{3}{4}$? $7\frac{3}{4}$?
18. How many eighths in 1? 2? $3\frac{3}{8}$? $4\frac{7}{8}$? $6\frac{5}{8}$? $7\frac{1}{8}$?
19. How many twelfths in 3? $4\frac{5}{12}$? $5\frac{7}{12}$? $7\frac{11}{12}$? $5\frac{1}{12}$?
20. How many fifteenths of a bu. in $10\frac{7}{15}$ bu.? in $20\frac{14}{15}$ bu.?
21. How many sixtieths of an hr. in $5\frac{17}{60}$ hr.? in $16\frac{35}{60}$ hr.?
22. Reduce to improper fractions: $7\frac{3}{7}$ oz., $12\frac{3}{4}$ gi., $24\frac{5}{12}$ yd.

23. Change to improper fractions: $10\frac{7}{32}$ bu., $25\frac{7}{30}$ mo., $2\frac{7}{8}$ pecks.

24. Change at sight to whole or mixed numbers the following quantities:

$\frac{39}{8}$ lb.; $\frac{76}{4}$ bu.; $\frac{100}{15}$ hr.; $\frac{71}{9}$ yd.; $\frac{86}{15}$ min.; $\frac{100}{11}$ rods; $\frac{83}{6}$ mi.; $\frac{96}{16}$ qt.; $\frac{93}{8}$ oz.

25. $\frac{5}{8}$ of a bushel equals how many 32nds of a bushel?

26. $\frac{7}{8}$ of a peck equals how many 48ths of a peck?

27. $\frac{11}{12}$ of an hour equals how many 48ths of an hour?

28. $\frac{15}{32}$ of a bushel equals how many 64ths of a bu.?

29. $\frac{7}{16}$ of an ounce equals how many 96ths of an ounce?

30. $\frac{17}{24}$ of a day equals how many 144ths of a day?

31. $\frac{17}{32}$ of a bushel equals how many 64ths of a bu.?

31. $\frac{25}{64}$ of a gallon equals how many 320ths of a gallon?

WRITTEN EXERCISE

Change each of the following to whole or mixed numbers. The resulting fractions should be in their lowest terms.

1. $\frac{428}{16}$ *6.* $\frac{3265}{48}$ *11.* $\frac{12654}{144}$

2. $\frac{965}{24}$ *7.* $\frac{27642}{8}$ *12.* $\frac{9365}{1728}$

3. $\frac{843}{75}$ *8.* $\frac{9327}{125}$ *13.* $\frac{8324}{12}$

4. $\frac{1278}{64}$ *9.* $\frac{275}{150}$ *14.* $\frac{36542}{625}$

5. $\frac{2376}{84}$ *10.* $\frac{9643}{16}$ *15.* $\frac{4597}{160}$

16. Reduce $18\frac{9}{16}$ to sixteenths.

17. Reduce $124\frac{7}{8}$ days to eighths of a day.

18. Reduce $127\frac{5}{16}$ to an improper fraction.

Reduce the following to improper fractions:

19. $124\frac{5}{7}$ weeks. *24.* $245\frac{3}{20}$ tons. *29.* $537\frac{5}{1728}$ cu. ft.

20. $54\frac{3}{4}$ gal. *25.* $396\frac{7}{16}$ lb. *30.* $245\frac{3}{8}$ gal.

21. $36\frac{5}{12}$ hr. *26.* $935\frac{20}{27}$ cu. yd. *31.* $96\frac{71}{144}$ sq. ft.

22. $72\frac{7}{30}$ mo. *27.* $842\frac{29}{60}$ min. *32.* $87\frac{19}{128}$ cd.

23. $185\frac{9}{100}$ rd. *28.* $376\frac{9}{25}$. *33.* $92\frac{9}{32}$ bu.

Reduce the following fractions to their lowest terms:

34. $\frac{375}{1000}$ *39.* $\frac{324}{640}$ sq. mi. *44.* $\frac{256}{2000}$ Ton *49.* $\frac{2520}{2240}$ Long T.

35. $\frac{85}{100}$ *40.* $\frac{1220}{2240}$ Long T. *45.* $\frac{4800}{5760}$ lb. *50.* $\frac{352}{1760}$ mi.

36. $\frac{625}{1000}$ *41.* $\frac{156}{160}$ A. *46.* $\frac{315}{7000}$ lb. *51.* $\frac{288}{640}$ sq. mi.

37. $\frac{875}{1000}$ *42.* $\frac{112}{160}$ A. *47.* $\frac{1296}{1728}$ cu. ft. *52.* $\frac{1008}{5760}$ lb.

38. $\frac{125}{10000}$ *43.* $\frac{144}{320}$ mi. *48.* $\frac{880}{5280}$ mi. *53.* $\frac{1430}{5280}$ mi.

RATIO OR RELATION

168. We have seen (90) that one number may be measured by another, and that there are several ways of expressing such a relation; also, (91) that this relation of numbers is indicated by the general term *ratio.*

169. In finding the relation or ratio of two numbers, either may be measured by the other. Thus, in comparing 12 and 3, we may say, "12 is 4 times 3," measuring 12 by 3, or we may say "3 is $\frac{1}{4}$ of 12," measuring 3 by 12. The term we use to express the measurement of one number by another is called the relation, or ratio.

170. In any case involving comparison or measurement, it is convenient to call the number measured the *base* and the number by which it is measured, the *part*. Thus, when we say "25 is $\frac{5}{6}$ of 30," the base is 30, the part is 25, and the relation or ratio is $\frac{5}{6}$. But if we say 30 is $\frac{6}{5}$ of 25, the *base* is 25, the *part* is 30, and the *ratio* is $\frac{6}{5}$.

Notice that reversing the order of the terms, reverses, or "inverts" the ratio. For example, 12 is $\frac{3}{4}$ of 16, and 16 is $\frac{4}{3}$ of 12. Also 9 is $\frac{1}{4}$ of 36, and 36 is 4 $(\frac{4}{1})$ times 9.

171. *Problem I. To find the part when the base and relation are given.*

What sum is $\frac{7}{9}$ of \$108?

ANALYSIS: $\frac{1}{9}$ of \$108 is \$12, and $\frac{7}{9}$ is 7 times \$12, or \$84.

FOR ORAL ANALYSIS

1. At \$200 an acre, what is the value of a lot containing $\frac{7}{8}$ of an acre?

2. Mr. A owns a $\frac{3}{16}$ interest in a company that is capitalized at \$80000. What is the amount of Mr. A's interest?

3. Of a farm of 600 acres, two-fifths is pasture land, and the remainder is under cultivation. How many acres are under cultivation?

4. Find $\frac{4}{5}$ of \$95; $\frac{7}{8}$ of 72 lb.; $\frac{3}{100}$ of \$27; $\frac{9}{17}$ of 85; $\frac{11}{12}$ of \$144; $\frac{13}{16}$ of 160 A.; $\frac{3}{4}$ of \$76; $\frac{5}{8}$ of 2000 lb.

5. How many minutes in $\frac{7}{15}$ of an hour?

6. How many hours in $\frac{5}{12}$ of a day?

7. How many pounds in $\frac{3}{8}$ of a ton of 2000 lb.?

8. How many cents in $\frac{8}{100}$ of \$12?

9. There are 320 rods in a mile; how many rods in $\frac{5}{8}$ of a mile?

10. There are 128 cu. ft. in a cord; how many cu. ft. in $\frac{3}{4}$ of a cord?

172. *Problem II. To find the relation or ratio when the part and base are given.*

A man owned 75 shares of stock and sold 45 shares, what part of his stock did he sell?

Analysis: Since 1 share is $\frac{1}{75}$ of his stock, 45 shares are $45 \times \frac{1}{75}$, or $\frac{45}{75} = \frac{3}{5}$ of his stock.

Note.—As the ratio is always the quotient obtained by dividing the part by the base, the analysis may be simplified thus: The part sold is equal to $45 \div 75$ or $\frac{45}{75}$ or $\frac{3}{5}$ of his stock.

ORAL EXERCISE

Analyze the following:

1. 25 is what part of 40?

2. 65 sq. rd. is what part of an acre?

Note.—In the case of this or similar problems involving denominate numbers, consult the tables in another part of the book.

3. 27 minutes is what part of an hour?

4. 35¢ is what part of $\$1\frac{1}{4}$?

5. 72 lb. is what part of 96 lb.?

6. A man having \$85, spent \$51; what part of his money did he spend?

7. 1200 lb. is what part of a ton?

Note.—Unless otherwise directed, consider a ton as 2000 lb.

8. Bought a book for 75¢ and sold it for 90¢. What part of the cost is the gain?

9. The damage by a fire was \$7500; the insurance amounted to \$6000. What part of the loss was covered by the insurance?

10. Goods bought at \$2.50 a yard, were sold at \$2.25. What part of the cost is the loss?

173. *Problem III. To find the base when the part and ratio are given.*

\$15 is $\frac{3}{8}$ of what sum?

Analysis: $\frac{1}{8}$ of the sum is $\frac{1}{3}$ of \$15 or \$5, and $\frac{8}{8}$, or the sum, is 8 times \$5, or \$40.

2nd Analysis: If \$15 is $\frac{3}{8}$ of a certain sum, that sum is $\frac{8}{3}$ of \$15 or \$40.

ORAL EXERCISE

Analyze the following:

1. 28 miles is $\frac{2}{7}$ of what distance?

2. Mr. B. pays $30 a month for rent. The rent per year is $\frac{2}{5}$ of his annual salary. What is his salary?

3. Frank paid $45 for a suit, which is $7\frac{1}{2}$ times what he paid for his shoes. What did he pay for his shoes?

SUGGESTION.—Reduce the ratio to an improper fraction.

4. Mr. C. earns $150 per month which is $2\frac{1}{2}$ times what his son Walter earns. What does Walter earn?

5. Two-fifths of a certain farm is pasture land. If there are 70 acres in pasture land, what is the area of the farm?

6. The rate of taxation in a town is $\frac{1}{25}$ of the assessed valuation of the property. If Mr. F's taxes amount to $30, for what sum is his property assessed?

7. A farmer loaned a sum of money for a year, receiving as interest an amount equal to $\frac{7}{100}$ of the sum loaned. The interest received was $9.10; what was the amount loaned?

8. If a lot containing $\frac{3}{8}$ of an acre was sold for $510, what was the value of the land per acre?

9. Mr. C. has $\frac{2}{5}$ of his capital invested in stocks and the remainder in real estate. If the amount invested in real estate is $12000, what sum is invested in stocks?

10. A man bought a piece of real estate and sold it, making a profit of $\frac{15}{100}$ of the cost. If his profits were $45.90, what was the cost?

ORAL TEST PROBLEMS

Study the following problems until you can give a clear and logical analysis of each.

1. $\frac{2}{3}$ of 12 is $\frac{1}{4}$ of what number?

2. $\frac{3}{4}$ of 16 is $\frac{1}{5}$ of what number?

3. $\frac{1}{5}$ of 60 is $\frac{2}{3}$ of what number?

4. $\frac{1}{6}$ of 96 is $\frac{4}{15}$ of what number?

5. Mr. A received $7.50 for the use of $150 for one year; how many hundredths of the principal did he receive?

6. If a three-eighths interest in a steamship is worth $21000, what is a one-fourth interest worth?

7. A real estate firm invested $8000 in a deal and gained $320. The gain was what part of the investment?

8. $\frac{2}{5}$ of $25 is how many hundredths of $50?

9. What is the cost of a yard of cloth, if $\frac{5}{8}$ of a yard cost $62\frac{1}{2}$¢?

10. What is the cost of 3 horses, if $\frac{2}{3}$ of the cost of 1 horse is $60?

11. What would 6 bushels of potatoes cost, if $\frac{1}{2}$ of a bushel cost $62\frac{1}{2}$¢?

12. If $\frac{1}{3}$ of a number is 15, what is $\frac{2}{3}$ of the number?

13. If $\frac{2}{3}$ of a number is 8, what is $\frac{3}{4}$ of the number?

14. If $\frac{3}{4}$ of a number is 27, what is $\frac{2}{3}$ of the number?

15. If $\frac{3}{2}$ of a number is 36, what is $\frac{4}{3}$ of the number?

16. If $\frac{3}{4}$ of a barrel of flour cost $9, what is the cost of $\frac{2}{3}$ of a barrel?

17. If $\frac{5}{6}$ of a yard of silk cost $62\frac{1}{2}$¢, what is the cost of $1\frac{1}{3}$ yards?

18. If $2\frac{1}{2}$ bushels of oats cost $1, what is the cost of $7\frac{1}{2}$ bushels?

19. If a train runs 3 mi. in 4 min., what is its speed per hour?

20. A farmer sold to some campers 250 lb. hay for $1.25. At that rate, what was the price per ton?

21. A house costs $\frac{1}{3}$ of $12000, and the cost of the house is $\frac{1}{4}$ of the cost of the lot. How much did the lot cost?

22. A carriage costs $\frac{2}{3}$ of $180, and the cost of the carriage is $\frac{3}{4}$ of the cost of a horse. What is the cost of both?

23. If $\frac{1}{2}$ of a number is 12, what is $\frac{1}{3}$ of the number?

24. 36 is what part of 42? of 50? of 54? of 60? of 84? of 96? of 100?

25. If I gain $24 on an investment of $800, my gain is how many hundredths of my investment?

26. If $6\frac{1}{4}$ gallons of wine cost $5.25, what is the cost of 10 gallons?

27. 20 is $\frac{1}{4}$ more than what number?

SUGGESTION.—20 is $\frac{5}{4}$ of what number?

28. 24 is $\frac{1}{7}$ more than what number?

29. 36 is $\frac{1}{8}$ more than what number?

30. $37\frac{1}{2}$ is $\frac{1}{2}$ more than what number?

31. $87\frac{1}{2}$ bushels is $\frac{1}{6}$ more than what quantity?

32. 30 is $\frac{2}{3}$ more than what number?

33. 49 is $\frac{3}{4}$ more than what number?

34. Sold butter at 25¢ per pound which was $\frac{1}{4}$ more than it cost. How much did it cost?

35. Sold a horse for $80, thereby gaining $\frac{2}{3}$ of the cost price. How much did the horse cost?

36. There are 30 sheep in a pasture, this being $\frac{1}{4}$ more than the sheep in another pasture. How many in the second pasture?

37. I paid $120 for a horse, which is $\frac{1}{3}$ more than I paid for a carriage. How much did I pay for both?

38. There are 48 bushels of wheat in a load, this being $\frac{1}{5}$ more than the bushels in another load. How many more bushels in the first load than in the second?

39. 6 is $\frac{1}{3}$ less than what number?

SUGGESTION.—6 is $\frac{2}{3}$ of the number.

40. 24 is $\frac{1}{5}$ less than what number?

41. 42 is $\frac{4}{7}$ less than what number?

42. $87\frac{1}{2}$ is $\frac{2}{9}$ less than what number?

43. $\frac{2}{3}$ of 60 is $\frac{1}{6}$ less than what number?

44. $\frac{3}{4}$ of 96 is $\frac{2}{5}$ less than what number?

45. A pair of gloves cost $2.50, which is $\frac{1}{6}$ less than the cost of a hat. How much did the hat cost?

46. I paid $120 for a carriage, and this sum is $\frac{1}{4}$ less than I paid for a horse. How much did I pay for both?

47. Henry's hat cost $\frac{2}{3}$ as much as his shoes, and he paid $7.50 for both. What did each cost?

SUGGESTION.—They both cost $\frac{5}{3}$ of the cost of the shoes.

48. A house and lot cost $1400. If the cost of the lot was $\frac{3}{4}$ the cost of the house, what did each cost?

49. John has $\frac{1}{4}$ more money than Frank, and both have $1.80. What sum has each?

SUGGESTION.—John's money is $\frac{5}{4}$ of Frank's.

50. Mr. A's carriage cost $\frac{5}{8}$ as much as his horse, and his horse cost $60 more than his carriage. What did each cost?

SUGGESTION.—$60 equals $\frac{3}{8}$ the cost of the horse.

51. Mary's money is $\frac{1}{5}$ less than Ella's and both have $2.70. What sum has each?

52. $\frac{5}{8}$ of Will's weekly salary is $3.50 more than $\frac{3}{8}$ of it. What is his salary?

WRITTEN TEST PROBLEMS

54. A man earned $85.60 in one month at piece-work in a factory, working eight hours a day. How much could he have earned had he worked nine hours a day?

55. If I buy property for $3561 and sell it for $3960, the gain is what part of the investment?

56. A money-lender received $38 for the use of $600 for four months. At that rate, how many hundredths of his principal would he receive for its use for one year?

57. If Arthur's expenses amount to $\frac{5}{8}$ of his wages, how much should he save in a year of fifty-two weeks out of a salary of $14 per week?

58. A lady receives an income of $125 per month from money loaned. If she receives $\frac{6}{100}$ per year for the use of her money, what is the amount loaned?

59. If a contractor can complete a job in 48 days when he employs 75 men, in what time should he complete it with 125 men?

60. A train left Springfield at 1:30 P. M., and reached Farmington, 91 miles away, at 3:15 P. M. What was the running rate per hour?

61. A real estate dealer sold two lots for $750 each. On one he gained $\frac{1}{4}$ of the cost and on the other he lost $\frac{1}{4}$ of the cost. Did he lose or gain on both transactions, and how much?

62. A farmer sold a horse for $250 and gained $\frac{1}{5}$ of the cost For what should he have sold the horse to gain $\frac{1}{10}$ of the cost?

63. Henry and his sister inherit an estate valued at $27000. Henry's share is $\frac{1}{4}$ greater than his sister's; what is the amount received by each?

64. Mr. A owns $\frac{3}{100}$ of the stock in a corporation. His share of a certain dividend was $543.30. Find the amount received by a man who owned $\frac{1}{8}$ of the stock?

65. The net profits of a firm for 1909 were $2956.24, which was $\frac{1}{8}$ less than the profits for 1908. What were the profits for 1908?

ADDITION OF FRACTIONS

174. We have seen (26 and 27) that only like numbers or those composed of the same kind of units, can be added. Hence, fractions can be added only when they have the same *fractional units* (158). For instance, we cannot add, in their present form, the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, since the numerators are *unlike numbers*. But if we reduce the series to a common denominator, obtaining $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$, we may add the fractions, since their numerators are now *like numbers*, having the same unit, "twelfths."

175. Even though two or more fractions have the same denominator, we cannot add them, unless they have the same *base* or *unit*. Thus, we cannot add the fractions $\frac{5}{16}$ of a mile, and $\frac{7}{16}$ of a foot, or $\frac{3}{4}$ of a pound, and $\frac{1}{4}$ of an ounce.

ORAL EXERCISE

176. The fractions most common in business are halves, fourths, eighths, and sixteenths, but thirds, fifths, and sixths occasionally occur. The efficient clerk or accountant must learn to compute such fractions mentally and without using the slow and formal processes required in the case of larger fractions.

Add $\frac{1}{8}$, $\frac{5}{16}$, $\frac{1}{4}$ and $\frac{1}{2}$.

SOLUTION.—Observe that the common denominator is 16. Then find the required numerators and add, thus: "2, 7, 11, 19 sixteenths, or $1\frac{3}{16}$.

Add the following mentally:

1. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.	*5.* $\frac{3}{8}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{1}{2}$.	*9.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$.
2. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{2}{3}$.	*6.* $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{4}{5}$.	*10.* $\frac{2}{3}$, $\frac{5}{12}$, $\frac{3}{8}$, $\frac{5}{6}$.
3. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{1}{2}$.	*7.* $\frac{7}{8}$, $\frac{3}{4}$, $\frac{9}{16}$, $\frac{1}{4}$.	*11.* $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$, $\frac{2}{3}$.
4. $\frac{5}{12}$ $\frac{1}{3}$, $\frac{3}{8}$, $\frac{1}{2}$.	*8.* $\frac{5}{6}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{12}$.	*12.* $\frac{5}{16}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{2}$.

Add, $2\frac{1}{2}$, $5\frac{3}{4}$, $7\frac{1}{8}$, $6\frac{1}{4}$.

OPERATION

$$\begin{array}{r} 2\frac{1}{2} \\ 5\frac{3}{4} \\ 7\frac{1}{8} \\ 6\frac{1}{4} \\ \hline 21\frac{5}{8} \end{array}$$

SOLUTION.—First add the fractions mentally as in the preceding exercise, obtaining $\frac{13}{8} = 1\frac{5}{8}$. Add this result to the integers, obtaining the complete result, $21\frac{5}{8}$.

Add the following mentally:

1. $3\frac{1}{4}$, $5\frac{1}{2}$, $11\frac{3}{4}$.	*4.* $5\frac{3}{16}$, $8\frac{7}{8}$, $10\frac{3}{4}$.	*7.* $23\frac{5}{16}$, $24\frac{3}{8}$, $16\frac{3}{4}$.
2. $6\frac{1}{8}$, $12\frac{3}{4}$, $9\frac{1}{2}$.	*5.* $6\frac{1}{8}$, $11\frac{1}{6}$, $7\frac{1}{12}$.	*8.* $32\frac{7}{8}$, $15\frac{1}{2}$, $24\frac{3}{4}$.
3. $3\frac{2}{3}$, $5\frac{1}{4}$, $3\frac{5}{6}$.	*6.* $12\frac{3}{4}$, $21\frac{1}{4}$, $42\frac{1}{2}$.	*9.* $12\frac{1}{6}$, $10\frac{5}{24}$, $9\frac{5}{6}$.

177. In adding any two simple fractions whose numerators are 1, and whose denominators are prime to each other, the numerator of the sum equals the sum of the denominators of the given fractions. Thus, $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$, $\frac{1}{7}+\frac{1}{8}=\frac{15}{56}$, $\frac{1}{8}+\frac{1}{9}=\frac{17}{72}$.

Using this short process, add the following:

$\frac{1}{4}+\frac{1}{5}$, $\frac{1}{6}+\frac{1}{7}$, $\frac{1}{6}+\frac{1}{11}$, $\frac{1}{9}+\frac{1}{10}$, $\frac{1}{8}+\frac{1}{5}$, $\frac{1}{7}+\frac{1}{13}$, $\frac{1}{4}+\frac{1}{15}$, $\frac{1}{2}+\frac{1}{9}$, $\frac{1}{5}+\frac{1}{16}$, $\frac{1}{4}+\frac{1}{19}$, $\frac{1}{5}+\frac{1}{17}$.

178. In the dry-goods trade, cloth comes in "pieces" (pc.) consisting either of even yards, or of yards and quarters. In billing such goods, the quarters are conveniently indicated by small figures written to the right and a little above the integers. Thus, 5^1, 6^3, 3^2 means $5\frac{1}{4}$, $6\frac{3}{4}$, and $3\frac{2}{4}$ yards.

Find the extensions in yards of the following:

1. 5 pc. 12^2, 15^3, 26^1, 17^2, 54^3.
2. 5 pc. 40^3, 35^1, 50^3, 20^2, 56^3.
3. 4 pc. 35^2, 40^3, 31^1, 35^1.
4. 5 pc. 48^1, 51^2, 40^3, 50^2, 35^1.

ORAL PROBLEMS

1. A man owning $\frac{2}{5}$ of a farm bought $\frac{3}{16}$ of it at another time. What part of the whole farm did he then own?

2. A farmer sold $\frac{1}{3}$ of his grain at one time, $\frac{1}{4}$ at another, and $\frac{1}{5}$ at another. What part of the whole did he sell?

3. A gardener has $1\frac{1}{2}$ A. in turnips, $2\frac{3}{4}$ A. in cabbage, and $1\frac{1}{4}$ A. in melons. How many acres in all?

4. A can do a piece of work in 5 days, and B can do it in 6 days. How much can both do in one day?

5. How much money has Arthur if $\frac{1}{6}$ of his money added to $\frac{1}{7}$ of it equals $26?

6. $\frac{1}{8}$ T. + $\frac{1}{2}$ T. + $\frac{3}{4}$ T. equals how many pounds?

7. How many minutes equal $\frac{5}{6}+\frac{1}{4}+\frac{2}{3}$ of an hour?

8. If Frank can saw a certain pile of wood in 12 hours, and George can saw it in 8 hours, in how many hours can both saw it?

9. How many rods in $\frac{1}{4}+\frac{3}{8}+\frac{7}{16}$ of a mile?

10. $\frac{1}{7}-\frac{1}{8}$ of A's income equals $7.50; what is the amount of his income?

11. Two lots, the one containing $\frac{3}{4}$ A. and the other $\frac{5}{8}$ A. were sold for $220. What was the price per acre?

12. Kate can fold a certain pile of papers in 12 hours, Nellie in 8 hours, and Clarice in 6 hours. In what time can the papers be folded if all work together?

WRITTEN PROBLEMS

What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$ and $\frac{11}{24}$?

SOLUTION

$\frac{3}{4} + \frac{7}{8} + \frac{9}{16} + \frac{11}{24} = ?$
$\frac{36}{48} + \frac{42}{48} + \frac{27}{48} + \frac{22}{48} = \frac{127}{48}$
$\frac{127}{48} = 2\frac{31}{48}$.

What is the sum of $14\frac{1}{2} + 25\frac{3}{8} + 9\frac{5}{6} + 11\frac{5}{12}$?

SOLUTION

$14\frac{1}{2} + 25\frac{3}{8} + 9\frac{5}{6} + 11\frac{5}{12} = ?$
$14 + 25 + 9 + 11 = 59$
$\frac{1}{2} + \frac{3}{8} + \frac{5}{6} + \frac{5}{12} = ?$
$\frac{12}{24} + \frac{9}{24} + \frac{20}{24} + \frac{10}{24} = \frac{51}{24}$
$\frac{51}{24} = 2\frac{3}{24}$, or $2\frac{1}{8}$
$59 + 2\frac{1}{8} = 61\frac{1}{8}$.

OR

	24 L. C. D.
$14\frac{1}{2}$	12
$25\frac{3}{8}$	9
$9\frac{5}{6}$	20
$11\frac{5}{12}$	10
59	$\frac{51}{24} = 2\frac{1}{8}$
$2\frac{1}{8}$	
$61\frac{1}{8}$	

66. What is the sum of $16\frac{1}{4}$, $18\frac{7}{8}$, $24\frac{5}{16}$, $19\frac{17}{24}$, $36\frac{5}{6}$, and $42\frac{2}{3}$?

67. A farmer has $36\frac{3}{4}$ A. in wheat, $44\frac{7}{8}$ A. in oats, $58\frac{3}{8}$ A. in corn, and $96\frac{1}{4}$ A. in pasture. How many acres has he?

68. A grocer packed in a box 36 lb. of sugar, $6\frac{7}{8}$ lb. of coffee, $2\frac{1}{2}$ lb. of tea, $16\frac{3}{4}$ lb. of beef, $12\frac{1}{4}$ lb. of bacon, and $16\frac{5}{8}$ lb. butter; what was the total weight, the box weighing $56\frac{3}{4}$ lb.?

69. A farmer sold grain as follows: $624\frac{3}{4}$ bu. of oats for $\$124\frac{7}{8}$, $473\frac{1}{4}$ bu. of wheat for $\$296\frac{5}{16}$, $324\frac{7}{8}$ bu. of barley for $\$96\frac{1}{4}$, $306\frac{7}{16}$ bu. of beans for $\$142\frac{7}{12}$, and $224\frac{1}{2}$ bu. of corn for $\$175\frac{1}{2}$. How many bushels did he sell and how much did he get for it all?

70. A merchant bought 30 bolts of dry goods containing the following number of yards: $40\frac{1}{4}$, $41\frac{1}{4}$, $42\frac{2}{4}$, 41, $43\frac{3}{4}$, $43\frac{1}{4}$, $44\frac{1}{4}$, $36\frac{3}{4}$, $37\frac{2}{4}$, $39\frac{2}{4}$, 37, 36, $35\frac{1}{4}$, $37\frac{1}{4}$, 34, $33\frac{2}{4}$, 37, $35\frac{1}{4}$, $36\frac{1}{4}$, $34\frac{3}{4}$, $34\frac{1}{4}$, $32\frac{3}{4}$, $35\frac{1}{4}$, $24\frac{3}{4}$, $27\frac{1}{4}$, $28\frac{1}{4}$, 29, $26\frac{2}{4}$, $22\frac{2}{4}$, 24. How many yards did he buy?

Copy and find the sum of the each of the following groups:

71.	*72.*	*73.*	*74.*	*75.*	*76.*
$364\frac{5}{8}$	$327\frac{4}{5}$	$643\frac{3}{16}$	$842\frac{2}{25}$	$634\frac{5}{12}$	$3426\frac{3}{4}$
$742\frac{7}{12}$	$645\frac{5}{6}$	$864\frac{7}{12}$	$375\frac{7}{10}$	$784\frac{4}{16}$	$3548\frac{5}{8}$
$436\frac{5}{6}$	$327\frac{2}{3}$	$963\frac{14}{24}$	$842\frac{4}{5}$	$643\frac{17}{20}$	$7263\frac{1}{2}$
$743\frac{3}{4}$	$846\frac{7}{10}$	$327\frac{5}{6}$	$963\frac{1}{2}$	$426\frac{7}{10}$	$4986\frac{1}{4}$
$375\frac{2}{3}$	$364\frac{8}{15}$	$842\frac{2}{3}$	$643\frac{3}{25}$	$962\frac{11}{30}$	$5432\frac{7}{8}$

77.	*78.*	*79.*	*80.*	*81.*
$21524\frac{11}{16}$	$34253\frac{9}{10}$	$22736\frac{7}{15}$	$13125\frac{5}{12}$	$12567\frac{5}{16}$
$13065\frac{5}{8}$	$25321\frac{3}{8}$	$16853\frac{4}{25}$	$24143\frac{3}{4}$	$45526\frac{7}{8}$
$43753\frac{3}{4}$	$12579\frac{7}{20}$	$40632\frac{3}{5}$	$13560\frac{7}{9}$	$30415\frac{3}{4}$
$10476\frac{1}{2}$	$40055\frac{4}{5}$	$12345\frac{11}{25}$	$30234\frac{5}{6}$	$57939\frac{4}{5}$
$44342\frac{7}{16}$	$54204\frac{7}{8}$	$43210\frac{4}{15}$	$45746\frac{2}{3}$	$21473\frac{11}{20}$
$13526\frac{1}{4}$	$13652\frac{3}{4}$	$25607\frac{18}{25}$	$25612\frac{1}{2}$	$65114\frac{7}{16}$
$26437\frac{3}{8}$	$31148\frac{3}{10}$	$34051\frac{1}{6}$	$14200\frac{1}{3}$	$13496\frac{1}{5}$
$10596\frac{5}{16}$	$26554\frac{1}{2}$	$15779\frac{8}{15}$	$17488\frac{7}{12}$	$28744\frac{3}{8}$

SUBTRACTION OF FRACTIONS

179. Fractions can be subtracted only when they are parts of like units and have a common denominator.

ORAL EXERCISE

1. Name the result of the following subtractions:

$\frac{3}{4}-\frac{1}{4}$; $\frac{5}{8}-\frac{1}{8}$; $\frac{11}{12}-\frac{5}{12}$; $\frac{6}{7}-\frac{2}{7}$; $\frac{3}{4}-\frac{1}{2}$; $\frac{5}{8}-\frac{1}{4}$; $\frac{1}{6}-\frac{1}{12}$.

2. Supply the missing term, *X*, in each of the following equations:

$\frac{1}{2}-\frac{1}{6}=X$; $\frac{3}{4}+X=\frac{9}{8}$; $X+\frac{2}{3}=\frac{5}{2}$; $\frac{5}{6}-\frac{1}{3}=X$; $\frac{7}{10}+X=\frac{4}{5}$; $\frac{1}{3}-\frac{1}{4}=X$; $X+\frac{3}{8}=\frac{7}{12}$; $\frac{5}{9}+X=\frac{5}{6}$.

3. $\frac{7}{8}-\frac{3}{4}=?$ $\frac{9}{10}-\frac{3}{5}=?$ $\frac{5}{7}-\frac{3}{5}=?$ $\frac{7}{8}-\frac{2}{3}=?$ $\frac{7}{12}-\frac{5}{16}=?$

4. Find the value of:

$2\frac{1}{2}-2$; $6\frac{1}{3}-4$; $3\frac{3}{4}-2\frac{1}{4}$; $5\frac{3}{4}-2\frac{1}{2}$; $17\frac{5}{8}-2\frac{2}{4}$; $12\frac{1}{2}-6\frac{1}{4}$; $5-\frac{3}{4}$; $7-\frac{3}{8}$; $12-\frac{2}{3}$; $16-\frac{5}{6}$.

5. Find the remainder in the following:

$5-1\frac{1}{4}$; $7-3\frac{1}{2}$; $14-12\frac{2}{3}$; $21-5\frac{3}{4}$; $16-7\frac{1}{8}$; $20-6\frac{5}{8}$; $32-5\frac{3}{5}$; $41-4\frac{2}{8}$; $56-7\frac{3}{10}$; $29-18\frac{7}{12}$.

Subtract $12\frac{5}{6}$ from $17\frac{3}{4}$.

ANALYSIS: $\frac{3}{4}=\frac{9}{12}$; $\frac{5}{6}=\frac{10}{12}$. As $\frac{10}{12}$ cannot be taken from $\frac{9}{12}$, we take 1 ($=\frac{12}{12}$) from 17, and combining it with $\frac{9}{12}$, we

have $\frac{21}{12}$. Then, $\frac{21}{12} - \frac{10}{12} = \frac{11}{12}$, $16 - 12 = 4$. Hence, the required remainder is $4\frac{11}{12}$.

In the same manner find the results, mentally, of the following:

6. $8\frac{3}{8}$ bu. $- 6\frac{7}{8} = ?$

7. $6\frac{3}{8}$ bu. $- 3\frac{5}{8}$ bu. $= ?$

8. $8\frac{1}{2}$ mi. $- 5\frac{5}{8}$ mi. $- ?$

9. $5\frac{1}{3}$ hr. $- 3\frac{1}{2}$ hr. $= ?$

10. $3\frac{1}{4}$ pt. $- 2\frac{1}{2}$ pt. $= ?$

11. $4\frac{2}{3}$ min. $- 2\frac{3}{4}$ min. $= ?$

12. A lady bought a hat for $\$6\frac{3}{4}$, a pair of shoes for $\$4\frac{1}{2}$, a parasol for $\$3\frac{2}{3}$, and a dress pattern for $\$1\frac{5}{6}$. How much change should she receive, if she gives the merchant two ten-dollar bills?

13. A can do a piece of work in 5 days, and B can do it in 6 days. How much more can A do in 1 day than B?

WRITTEN EXERCISE

Find the difference between the following:

82.
$$\begin{array}{r} 24 = 23\frac{7}{7} \\ 14\frac{2}{7} = 14\frac{2}{7} \\ \hline 9\frac{5}{7} \end{array}$$

83.
$$\begin{array}{r} 371 \\ 245\frac{8}{9} \\ \hline \end{array}$$

84.
$$\begin{array}{r} 237 \\ 99\frac{3}{32} \\ \hline \end{array}$$

85.
$$\begin{array}{r} 178 \\ 25\frac{7}{16} \\ \hline \end{array}$$

86. From 85 take $14\frac{7}{8}$.

87. From 69 take $63\frac{5}{24}$.

88. Subtract 84 from $125\frac{5}{14}$.

89. Subtract 92 from $320\frac{7}{17}$.

Find the difference between the following:

90.
$$\begin{array}{r} 27\frac{2}{3} = 26\frac{20}{12} \\ 18\frac{3}{4} = 18\frac{9}{12} \\ \hline 8\frac{11}{12} \end{array}$$

91.
$$\begin{array}{r} 85\frac{9}{16} \\ 72\frac{3}{4} \\ \hline \end{array}$$

92.
$$\begin{array}{r} 97\frac{5}{14} \\ 36\frac{5}{8} \\ \hline \end{array}$$

93.
$$\begin{array}{r} 33\frac{1}{16} \\ 28\frac{7}{12} \\ \hline \end{array}$$

94. From $84\frac{1}{6}$ take $27\frac{1}{4}$.

95. Take $37\frac{5}{6}$ from $89\frac{3}{4}$.

96. Subtract $75\frac{9}{16}$ from $126\frac{1}{640}$.

97. From $97\frac{1}{16}$ take $8\frac{9}{14}$.

98. What is the difference between $245\frac{7}{12}$ and $124\frac{9}{16}$?

99. From a cask of wine containing $63\frac{1}{2}$ gallons, $24\frac{3}{4}$ gallons were sold at one time, and $16\frac{2}{3}$ gallons at another time. How many gallons remained?

100. A farmer bought $124\frac{7}{8}$ acres of land at one time, and $246\frac{9}{16}$ acres at another time. He afterward sold $184\frac{3}{4}$ acres. How many had he left?

MULTIPLICATION OF FRACTIONS

180. It has been shown (162) that a fraction is multiplied by a whole number, either by multiplying its numerator or dividing its denominator by the whole number. Thus, $\frac{7}{12} \times 4 = \frac{7}{3}$ or $\frac{7}{3}$.

The process of multiplying together several fractions is therefore the same as multiplying together their several numerators and dividing the result by the product of their several denominators. Thus,

$\frac{3}{4} \times \frac{5}{6} \times \frac{7}{9} = \frac{3 \times 5 \times 7}{4 \times 6 \times 9}$ This is merely a compound division (as illustrated in 125) and the process may be shortened by cancellation.

What is $\frac{4}{7} \times \frac{14}{15} \times \frac{5}{8} \times \frac{3}{8} \times \frac{4}{9} \times \frac{8}{11}$?

OPERATION

$$\frac{\cancel{2} \qquad\qquad\qquad\qquad \cancel{4}}{\cancel{4} \times \cancel{14} \times \cancel{5} \times \cancel{3} \times \cancel{4} \times \cancel{8}} = \frac{4}{99}$$
$$\cancel{7} \times \cancel{15} \times \cancel{8} \times \cancel{8} \times 9 \times 11$$
$$\cancel{3} \quad \cancel{2} \quad \cancel{2}$$

We simply arrange the numerators as a compound dividend and the denominators as a compound divisor and shorten the operation by cancellation (as shown in 126).

181. In practical arithmetic it is rarely necessary to multiply together a series of fractional numbers, but cases are frequent when two numbers are to be multiplied together, one or the other of them being either a fraction or a mixed number. Sometimes both factors are fractional numbers. There are several methods of multiplying together such fractional numbers, the method to be selected in a given case depending on the numbers involved.

1. Multiply $7\frac{3}{4}$ by $5\frac{2}{3}$.

OPERATION

$7\frac{3}{4} = \frac{31}{4}$
$5\frac{2}{3} = \frac{17}{3}$
$\frac{31}{4} \times \frac{17}{3} = \frac{527}{12} = 43\frac{11}{12}$

Here the multiplication is most readily performed by reducing both factors to improper fractions, and divding the product of the numerators by the product of the denominators.

2. Multiply $48\frac{4}{5}$ by $25\frac{3}{8}$.

OPERATION

$48 \times 25 = 1200$
$48 \times \frac{3}{8} = 18$
$\frac{4}{5} \times 25 = 20$
$\frac{4}{5} \times \frac{3}{8} = \frac{3}{10}$

$1238\frac{3}{10}$

Here, as the whole numbers are readily multiplied by the fractions, the operation is much shortened by multiplying the parts of the factors separately and adding the results.

3. Multiply 29 by $6\frac{7}{8}$.

OPERATION

$29 \times 6 = 174$
$29 \times \frac{7}{8} = \frac{203}{8} = 25\frac{3}{8}$

$199\frac{3}{8}$

EXPLANATION.—First multiply together the whole numbers, then the whole number and the fraction. This method is usually to be preferred where one of the factors is a whole number.

4. Multiply $125\frac{3}{8}$ by $\frac{3}{4}$.

OPERATION

$125 \times \frac{3}{4} = \frac{375}{4} = 93\frac{3}{4}$

$\frac{3}{8} \times \frac{3}{4} = \frac{9}{32}$

$94\frac{1}{32}$

EXPLANATION.—First multiply the whole number by the fraction, then the fraction by the fraction, and unite the results.

ORAL EXERCISE

Selecting the shortest method, find the results of the following multiplications:

1.	$5 \times 4\frac{3}{5}$	*4.*	$3\frac{1}{2} \times 2\frac{1}{2}$	*7.*	$12\frac{1}{4} \times 2\frac{1}{3}$	*10.*	$3\frac{1}{8} \times 18$
2.	$18 \times 1\frac{1}{2}$	*5.*	$6\frac{1}{8} \times \frac{1}{4}$	*8.*	$16\frac{1}{2} \times \frac{3}{4}$	*11.*	$5\frac{1}{2} \times 2\frac{1}{4}$
3.	$12 \times 2\frac{3}{4}$	*6.*	$32 \times 2\frac{1}{4}$	*9.*	$7\frac{5}{8} \times 10$	*12.*	$8\frac{2}{3} \times 3\frac{1}{4}$

Find the cost of:

13.	$8\frac{3}{4}$ yd. cloth at $12¢$.	*17.*	$16\frac{3}{4}$ bu. bran at $30¢$.
14.	200 lb. pork at $6\frac{1}{4}¢$.	*18.*	$9\frac{1}{4}$ gal. milk at $25¢$.
15.	$7\frac{1}{2}$ yd. ribbon at $12\frac{1}{2}¢$.	*19.*	$15\frac{1}{2}$ qt. berries at $8¢$.
16.	$8\frac{7}{16}$ lb. fish at $8¢$.	*20.*	$26\frac{1}{2}$ lb. sugar at $5¢$.

21. At $12\frac{1}{2}$ cents per yard, what will 3 yards of cloth cost? 4 yards?

22. At $7\frac{1}{2}$ cents per pound, what will 6 pounds of rice cost? 7 pounds? 8 pounds? 10 pounds?

23. If a jar of butter holds $6\frac{1}{4}$ pounds, how many pounds will 2 jars hold? 3? 4? 5? 6? 7? 8? 9? 10?

24. At $14\frac{1}{2}$ cents per yard, what will 2 yards cost? 3 yd.? 4 yd.? 5 yd.? 6 yd.? 7 yd.?

25. A can do a piece of work in 5 days, and B can do it in 6 days. How much can both do in 2 days?

Find the value of the following:

26.	$3 \times \frac{3}{4}$	*35.*	$12 \times \frac{5}{7}$	*44.*	$5 \times 6\frac{1}{4}$
27.	$4 \times \frac{5}{6}$	*36.*	$15 \times \frac{4}{5}$	*45.*	$6 \times 12\frac{1}{2}$
28.	$5 \times \frac{6}{7}$	*37.*	$18 \times \frac{3}{7}$	*46.*	$4 \times 16\frac{2}{3}$
29.	$8 \times \frac{3}{8}$	*38.*	$24 \times \frac{4}{9}$	*47.*	$8 \times 8\frac{1}{3}$
30.	$\frac{1}{2}$ of 14	*39.*	$\frac{4}{7}$ of 100	*48.*	$\frac{2}{3} \times 8$
31.	$\frac{2}{3}$ of 16	*40.*	$\frac{3}{16}$ of 100	*49.*	$\frac{5}{6} \times 7$
32.	$\frac{3}{4}$ of 15	*41.*	$\frac{5}{12}$ of 150	*50.*	$\frac{3}{8} \times 15$
33.	$\frac{3}{8}$ of 50	*42.*	$\frac{3}{20}$ of 75	*51.*	$\frac{7}{12} \times 11$
34.	$\frac{5}{6}$ of 75	*43.*	$\frac{7}{15}$ of 50	*52.*	$\frac{9}{16} \times 7$

53. What is the value of $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{7}$ of $\frac{7}{10}$?

54. What is the cost of $\frac{1}{4}$ lb. of tea, if 1 lb. cost $\$\frac{4}{5}$?

55. A boy has $\frac{1}{2}$ of a dollar, and gives $\frac{1}{2}$ of it to his friend. What part does he give away?

56. A farmer has $\frac{7}{8}$ of a bushel of cloverseed, and sows $\frac{1}{3}$ of it. What part of a bushel does he sow, and what part of a bushel does he have left?

57. What is the value of $\frac{3}{5}$ A., if $1\frac{1}{2}$ A. cost $75?

58. If $\frac{3}{4}$ of a bushel of potatoes costs $18\frac{3}{4}$ cents, what will $2\frac{1}{4}$ bu. cost? $1\frac{1}{2}$ bu.? 3 bu.? $2\frac{1}{4}$ bu.? $\frac{1}{2}$ bu.?

59. What will $\frac{2}{3}$ of a barrel of flour cost at $\$6\frac{3}{5}$ per barrel? $\frac{3}{4}$ of a barrel? $\frac{5}{6}$ of a barrel? $\frac{3}{8}$ of a barrel? $\frac{5}{12}$ of a barrel?

SUGGESTION.—$6\frac{1}{4} = \frac{25}{4}$.

Find the value of the following:

60. $\frac{1}{2}$ of $\frac{3}{8}$	*65.* $\frac{3}{4} \times \frac{2}{3}$	*70.* $\frac{1}{2}$ of $3\frac{1}{2}$
61. $\frac{1}{3}$ of $\frac{7}{8}$	*66.* $\frac{3}{5} \times \frac{5}{6}$	*71.* $\frac{2}{3}$ of $6\frac{1}{4}$
62. $\frac{1}{4}$ of $\frac{4}{7}$	*67.* $\frac{5}{6} \times \frac{6}{7}$	*72.* $\frac{3}{4}$ of $8\frac{1}{3}$
63. $\frac{3}{4}$ of $\frac{7}{8}$	*68.* $\frac{7}{8} \times \frac{5}{12}$	*73.* $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{3}{8}$
64. $\frac{3}{4}$ of $\frac{4}{7}$	*69.* $\frac{3}{6} \times \frac{8}{5}$	*74.* $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{16}$

WRITTEN EXERCISE

Perform the following multiplications, selecting the shortest process. Each result should be either a whole number or a mixed number with the fraction in its lowest terms.

101. $36\frac{1}{2} \times 15$	*106.* $75 \times 24\frac{3}{14}$	*111.* $24\frac{3}{4} \times 16\frac{2}{3}$
102. $84\frac{3}{4} \times 16$	*107.* $84 \times 25\frac{5}{6}$	*112.* $36\frac{2}{3} \times 18\frac{3}{4}$
103. $37\frac{5}{8} \times 24$	*108.* $12\frac{1}{2} \times 12\frac{1}{2}$	*113.* $75\frac{3}{4} \times 36\frac{2}{5}$
104. $96\frac{3}{32} \times 64$	*109.* $25\frac{1}{2} \times 25\frac{1}{2}$	*114.* $25\frac{2}{3} \times 48\frac{3}{4}$
105. $25\frac{1}{4} \times 9.$	*110.* $35\frac{1}{4} \times 36\frac{3}{5}$	*115.* $96\frac{3}{8} \times 28\frac{5}{12}$

Find the cost of:

116. 164 bu. wheat at $90\frac{1}{8}¢$.
117. $41\frac{3}{4}$ cords of wood at $\$6\frac{1}{2}$.
118. 586 bu. oats at $36\frac{3}{8}¢$.
119. $12\frac{1}{2}$ M. feet of lumber at $\$13\frac{3}{4}$.
120. $29\frac{1}{4}$ T. coal at $\$5\frac{1}{4}$.
121. $32\frac{3}{4}$ yd. silk at $\$2\frac{1}{2}$.
122. 172 lb. butter at $28\frac{3}{4}¢$.
123. $19\frac{11}{16}$ lb. meat at $16\frac{1}{2}$c.
124. $4\frac{1}{2}$ boxes soap at $3.75.
125. 525 lb. fish at $5.75 per hundred.

126. A commission merchant receives 4 hogs weighing respectively $246\frac{1}{4}$, $275\frac{3}{8}$, $321\frac{7}{16}$ and $302\frac{5}{8}$ pounds to be sold on commission. He sells at $7\frac{1}{2}$ cents per pound. How much does he remit, if he deducts $16.50 as his commission?

127. A merchant sold 20 pieces of denim containing respectively 40^2, 41^1, 41^1, 43, 42^2, 44, 43^3, 42, 45, 42^1, 40^1, 41^1, 42^2, 44, 41^3, 43, 47^2, 43^1, 42, 43^1 yards, at $12\frac{1}{2}$ cents per yard. How much did he receive?

128. A merchant sold 20 yards of silk at $\$2\frac{1}{2}$ per yard, $26\frac{1}{4}$ yards of satin at $\$1\frac{1}{2}$ per yard, $36\frac{3}{4}$ yards of denim at $\$\frac{1}{2}$ per yard, $24\frac{3}{4}$ yards of carpet at $\$\frac{3}{4}$ per yard. What was the amount of the sale?

129. A wood and coal dealer sold $16\frac{3}{4}$ cords of hard wood at $\$6\frac{1}{2}$ per cord, $20\frac{1}{2}$ cords of soft wood at $\$3\frac{1}{4}$ per cord, $18\frac{3}{4}$ tons of hard coal at $\$7\frac{1}{2}$ per ton, $23\frac{7}{8}$ tons of soft coal at $\$2\frac{1}{4}$ per ton. Find the amount of the sales.

DIVISION OF FRACTIONS

182. The methods of dividing a fraction by a whole number have been considered in 162. We there learned that a fraction is divided by dividing its numerator or multiplying its denominator. We are now to consider divisions in which the divisor is a fracton.

1. Divide 12 by $\frac{1}{4}$.

ANALYSIS: $\frac{1}{4}$ is contained in 1, 4 times; hence, $\frac{1}{4}$ is contained in 12, 12 times 4, or 48 times.

2. Divide 12 by $\frac{3}{4}$.

ANALYSIS: $\frac{1}{4}$ is contained in 1, 4 times, $\frac{3}{4}$ is 3 times $\frac{1}{4}$; hence, $\frac{3}{4}$ is contained in 1, $\frac{1}{3}$ of 4 times, or $\frac{4}{3}$ times, and in 12, 12 times $\frac{4}{3}$ or 16 times.

183. The **reciprocal** of a number is the quotient that results from dividing 1 by that number. Thus, the reciprocal of 4 is $\frac{1}{4}$; the reciprocal of $\frac{1}{4}$ is 4, and the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

184. It will be noticed that the reciprocal of a fraction is obtained by interchanging its terms, or inverting the fraction. Thus, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of 4, it is $\frac{1}{4}$, of $\frac{1}{5}$, it is 5 $(\frac{5}{1})$.

185. From the analysis of the preceding example, we may derive the following general formula for dividing any number by a fraction:

Multiply the dividend by the reciprocal of the divisor, or the divisor inverted.

3. Divide $7\frac{1}{3}$ by $2\frac{1}{2}$.

ANALYSIS: $7\frac{1}{3} = \frac{22}{3}$, $2\frac{1}{2} = \frac{5}{2}$. The reciprocal of $\frac{5}{2}$ is $\frac{2}{5}$ (that is, $\frac{5}{2}$ is contained in 1, $\frac{2}{5}$ times) $\frac{22}{3} \times \frac{2}{5} = \frac{44}{15} = 2\frac{14}{15}$.

ORAL EXERCISE

Analyze the following:

1.	$9 \div \frac{2}{3}$	*5.*	$7 \div 2\frac{1}{2}$	*9.*	$\frac{2}{3} \div \frac{1}{4}$	*13.*	$2\frac{1}{2} \div \frac{2}{5}$
2.	$12 \div \frac{1}{5}$	*6.*	$11 \div 3\frac{2}{3}$	*10.*	$\frac{3}{4} \div \frac{2}{5}$	*14.*	$3\frac{1}{4} \div \frac{13}{16}$
3.	$18 \div \frac{2}{7}$	*7.*	$12 \div 2\frac{1}{4}$	*11.*	$\frac{5}{6} \div \frac{5}{7}$	*15.*	$3\frac{1}{2} \div 2\frac{1}{2}$
4.	$5 \div \frac{3}{4}$	*8.*	$15 \div 2\frac{1}{2}$	*12.*	$\frac{3}{8} \div \frac{9}{10}$	*16.*	$4\frac{1}{2} \div \frac{9}{10}$

WRITTEN EXERCISE

Find the value of the following:

130.	$\frac{16}{1} \div \frac{3}{4}$	*137.*	$7 \div 1\frac{1}{2}$	*144.*	$640 \div \frac{8}{9}$
131.	$\frac{24}{1} \div \frac{7}{8}$	*138.*	$11 \div 1\frac{3}{8}$	*145.*	$560 \div 2\frac{1}{4}$
132.	$36 \div \frac{5}{6}$	*139.*	$15 \div 2\frac{5}{6}$	*146.*	$420 \div 3\frac{1}{3}$
133.	$120 \div \frac{15}{16}$	*140.*	$62 \div 3\frac{7}{8}$	*147.*	$144 \div 5\frac{1}{4}$
134.	$18 \div \frac{3\frac{1}{3}}{3}$	*141.*	$300 \div 6\frac{2}{3}$	*148.*	$28 \div 2\frac{2}{3}$
135.	$11 \div \frac{5\frac{5}{24}}{2}$	*142.*	$105 \div 7\frac{1}{2}$	*149.*	$120 \div 12\frac{1}{2}$
136.	$11 \div \frac{6}{25}$	*143.*	$195 \div 9\frac{3}{4}$	*150.*	$1720 \div 18\frac{3}{4}$

Find the value of the following:

151.	$\frac{5}{6} \div \frac{3}{4}$	*158.*	$1\frac{1}{2} \div \frac{2}{3}$	*165.*	$2\frac{1}{2} \div 2\frac{1}{3}$
152.	$\frac{9}{16} \div \frac{3}{4}$	*159.*	$2 \div \frac{3}{4}$	*166.*	$3 \div 2\frac{1}{5}$
153.	$\frac{1}{4} \div \frac{1}{8}$	*160.*	$3\frac{1}{3} \div \frac{5}{6}$	*167.*	$6\frac{2}{3} \div 3\frac{1}{3}$
154.	$\frac{9}{24} \div \frac{3}{4}$	*161.*	$6\frac{1}{4} \div \frac{5}{7}$	*168.*	$8\frac{1}{3} \div 2\frac{1}{2}$
155.	$\frac{15}{16} \div \frac{5}{6}$	*162.*	$12\frac{1}{2} \div \frac{3}{8}$	*169.*	$16\frac{2}{3} \div 6\frac{2}{3}$
156.	$\frac{9}{16} \div \frac{5}{6}$	*163.*	$16\frac{2}{3} \div \frac{5}{6}$	*170.*	$18\frac{3}{4} \div 6\frac{1}{4}$
157.	$\frac{8}{128} \div \frac{1}{12}$	*164.*	$21\frac{7}{8} \div 2$	*171.*	$202\frac{1}{4} \div 125$

$7\frac{1}{2}$ is what part of $9\frac{1}{4}$?

SOLUTION.—The required relation is expressed by

$$\frac{7\frac{1}{2}}{9\frac{1}{4}} = 7\frac{1}{2} \div 9\frac{1}{4} = \frac{15}{2} \div \frac{37}{4} = \frac{15}{2} \times \frac{4}{37} = \frac{30}{37}$$

Such expressions as $\frac{7\frac{1}{2}}{9\frac{1}{4}}$, $\frac{\frac{5}{6}}{\frac{3}{8}}$, $\frac{5\frac{1}{2}}{8}$, $\frac{9}{\frac{3}{5}}$, etc., have by certain authors been termed *complex fractions*. These so-called "fractions" are best treated as the expressions of unperformed divisions.

WRITTEN EXERCISE

172. $19\frac{1}{2}$ is what part of 25?

173. 47 is what part of $65\frac{1}{2}$?

174. $28\frac{1}{4}$ is what part of $42\frac{1}{2}$?

175. What part of $19\frac{3}{4}$ is $13\frac{2}{7}$?

176. What part of 100 is $13\frac{2}{3}$?

177. Find the value of $\frac{8\frac{1}{2}}{9\frac{3}{5}}, \frac{7\frac{1}{6}}{14\frac{1}{2}}, \frac{21\frac{1}{3}}{25}$

178. Reduce to simplest form $\frac{\frac{4}{9}}{7\frac{1}{3}}, \frac{9}{5\frac{1}{2}}, \frac{16\frac{1}{2}}{19\frac{1}{5}}$

ORAL TEST PROBLEMS

1. Seven and one-fourth acres of land was sold for \$87; what was the price per acre?

2. At $\$7\frac{1}{2}$ per ton, how much coal can be bought for \$25?

3. If 81 bushels of wheat was harvested from a lot containing $4\frac{1}{2}$ acres, what was the yield per acre?

4. At $1\frac{1}{4}¢$ per lb., how much bran should be sold for \$1?

5. From a piece of land containing 10 acres, $3\frac{1}{2}$ acres of the land was sold, what part of the original lot remains unsold?

6. Three men working together for $5\frac{1}{2}$ days cut 22 cords of wood. What was the average per day for each man?

7. At $3\frac{1}{2}¢$ per lb., how much buckwheat flour can be bought for \$7?

8. From a piece of cloth containing $15\frac{3}{4}$ yd., $10\frac{1}{2}$ yd. was sold; what part of the cloth remains?

9. A can do a certain piece of work in 12 days and B in 15 days. In how many days can both do it, working together?

10. How many chestnuts are worth 10 walnuts if 6 chestnuts are worth 12 walnuts?

11. If Frank can cut a cord of wood in 2 days, and George can cut a cord in 3 days, how long should it take both of them to cut 5 cords?

12. If $\frac{3}{4}$ of a barrel of pork was sold for \$13.15, what is the value of 3 barrels?

SUGGESTION.—3 barrels is 4 times $\frac{3}{4}$ barrels.

13. What is the value of 5 acres of land, if $\frac{5}{8}$ of an acre is sold for \$13.50?

14. When hay is $8\frac{1}{2}$ a ton, what part of a ton can be bought for $6\frac{1}{4}$?

15. If a retail dealer sells 75-pound sacks of coal for $1, what does he receive per ton?

16. Charles earns $12\frac{1}{2}$ per week, spends $9\frac{1}{4}$ for expenses, and puts the rest in the bank. What part of his salary is he saving?

17. If 375 lb. flour was bought for $12, what is the price per hundred?

SUGGESTION.—375 lb. equals $3\frac{3}{4}$ hundreds.

18. A man paid $41\frac{1}{2}$ for 750 lb. hay. What is the price per ton?

19. Walter is $2\frac{1}{2}$ times as old as Frank, and the sum of their ages is 21 years. How old is each?

20. James is $2\frac{2}{3}$ times as old as Harvey, and he is 15 years older than Harvey. How old is each?

21. If a train runs 49 miles in 2 hr. 20 min., what is the running rate per hour?

22. If a jar containing $6\frac{1}{4}$ lb. butter was sold for $2, what should be paid for a jar containing $4\frac{1}{2}$ lb.?

23. If a certain brand of coffee is retailed at $3\frac{1}{2}$ lb. per $1, what should be paid for 8 lb.?

24. If $3\frac{1}{2}$ was paid for $\frac{7}{10}$ of a ton of coal, what should be paid for $\frac{3}{5}$ of a ton?

25. Henry is $\frac{1}{3}$ older than Clyde, and the sum of their ages is 28 years. How old is each?

26. A merchant sells suits for $30, which is $\frac{1}{4}$ more than they cost; what does he gain on each suit?

27. By selling oranges at $1\frac{3}{5}$ a box, a fruit dealer loses $\frac{1}{5}$ of the cost. What is the cost?

28. A piece of cloth containing $2\frac{5}{8}$ yd. was sold for $3\frac{1}{2}$. How many yards should be sold for $5?

29. Twice Arthur's money is three times Frank's, and both have $20. What sum has each?

30. If $7\frac{1}{2}$ is paid for $\frac{7}{16}$ of an acre of land, what amount of land should be bought for $17\frac{1}{2}$?

WRITTEN TEST PROBLEMS

179. What is the sum of $9\frac{2}{3}$, $18\frac{7}{8}$, $24\frac{3}{4}$, $21\frac{5}{6}$, $31\frac{3}{8}$, $14\frac{7}{24}$, and $18\frac{11}{12}$?

180. From the sum of $21\frac{3}{4}$ and $36\frac{7}{8}$ take the sum of $25\frac{2}{3}$ and $16\frac{5}{6}$.

181. What is the total of the sum, difference, product and quotient of $8\frac{3}{4}$ and $5\frac{1}{2}$?

182. The sum of three numbers is $175\frac{15}{16}$, two of the numbers are $26\frac{1}{4}$ and $120\frac{7}{8}$ respectively. What is the other number?

183. The difference between two numbers is $87\frac{1}{2}$, the larger is $256\frac{1}{4}$. What is the less number?

184. The product of two numbers is $364\frac{3}{4}$, one of the numbers is $36\frac{1}{2}$. What is the other number?

185. The quotient is $12\frac{2}{3}$, the divisor is $8\frac{3}{4}$. What is the dividend?

186. The dividend is $316\frac{1}{3}$, the quotient is 15, the remainder is $3\frac{5}{6}$. What is the divisor?

187. If a man earns $\$3\frac{1}{4}$ per day, how much will he earn in $25\frac{1}{2}$ days?

188. If a man earns $\$2\frac{3}{4}$ per day, how long will it take him to earn $\$23\frac{3}{8}$?

189. There are $268\frac{4}{5}$ cubic inches in one gallon, dry measure. How many cubic inches in $\frac{3}{4}$ of a gallon?

190. At $\$2\frac{1}{2}$ per day, how much will 11 masons earn in $7\frac{1}{2}$ days?

191. I gave $\frac{1}{4}$ of my money to A, $\frac{1}{3}$ to B, and $\frac{1}{5}$ to C. What part of my money have I left?

192. If 15 cords of wood cost $\$71\frac{1}{4}$, what will $16\frac{3}{4}$ cords cost?

193. If $7\frac{1}{2}$ cords of wood cost $\$35\frac{5}{8}$, how many cords can be bought for $\$83\frac{3}{4}$?

194. I bought a horse for \$125 and found that I had $\frac{3}{5}$ of my money left. How much had I at first?

195. I paid \$7500 for a house and lot. The house cost $1\frac{1}{2}$ times as much as the lot. Find the cost of each.

196. A man invested $\frac{1}{3}$ of his money in real estate, $\frac{1}{2}$ in bonds, $\frac{1}{8}$ in railroad stock, and put the balance, \$2400, in a bank. How much did he have in all, and how much in each investment?

197. I sold two horses for $140 each. On one I lost $\frac{1}{6}$ of the cost price, and on the other I gained $\frac{1}{6}$. Did I gain or lose by the sale of the horses, and how much?

198. A merchant sold 16 pieces of flannel containing 29^1, 28, 28^3, 26^2, 27^3, 33^3, 36^2, 34, 37^1, 32^2, 30^2, 31, 32^1, 34^3, 35, and 36^2 yards respectively, at $6\frac{1}{4}$ cents per yard. How much did he receive?

199. A farm consists of 6 fields as follows: The first contains $24\frac{3}{4}$ acres, the second $43\frac{1}{3}$ acres, the third $37\frac{1}{2}$ acres, the fourth $14\frac{1}{6}$ acres, the fifth $23\frac{2}{3}$ acres, and the sixth $19\frac{1}{16}$ acres. How much was received for the farm, it being sold at $\$75\frac{3}{4}$ per acre?

200. B and C have $\$1200$; B has $2\frac{1}{2}$ times as much as C. How many dollars has each?

201. A speculator bought 250 acres of land at $\$24\frac{3}{4}$ per acre, and $175\frac{3}{8}$ acres at $\$18\frac{3}{4}$ per acre. He sold $\frac{3}{4}$ of the first piece at a profit of $\$2\frac{1}{4}$ per acre, and $\frac{2}{3}$ of the second piece at $20 per acre. What should be the gain on the whole by selling the balance of the land at $25 per acre?

202. What will $62\frac{1}{2}$ yards of cloth cost, if 25 yards cost $\$6\frac{1}{4}$?

203. A husks $7\frac{1}{2}$ bushels of corn in $\frac{3}{4}$ of an hour, and B husks $5\frac{3}{4}$ bushels in $\frac{3}{5}$ of an hour. How many bushels can both husk in 8 hours?

204. Land averaging $3\frac{1}{2}$ tons per acre will yield how many tons on $32\frac{1}{4}$ acres?

205. A man receives $\$4\frac{1}{5}$ per day, and his daily expenses are $\$2\frac{1}{4}$. How long will it take him to save $156?

206. Find the cost of $11\frac{1}{2}$ tons of coal at $\$4\frac{1}{5}$ per ton with drayage at $\$\frac{1}{6}$ per ton?

207. A merchant bought 200 crates of peaches at $62\frac{1}{2}$ cents per crate. He sold $\frac{1}{4}$ of them at an advance of 11 cents per crate, $\frac{1}{2}$ of the remainder at $87\frac{1}{2}$ cents per crate, and the balance at a loss of $4\frac{1}{4}$ cents per crate. How much did he gain?

208. A farmer exchanged $10\frac{1}{2}$ tons of hay at $\$9\frac{3}{4}$ per ton, for coal at $\$5\frac{1}{4}$ per ton. How many tons of coal did he receive?

209. Bought $48\frac{1}{2}$ cords of wood at $\$7$ per cord; sold $\frac{1}{2}$ of it at $\$7\frac{1}{2}$ per cord, and the balance for $175. How much did I gain?

210. A speculator invested $\frac{1}{2}$ of his money in real estate, $\frac{1}{2}$ of the remainder in city bonds, $\frac{1}{2}$ of the second remainder in rail-

road stocks, and the balance, $7500, was deposited. How much had he at first?

211. Bought $9\frac{1}{2}$ yards of silk at $\$1\frac{3}{4}$ per yard, $4\frac{3}{4}$ yards of cassimere at $\$2\frac{1}{2}$ per yard, $22\frac{1}{2}$ yards of cashmere at $\$1\frac{1}{4}$ per yard, and paid for it all in wood at $8 per cord. How many cords were required?

212. A grocer purchased 12 firkins of butter weighing respectively $62\frac{1}{2}$, $60\frac{3}{4}$, $61\frac{1}{4}$, $63\frac{3}{8}$, $58\frac{1}{4}$, $59\frac{9}{16}$, $49\frac{1}{16}$, $63\frac{7}{8}$, $65\frac{5}{16}$, $60\frac{3}{4}$, $62\frac{1}{8}$ and $64\frac{3}{8}$ pounds, at $16\frac{3}{4}$ cents per pound. What is the total cost, allowing $5\frac{1}{4}$ pounds for each tub?

213. A man performed $\frac{1}{4}$ of his journey the first day, $\frac{1}{2}$ of the remainder the second day, and found that he still had $36\frac{3}{8}$ miles to go. How far did he travel the first day?

214. A wholesale merchant sold 32 pieces of denim as follows: 44^1, 45^1, 47^1, 48^2, 43^2, 42^3, 41^3, 44^1, 52^1, 53^2, 54, 52, 48, 49^3, 41, 42^2, 50, 48^1, 46, 51^1, 52, 52^3, 53, 47^2, 49^3, 53^2, 43^1, 48^2, 49^1, 48^3, 52^1, 51^2, at $12\frac{1}{2}$ cts. per yard. How much did he receive?

215. I sold 5 jars of butter weighing $35\frac{1}{2}$, $32\frac{3}{4}$, $31\frac{7}{8}$, $29\frac{5}{8}$, and $36\frac{1}{4}$ pounds respectively, at $18\frac{3}{4}$ cents per pound. The jars weighed $5\frac{1}{2}$, $6\frac{1}{4}$, $5\frac{5}{16}$, $6\frac{1}{4}$, and $7\frac{7}{8}$ pounds respectively. I received in payment 20 pounds of coffee at $33\frac{1}{3}$ cts. per pound, 5 pounds of tea at $62\frac{1}{2}$ cts. per pound, and the balance in sugar at 4 cts. per pound. How many pounds of sugar did I receive?

CHAPTER IX

DECIMALS

186. The student has already learned (17) that a decimal is an abbreviated expression of a fraction whose denominator is some power of ten, as 10, 100, 1000, 10000, etc. Thus, $\frac{9}{10}$, $\frac{7}{100}$, $\frac{16}{1000}$, and $\frac{125}{10000}$, may be written, .9, .07, .016 and .0125 respectively.

187. The period (.) that is used to indicate a decimal is called a decimal point. The number of figures to the right of the decimal point equals the number of ciphers in the denominator of the decimal.

188. If there are more ciphers in the denominator of the decimal than there are figures in the numerator, the required places are made up by prefixing ciphers. Thus, $\frac{13}{10000}$ is written, .0013.

189. In order to read decimals readily the student should learn thoroughly the following:

TABLE OF DECIMAL PLACES

One decimal place indicates tenths.

Two decimal places indicates hundredths.

Three decimal places indicates thousandths.

Four decimal places indicates ten thousandths.

Five decimal places indicates hundred-thousandths.

Six decimal places indicates millionths.

NOTE.—Decimals beyond millionths seldom occur in practical arithmetic.

190. The unit value of the figures in a decimal increases in a ten-fold ratio *from right to left* as in whole numbers. Thus, in the decimal, .123456, beginning at the right, the figures and their unit values are as follows:

6 millionths; 5 hundred-thousandths; 4 ten-thousandths; 3 thousandths; 2 hundredths; 1 tenth.

191. A *mixed decimal* is one that consists of a whole number and a decimal, as, 7.8, 15.007, 125.86, etc.

In reading a mixed decimal, read first the whole number then the decimal.

ORAL EXERCISE

Read the following at sight:

SUGGESTION.—First read the numerator as a whole number; then determine the denominator by the number of decimal places. For instance, to read .01046, we say "one thousand, forty-six;" then, as there are five decimal places, we give the denominator "hundred-thousandths." In reading mixed decimals, separate the whole number from the decimal by the word "and." Thus, read 26.016, "twenty-six *and* sixteen thousandths."

.5	.7625	.00625	.365	5.5
.25	.0625	.00075	.101	25.25
.08	.0055	.00001	.3245	125.325
.125	.0005	.325625	.000625	300.065
.025	.32467	.025625	.000025	3000.0245
.005	.03254	.005625	.000005	500.0086

192. A *complex decimal* is one that terminates in a common fraction, as, $.05\frac{1}{3}$, $.106\frac{1}{7}$, $.00\frac{1}{4}$. The fraction is not considered as occupying one of the decimal places, but is read as a fractional part of the last decimal place. Thus, $.01\frac{1}{7}$ is read, "one and $\frac{1}{7}$ hundredths," and $.000\frac{1}{3}$ is read, "one-third thousandths."

Read the following complex decimals:

$.205\frac{1}{6}$	$.0\frac{1}{3}$	$201.005\frac{1}{3}$
$.1358\frac{2}{9}$	$.00\frac{1}{6}$	$1108.0011\frac{1}{3}$
$.00015\frac{1}{7}$	$.000\frac{1}{9}$	$759.0000\frac{1}{6}$

193. When decimals are expressed in words, it is important to note carefully the punctuation. Thus, "eight and one third millionths," is written, $.0000008\frac{1}{3}$; but "eight, and one third millionths," is written $8.0000001\frac{1}{3}$. To avoid confusion, a comma is used to set off the word that indicates the whole number, as, "five hundred two, and seven millionths" (502.000007).

WRITTEN EXERCISE

Write in figures the following decimals:

1. Twenty-one, hundredths.
2. Seven, hundred-thousandths.

NOTE.—Observe the distinction between *"seven, hundred-thousandths,"* and *"seven hundred, thousandths."*

3. Three hundred twenty-seven, thousandths.
4. Three thousand twenty eight, ten thousandths.
5. Four hundred one, millionths.
6. One hundred six, and fifty four, thousandths.

7. Forty, and nineteen, hundred -thousandths.
8. Eight and one third, millionths.
9. Twelve, and one sixth, hundredths.
10. Twelve and one sixth, hundredths.
11. Forty two, and two hundred four, ten-thousandths.
12. Two hundred, and four and one third, millionths.
13. Five hundred two, thousandths.
14. Five hundred, and two, thousandths.
15. Twenty one hundred, and seventeen, hundredths.
16. Eighty nine, and nine thousand six, hundred thousandths.

REDUCTION OF DECIMALS

194. The principles governing the reduction of decimals are the same as those governing the reduction of common fractions. (162 Prin. III.)

195. A decimal is reduced to a common fraction, simply by supplying its denominator. Thus, $.0125 = \frac{125}{10000} = \frac{1}{80}$.

196. A decimal is in its lowest *decimal terms* when there are no ciphers to the right of its numerator. Thus, .1245, and .008 are in their lowest decimal terms. A decimal having ciphers at the right of its numerator is reduced to its lowest decimal terms by dropping the ciphers. .18100 = .181. Since $\frac{18100}{100000} = \frac{181}{1000} = .181$. Dropping a cipher from the right of a decimal is equivalent to dividing both terms by 10; dropping two ciphers, divides both terms by 100 and so on.

Conversely, annexing a cipher to the right of a decimal is equivalent to multiplying both terms by 10, or reducing it to higher terms. Thus, $.12 = \frac{12}{100}$ and $.120 = \frac{120}{1000}$. Annexing two ciphers multiplies both terms by 100 and so on.

197. A decimal is reduced to any required higher denomination by annexing the required number of ciphers. Thus, .175 is changed to millionths by annexing three ciphers. (.175000.)

It follows, therefore, that decimals not having the same denominator, may be reduced, or changed to equivalent decimals having the same denominator, by the simple process of annexing or dropping ciphers. Thus, .121 and .00149 are reduced to a common denominator by annexing two ciphers to .121 = .12100. Also, .2500 and .19 may be reduced to a common denominator, either by dropping the ciphers in .2500 (.25), or by annexing two ciphers to .19 (.1900).

198. From the foregoing it is evident that dropping ciphers from the right of a decimal or annexing them to it does not affect the value of the decimal.

WRITTEN EXERCISE

Reduce the following decimals to common fractions in their lowest terms:

17. .16; .125; .95; .0025; .075; .0144; .00625; .0288.

18. Change the following to ten-thousandths:

.6; .15; .125; .01; .004; .567000; .001000; .007.

19. Change the following to decimals, having a common denominator (that is, having the same number of decimal places):

.11; .0057; .9; .12345; .650000; .009; .0056; .000160.

20. Read the following, then reduce to their lowest *decimal* terms and read:

.8600; .00120; .7000; .85000; .000100; .010100.

NOTE.—Ordinarily, in reading decimals with ciphers to the right, the ciphers are not taken into account. Thus, .07500 would be read simply "seventy-five thousandths."

REDUCING COMPLEX DECIMALS TO FRACTIONS

199. This process is illustrated in the following problem and solution.

EXAMPLE: Reduce $.83\frac{1}{3}$ to a fraction.

OPERATION

$.83\frac{1}{3} = \frac{250}{3}$

$\frac{250}{3} \div 100 = \frac{250}{300} = \frac{5}{6}$

EXPLANATION.—The mixed number which is the numerator of the decimal is reduced to an improper fraction. We then divide this fraction by the denominator of the decimal, by annexing to the denominator as many ciphers as there are in the denominator of the decimal. Finally, we reduce this fraction to its lowest terms.

WRITTEN EXERCISE

Reduce to fractions in their lowest terms, the following complex decimals:

21. $.75\frac{1}{6}$	*25.* $.58\frac{1}{3}$	*29.* $.66\frac{2}{3}$
22. $.08\frac{2}{3}$	*26.* $.00\frac{5}{7}$	*30.* $.123\frac{1}{3}$
23. .009	*27.* $.13\frac{1}{3}$	*31.* $.15\frac{7}{7}$
24. $.857\frac{1}{7}$	*28.* $.98\frac{8}{9}$	*32.* $.022\frac{2}{9}$

REDUCING COMMON FRACTIONS TO DECIMALS

200. This process is illustrated and explained in the following problem and solution.

EXAMPLE: Reduce $\frac{7}{8}$ to a decimal.

OPERATION

$$8)\overline{1000} \quad \frac{7}{8} = \frac{7 \times 125}{8 \times 125} = \frac{875}{1000} = .875$$
$$\overline{125}$$

The operation involves the reduction of a fraction to a given higher denominator (166). Since the higher denominator must be a power of 10, we divide until the division is exact and find that the decimal denominator is 1000 and that the denominator of the fraction is contained in this number 125 times. Hence, to change $\frac{7}{8}$ to a decimal, *both terms* must be multiplied by 125, giving .875 for the required decimal. In practice this operation is conveniently shortened by annexing ciphers to the numerator and dividing by the denominator, thus, 8)7.000

.875

201. Since powers of 10 contain no other prime factors than 2 and 5, it follows that a common fraction *in its lowest terms* cannot be changed to an exact decimal unless its denominator contains no other prime factors than 2 or 5. Thus $\frac{3}{8}$ or $\frac{7}{16}$ or $\frac{3}{5}$ can each be reduced to an exact decimal, but $\frac{2}{3}$ or $\frac{5}{7}$ cannot be, since there is no power of 10 that will exactly contain 3 or 7.

202. Exact division fractions are those that can be changed to exact decimals, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$, $\frac{4}{5}$, etc.

203. Inexact division fractions are those that cannot be changed to exact decimals, as $\frac{5}{7}$, $\frac{1}{12}$, $\frac{4}{11}$, $\frac{7}{15}$, etc.

204. In changing an inexact division fraction to a decimal, the result is always a complex decimal. Thus, $\frac{1}{3} = .3\frac{1}{3}$, $.33\frac{1}{3}$, or $.333\frac{1}{3}$, etc. $\frac{1}{6} = .16\frac{2}{3}$, $.166\frac{2}{3}$, etc.

205. It should be noted that a fraction *not in its lowest terms*, may have a denominator divisible by other prime factors than 2 or 5, and still be reducible to an exact decimal. Thus either $\frac{9}{12}$ or $\frac{6}{15}$ or $\frac{15}{48}$, can be reduced to an exact decimal, since these fractions reduced to their lowest terms, become respectively, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{3}{8}$.

206. If the reduction of an inexact division fraction to a decimal be continued indefinitely, there will always be a figure or series of figures that will be repeated in the decimal. Thus, in

changing $\frac{1}{3}$ or $\frac{5}{6}$ to a decimal, we get a succession of 3's and in changing $\frac{2}{3}$ or $\frac{1}{6}$ we get a succession of 6's. If we reduce the fraction $\frac{1}{11}$ to a decimal of six places, we get $.090909\frac{1}{11}$, the figures 09, being indefinitely repeated. $\frac{1}{7}$ carried out to ten or more places, gives .1428571428 5, etc., the figures .142857 being repeated indefinitely.

207. A **repetend** is the repeated figure or succession of figures, that result from reducing an inexact division fraction to a decimal.

208. The sign + is used to indicate an incomplete decimal, that is, one that may be still further extended. Thus, $\frac{2}{3} = .66+$, or $\frac{1}{7} = .142+$, or $\frac{1}{8} = .12+$.

WRITTEN EXERCISE

33. Of the following fractions, state which will reduce to exact and which to inexact decimals:

$\frac{17}{25}$, $\frac{29}{75}$, $\frac{15}{24}$, $\frac{49}{64}$, $\frac{17}{128}$, $\frac{51}{85}$, $\frac{41}{96}$, $\frac{33}{44}$, $\frac{36}{60}$, $\frac{59}{60}$, $\frac{71}{80}$, $\frac{29}{40}$, $\frac{129}{500}$, $\frac{19}{250}$, $\frac{11}{16}$, $\frac{23}{32}$, $\frac{52}{65}$, $\frac{30}{36}$.

34. Extend the following fractions to four decimal places. Write + after the last figure of each result to show that the decimals are incomplete:

$\frac{5}{7}$, $\frac{11}{13}$, $\frac{15}{23}$, $\frac{5}{12}$, $\frac{1}{6}$, $\frac{28}{79}$, $\frac{116}{223}$, $\frac{185}{300}$.

35. Find the repetend in changing each of the following to decimals:

$\frac{2}{3}$, $\frac{7}{11}$, $\frac{6}{7}$, $\frac{5}{13}$, $\frac{2}{9}$, $\frac{5}{12}$.

36. Extend each of the following to the fifth decimal place:

$.05\frac{1}{6}$, $13.7\frac{1}{11}$, $.00\frac{1}{8}$, $9.0007\frac{1}{9}$, $.14\frac{1}{12}$; $.7\frac{6}{7}$.

37. Reduce each of the following to a common fraction in its lowest terms:

$.13\frac{1}{3}$, $.00\frac{5}{6}$, $2.7\frac{2}{5}$, $.016\frac{1}{2}$, .025, .1248, .0625, .256, 1.28.

38. Change the following to hundredths, and fractions of a hundredth:

$\frac{11}{14}$, $\frac{29}{45}$, $\frac{12}{28}$, $\frac{5}{13}$, $\frac{78}{85}$, $\frac{39}{55}$, $\frac{11}{17}$, $\frac{23}{24}$.

39. \$17 is what decimal part of \$64.20, correct to four places? \$13 is how many hundredths of \$19? How many thousandths? How many millionths?

40. $\frac{7}{9}$ is what decimal part of $\frac{11}{12}$ correct to three places?

41. 79 is what decimal part of 500? of 1000? of 5000? (correct to an exact decimal).

ADDITION OF DECIMALS

209. To add decimals, the several numbers are so written that the decimal points stand in a vertical column. This is necessary in order that like orders may fall in the same vertical column, tenths under tenths, hundredths under hundredths, etc.

EXAMPLE: Find the sum of 23.007, 101.125, .00017, 8.000005, 206.86, 70.1234, .078679.

OPERATION

23.007
101.125
.00017
8.000005
206.86
70.1234
.078679
409.194254

EXPLANATION.—The numbers are written so that the decimal points and figures of the same order fall in the same vertical columns. Unless the missing orders are filled in with ciphers, care must be taken not to add figures of different orders.

WRITTEN EXERCISE

42. Add .325, 300.025, .9375, 32.125462, and 7.5.

43. Add three, and seven tenths; twenty-five, and one-hundred twenty-five, thousandths; ninety-five, millionths; two-hundred-two, ten-thousandths; two-hundred, and two, ten-thousandths.

44. Add $16\frac{3}{4}$, $25\frac{1}{2}$, $13\frac{7}{8}$, $4\frac{7}{16}$, $5\frac{1}{32}$, and $3\frac{5}{8}$.

SUGGESTION.—Change common fractions to decimal fractions before adding.

45. What is the sum of \$$16\frac{1}{2}$, \$$13\frac{7}{8}$, \$$12\frac{3}{4}$, \$$8\frac{3}{4}$, and \$7?

46. What is the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, $\frac{5}{32}$, $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{31}{32}$. Add as common fractions and express the result in a decimal fraction.

47. What is the sum of .125, .75, .625, .5625, .9375, .0025, .075, .00875, and .00005, expressing the result in a mixed number?

48. A farmer has $48\frac{1}{2}$ acres in oats, $52\frac{3}{4}$ acres in wheat, $36\frac{7}{8}$ acres in barley, $4\frac{4}{16}$ acres in potatoes, and $75\frac{5}{32}$ acres in corn. How many acres in all, expressed decimally?

SUBTRACTION OF DECIMALS

210. Decimals are subtracted in the same manner as are whole numbers.

EXAMPLE: From 162.00128 subtract 79.976215.

OPERATION

162.00128
79.976215
82.025065

EXPLANATION.—Write the numbers so that tenths are under tenths, hundredths are under hundredths, etc., and subtract as in whole numbers.

WRITTEN EXERCISE

49. From 32.049 take 14.753.

50. From 4.0325 take .943.

51. From 31.042 take .000025.

52. From 25 thousandths subtract 25 millionths.

53. Subtract 14 dollars 4 cents from 25 dollars 16 cents 7 mills.

54. From one-tenth take one-millionth.

55. From twenty-five, and nine-tenths, subtract twelve, and one-hundred-two, millionths.

56. A merchant having 94.875 yards of silk, sells $36\frac{3}{4}$ yards at one time, and $23\frac{2}{32}$ yards at another time. How many yards has he remaining?

57. A coaling station having 4938.3125 tons of coal delivered $1371\frac{3}{16}$ tons at one time and $2142\frac{1}{8}$ at another. How many tons were left?

58. From $17.012\frac{1}{3}$ subtract $5.87\frac{1}{6}$ correct to the fifth decimal place.

59. A owned a tract of land containing $158\frac{58}{160}$ acres. From this tract he sold a lot containing $29\frac{97}{160}$ acres. Find the number of acres remaining, correct to the fourth decimal place.

MULTIPLICATION OF DECIMALS

211. The product of any series of decimals will have as many decimal places as there are in all the factors. The reason for this is shown in the following solution:

EXAMPLE: Multiply .16 by .021.

OPERATION

$.16 = \frac{16}{100}$

$.021 = \frac{21}{1000}$

$\frac{16}{100} \times \frac{21}{1000} = \frac{336}{100000} = .00336.$

Supplying the denominators and multiplying as in common fractions, we find that the denominator of the product has 5 ciphers, or as many ciphers, as there are in both the denominators of the factors. Hence the decimal point has 5 decimal places, or as many as there are in both factors.

212. Moving the decimal point *to the right,* multiplies the decimal by 10, 100, 1000, etc., according to the number of places it is moved. For instance, $.123 \times 10 = 1.23$; $.123 \times 100 = 12.3$;

$.123 \times 1000 = 123$. This process is equivalent to dividing the denominator of the fraction by 10, 100, 1000, etc., which (162 Prin. II) multiplies the fraction.

Comparing $\frac{123}{10000}$, $\frac{123}{100}$, $\frac{123}{10}$, and 123, it is seen that diminishing the number of decimal places, is equivalent to diminishing the number of ciphers in the denominator and therefore multiplies the decimal.

ORAL EXERCISE

1. Multiply .1 by 10; by 100; by 1000.
2. Multiply .645 by 10; by 100; by 1000; by 10000.
3. 13 is how many times 1.3? .13? .013?
4. Multiply 65.1234 by 100; by 10000.
5. Multiply .0014 by 100000; by 10; by 1000; by 1000000.
6. Multiply 12.5 by one million; by ten thousand; by ten.
7. .08 is what part of 8? of 80? of 800?
8. .17 is how many times .017? 1.7? .17? .0017? .000017?
9. Multiply 12 by .1; by .01; by .001.
10. Multiply the following by .01: 64, \$7.15, 85¢, 19.7 acres, 17.25 lb., \$ .006.
11. Multiply \$18.375 by 100; by .01; by 1000; by .1; by .001; by 10000.
12. State at sight the results of the following multiplications: $.07 \times .009$; $1.5 \times .04$; $\$7.50 \times .02$; $\$9 \times .06$; $.019 \times .005$; $.12 \times 1200$; $\$1.60 \times .06$; $130 \times .007$.

PER CENT

213. The term **per cent** (%) is a commonly used expression, equivalent to "hundredths." Thus, 6 per cent means .06; $7\frac{1}{2}$ per cent is the same as $.07\frac{1}{2}$ or .075.

ORAL EXERCISE

1. Find 8 per cent of \$90; of \$100; of \$12; of 60¢; of 1¢; of \$11; of 11¢; of \$1.20.
2. A man borrowed \$500, paying interest at the rate of 7 per cent a year. What was the amount of interest for 1 year? 5 years? 6 months? for 10 years?
3. A man bought a farm for \$4000 and sold it so as to gain 12% on the cost; for what sum did he sell it?

4. Mr. R's property is assessed at $7000 and the tax rate is $2\frac{1}{2}\%$ of the assessment. What is the amount of his taxes?

5. Express the following in decimals: 9% ; 15% ; 117% ; 1% ; $2\frac{1}{2}\%$; $6\frac{1}{4}\%$; $12\frac{1}{2}\%$; $\frac{1}{4}\%$ ($= \frac{1}{4}$ of 1%, or $\frac{1}{4}$ of .01) ; $\frac{3}{4}\%$.

6. Express the following as common fractions: 25% ; $12\frac{1}{2}\%$; $16\frac{2}{3}\%$; 20% ; 10% ; 6% ; 75% ; 5%.

7. What is 75% of $12? of $40? of $1.20? of 20¢? of 1¢? of 5¢?

214. In money computations a final result is expressed in dollars and cents, omitting fractional parts of a cent. Fractional parts either in common or decimal form are disregarded if less than $\frac{1}{2}$ cent; if greater, the resulting fraction is regarded as an additional cent; thus, $15\frac{1}{3}$ cts. = 15 cts.; $15\frac{1}{2}$ cts. = 16 cts.

WRITTEN TEST PROBLEMS

60. Mr. Wallace's assessment is $6596.15 and the tax rate is .0315. What is the amount of his taxes?

61. A firm borrowed $7500 for 4 months at an interest rate of 7% a year. What is the amount of the interest?

62. Find $.07\frac{1}{3}$ of $95.10.

63. Find the product of $.17\frac{1}{6} \times 5.2\frac{1}{3}$ correct to four decimal places.

64. A firm's books shows debts owed to the firm amounting to $9654.35. It is estimated that 15% of this amount is not collectible. What is the amount collectible?

65. The circumference or distance around any circle is 3.1416 times the diameter, or distance across. If the diameter of a circular park is 640 ft., what is the circumference?

66. According to the reports, a party of French soldiers marched for 10 days at an average of 48.65 kilometers per day. One kilometer equals .621 mi. How many miles did the soldiers march?

67. A tower in Paris is 121.6 meters high. A meter equals about 3.2808 ft. How many feet high is the tower?

68. The standard English pound sterling or sovereign is equal to $4.8665 in American gold. What is the cost in American money, of an invoice of English broadcloth amounting to 748.5 pounds sterling?

69. The Harper Tool Company's sales for one year amounted to $642,594.90. If their net profits average $9\frac{1}{4}\%$ of their sales, what is the amount of their profits?

70. The annual death rate in a certain city is 5.16 persons per 1000. If the population of the city is 35768, what is the number of deaths per year, correct to the nearest unit?

71. Standard United States gold is nine tenths pure, (that is, it consists of 9 parts pure gold and 1 part baser metal, or alloy). If the gold dollar weighs 25.8 grains, how many grains of pure gold are there in a ten dollar gold piece?

DIVISION OF DECIMALS

215. In problems involving the division of decimals, either the dividend or the divisor may be a decimal. Also, in dividing one whole number by another it is often desirable to express the remainder as a decimal part of the quotient.

216. The process of dividing decimals is the same as dividing whole numbers, except in the matter of determining the number of decimal places in the quotient.

217. Case I. *When the divisor is a whole number.*

Study the following solutions:

EXAMPLE: Divide 1.728 by 12.

OPERATION IN FULL	CONTRACTED OPERATION
$1.728 = \frac{1728}{1000}$	$12)1.728$
$\frac{1728}{1000} \div 12 = \frac{144}{1000}$	$.144$
$\frac{144}{1000} = .144$	

EXPLANATION.—The numerator of the decimal is divided by the whole number. The quotient is then to be divided by the denominator of the dividend, 1000. There are therefore three decimal places to point off in the quotient. Hence, in the division of any decimal by a whole number, there must be as many decimal places pointed off in the quotient as there are places in the dividend.

EXAMPLE: Divide 31 by 7, the quotient to be correct to four decimal places.

OPERATION

$7)31.0000$

$4.4285+$

EXPLANATION.—As the quotient is to be extended to four decimal places, we place a decimal point after the dividend and annex four ciphers.

EXAMPLE: $8\frac{1}{3}$ is what part of $9\frac{2}{7}$ correct to four decimal places?

SOLUTION.—The relation is expressed by

$\frac{8\frac{1}{3}}{9\frac{2}{7}} = 8\frac{1}{3} \div 9\frac{2}{7} = \frac{25}{3} \div \frac{65}{7} = \frac{25}{3} \times \frac{7}{65} = \frac{35}{39}$. $35.0000 \div 39 = .8974+$

218. A decimal is divided by 10, 100, 1000, etc., by moving the decimal point as many places to the left as there are ciphers in the divisor. Thus, $123.45 \div 100 = 1.2345$; $.009 \div 1000 = .000009$.

WRITTEN EXERCISE

Perform the following divisions, extending your results to three decimal places:

72. $79.14 \div 9$
73. $101.06 \div 11$
74. $16.5 \div 27$
75. $147 \div 13$
76. $65.005 \div 28$
77. $250 \div 21$
78. $\frac{3}{7} \div \frac{2}{9}$
79. $8\frac{1}{2} \div 7\frac{1}{3}$
80. $7\frac{1}{4} \div 19$
81. $.05\frac{1}{3} \div 3\frac{1}{7}$

Solve the following correct to four decimal places:

82. $5\frac{1}{4}$ is what part of $6\frac{2}{3}$?
83. $15\frac{1}{2}$ is what part of 47?
84. $9\frac{5}{6}$ is what part of $15\frac{1}{4}$?
85. $\frac{7}{9}$ is what part of $\frac{9}{11}$?

219. Case II. *When the Divisor is a Decimal.*

Study the following solutions:

EXAMPLE: Divide 9.375 by .25.

$$25)\overline{937.5}$$
$$37.5$$

The divisor is changed to a whole number by moving the decimal point two places to the right. This multiplies the divisor by 100 (212). The decimal point of the dividend is also moved the same number of places to the right. This multiplies the dividend by 100. Multiplying both divisor and dividend by the same number does not change the value of the quotient (98 Prin. V). The division is then performed as in Case I.

EXAMPLE: Divide 32.97 by .007.

OPERATION

$$7)\overline{32970}$$
$$4710$$

EXPLANATION.—Here, in order to reduce the divisor to a whole number, the decimal point is moved three places to the right. To remove the decimal point of the dividend three places to the right, one cipher must be annexed.

GENERAL RULE FOR DIVISION OF DECIMALS

220. The following rules apply to all decimals.

I. *If the divisor is a decimal, reduce it to a whole number by moving the decimal point the required number of places to the right.*

II. *Move the decimal point of the dividend the same number of places to the right, annexing ciphers if necessary.*

III. *Divide as in whole numbers, pointing off as many decimal places in the quotient as there are decimal places in the dividend, when the division is performed.*

221. If the divisor is a whole number ending in ciphers, shorten the operation, as in simple division, by dropping the ciphers and moving the decimal point of the dividend the same number of places to the left. (See 218.)

EXAMPLE: Divide 7.2 by 600.

OPERATION

$$6).072$$
$$.012$$

EXPLANATION.—As there are two ciphers to be dropped from the divisor, the decimal point of the dividend is moved two places to the left, the dividend, 7.2, becoming .072. This is equivalent to dividing both dividend and divisor by 100, which (98 Prin. V) does not affect the quotient.

WRITTEN EXERCISE

Perform the following divisions. Incomplete answers should be correct to three decimal places:

86.	$.84375 \div .25$	*91.*	$.635 \div .000005$	*96.*	$.08\frac{1}{3} \div .2\frac{2}{3}$
87.	$.855 \div 2.25$	*92.*	$.1728 \div 1.2$	*97.*	$.005\frac{1}{4} \div 12.5$
88.	$9.45 \div .075$	*93.*	$324.36 \div .0009$	*98.*	$.17\frac{1}{3} \div .008\frac{1}{7}$
89.	$94.3 \div .0004$	*94.*	$.02135 \div .007$	*99.*	$21.432 \div .00008$
90.	$6.15 \div 3500$	*95.*	$124.8 \div 9000$	*100.*	$132.56 \div 2900$

BUYING OR SELLING BY THE HUNDRED OR BY THE THOUSAND

222. Many commodities are bought and sold in the markets, at so much per hundred, or per hundred pounds. A hundred pounds in commerce is called a "hundred weight" (cwt.) or "Cental" (C). Lumber is sold by the thousand board feet, and shingles and some other articles, are sold by the thousand (M).

ORAL EXERCISE

1. When flour sells at $3.50 per cwt., what is the price per lb.?

2. What is the cost of 25 thousand shingles at $2.50 per M?

3. What is the cost of 140 lb. nails at $2.50 per cwt.?

4. At $8 per M, how many bricks can be bought for $50?

5. Tiles at \$9 per M are worth how much per hundred? How much per piece?

6. What will 15 bunches of shingles cost (4 bunches to the M) at \$6 per M?

7. At \$25 per M, how many feet of clear pine lumber can be had for \$40?

8. What is the cost of 2400 feet of boards at \$15 per M?

9. What is the cost of 450 lb. hay at 80¢ per hundred?

10. What is the charge for 250 lb. mess pork at \$11 per cwt.?

11. Find the storage charge on 60,000 lb. wheat at $2\frac{1}{2}$¢ per C.

12. At 35¢ per cwt., what is the freight charge on 450 lb. of household goods?

WRITTEN EXERCISE

101. At \$1.80 per cwt., what is the cost of 1275 lb. nails?

102. Find the freight charge on a car-load of Holyoke paper, wt. 21,560 lb. at \$1.16 per hundred.

103. Find the total cost of the following bill of lumber:

1275 feet pine flooring	at \$26.75	per M
2500 feet rough pine	at \$14.50	per M
648 feet hemlock	at \$18.25	per M

104. Find the value in U. S. money of a shipment of 200 tons of American wheat sold in London at 5 shillings a cental. (1 shilling = \$ .2433.)

105. What is the cost of 3256 brick at \$7.50 per M?

106. In fertilizing a 12-acre field, a farmer used 875 lb. of guano, at \$4.65 per C. What was the cost per acre?

107. What is the freight on a shipment of furniture, wt. 1646 lb., at 38¢ per hundred?

108. In fencing an 80-acre field, the owner used 485 locust posts at \$7.25 per 100, 23,800 feet of fencing boards at \$8.75 per M, and 350 lb. nails at \$3.75 per cwt., and paid \$145.80 for labor. What does the fencing of the field cost per acre?

109. At \$12.25 per cwt., how many pounds of bacon can be bought for \$75?

110. At \$18.50 per M, how many feet of lumber can be had in exchange for 3 beef cattle, total weight 2865 lb., at \$6.50 per cwt.?

111. Through an illegal arrangement with a railroad agent, a firm was permitted to pay $11\frac{1}{2}¢$ per cwt., for its freight shipments instead of $15¢$ per cwt., the regular tariff. If the firm's shipments under the illegal rate amounted to 275,000 lb., what was the amount of their rebate?

ORAL TEST PROBLEMS—REVIEW

1. Divide 1.8 by .09; by .003; by .6; by 20.

2. 17 is what decimal part of 68? of 850? of 51?

3. \$9.10 is .07 of what sum?

4. \$1.20 is 6% of what sum?

5. A man gained 25% of the cost by selling some goods at a profit of \$50. What was the cost?

6. Fred sold his skates for \$2.40 which was .75 of what they cost. What was the cost?

7. Express in decimals the following: 30%; 2%; $2\frac{1}{2}\%$; 125%; $19\frac{1}{4}\%$; $\frac{1}{2}\%$; $\frac{1}{4}\%$; $\frac{1}{8}\%$; $5\frac{1}{8}\%$; $3\frac{3}{4}\%$.

8. \$1.75 is $2\frac{1}{2}\%$ of what sum?

9. \$16 is how many hundredths of \$300? of \$400? of \$500?

10. \$12 is what per cent of \$100? of \$200? of \$60? of \$48?

11. $\frac{1}{8}$ is what decimal part of $\frac{1}{4}$? of $\frac{1}{2}$? of 1?

12. $\$\frac{1}{2}$ is what per cent of \$5? of \$10? of \$8?

13. Change $\frac{5}{8}$ to a decimal.

14. What is the sum of 5%, $7\frac{1}{2}\%$, $8\frac{1}{4}\%$, and $12\frac{3}{4}\%$, expressed as a decimal?

15. A man sold a watch for \$60, gaining 25% of the cost. What did the watch cost?

16. Express the following decimals in terms of per cent: .05; .025; .125; .0925; .06125; .072; .186.

17. \$2.50 is $\frac{1}{8}\%$ of what sum?

18. Frank bought a pony for \$160 and sold it for \$200. What per cent of the cost did he gain?

19. A lady sold subscription books at \$4 each, receiving a commission of 20% of the sale price. What is her commission on each hundred books sold? How many books must she sell to earn \$100?

20. A man received \$12.50 as interest on a sum loaned for 6 months, at the rate of 5% per annum. What sum did he loan?

WRITTEN TEST PROBLEMS—REVIEW

112. Add nine, and nine-tenths; twenty-five, and twenty-five thousandths; five-thousand, and six-hundred-twenty-five ten-thousandths; six-thousand-six-hundred-twenty-five ten-thousandths; five, and five-hundred-five thousandths; forty-eight tenths; and one-hundred, and one-hundred-six millionths.

113. What is the sum of 6.25, 9.5, 300.00035, $4\frac{1}{6}$, $3\frac{3}{4}$, $9\frac{1}{7}$, 3.005, $2.1403\frac{1}{3}$, 41520.325161, $3.0\frac{1}{6}$, and $400.0011\frac{2}{7}$, correct to six decimal places?

114. Reduce to decimals $\frac{7}{8}$, $\frac{9}{16}$, $\frac{3}{32}$, and $\frac{7}{64}$.

115. Reduce to common fractions .125, .0625, $.008\frac{3}{4}$, $.06\frac{2}{3}$, and $.011\frac{1}{9}$.

116. From one, take one-millionth.

117. Find the product of $.13\frac{1}{3}$ and 304.569.

118. Divide 9 by .01; by .001; by .00001.

119. A man spent .55 of his yearly salary, and had $540 left. What was his monthly salary?

120. A has a flock of 720 sheep, and sells .625 of the flock. What is the value of the remainder of the flock at $\$3\frac{1}{4}$ per head?

121. My profits in business for two years were $5312.50. What were my profits each year, if the profits this year were .125 greater than last year?

122. A man invests .375 of his money in real estate, .2 in bonds, .25 in stocks, and deposits the remainder, $3500, in a bank. How much does he invest in each?

123. If $.33\frac{1}{3}$ of 75 cords of wood cost $212.50, what will .75 of a cord cost?

124. A has $1500, which is .25 more than B has. How much have they together?

125. A has $1200, which is $.33\frac{1}{3}$ less than B has. How much have they together?

126. A speculator has 640 acres of land; he sells .25 of it at $43 per acre; $.16\frac{2}{3}$ of the remainder at $30 per acre, as many acres as he sold the first and second times at $\$37\frac{1}{2}$ per acre, and the balance at $24 per acre. How much did he gain if he paid $25 per acre for the land?

127. If 72.125 bushels of corn cost $28.85, what is the cost of one bushel?

128. A man left .375 of his estate to his wife, .4 of the remainder to his daughter, and the remainder to his son, who received $7678.50. How much did the wife and daughter each receive?

129. At $7.50 per ton, what is the cost of 1447 lb. coal?

SUGGESTION.—1.447 ÷ 2 = .7235, the number of tons.

130. Find the exact result of dividing .05$\frac{1}{6}$ by .008$\frac{2}{3}$.

131. 19$\frac{5}{8}$ is what part of 32, correct to three decimal places?

132. $11.25 is how many hundredths (that is, what per cent) of $75?

133. The amount owed to Brown & Co., as shown by their books, is $7587.15. Allowing 7% for bad debts, what sum is collectible?

134. There are 1728 cu. in. in one cubic foot and 231 cu. in. in one gallon. A gallon is what decimal part of a cubic foot, correct to three places? A cubic foot equals how many gallons, correct to three decimal places?

135. The area of the largest circle that can be inscribed in any square is .7854 + of the square. What is the area in square feet (correct to four decimal places) of the largest circle that can be sawed out of a board three feet square?

136. A kilometer equals .621 of a mile; how many kilometers in 1 mile, correct to three places?

137. At $8.25 per M, what is the value of 27 bunches of shingles, each containing 250 shingles?

138. If water weighs 62.5 lb. per cubic foot, and iron weighs 7.25 as much as water, what is the weight of 15 cu. ft. of iron?

139. If the weight of lead is 11.37 times the weight of water, and the weight of pure gold is 19.55 times the weight of water, the weight of a piece of gold would be how many times the weight of a piece of lead of the same size, correct to three decimal places?

140. In a city having a population of 45,968, the deaths in one year were 2169. Find the annual death rate (per cent) correct to two decimal places.

141. A firm's total investment is $21,538, and their net profits for one year were $2795. Find the per cent of profits, correct to hundredths.

142. The metre contains 39.374 inches, and the English yard 36 inches. The yard is what decimal part of the metre, correct to thousandths?

143. The theoretical horse-power of an engine as calculated by the manufacturers is 168.75. The actual horse-power developed on trial was 141.6. What per cent (correct to hundredths) of the theoretical horse-power was lost? What per cent of the theoretical horse-power was developed?

144. The weather records of a California city showed 79 cloudy days during a certain year. What is the per cent of fair days, correct to hundredths?

145. A fast train has been making the run from Chicago to New York in 22 hours. By a new schedule, the run is made in 20 hours. The average speed has been increased what per cent?

146. A farmer sells to a lumber dealer a load of hay, weighing 2158 lb., at $12 a ton, and receives payment in fencing lumber at $9.50 per M. How many feet of lumber should he receive?

147. A firm fails, with liabilities amounting to $15,578. If the assets amount to $11,247, what per cent (correct to thousandths) of their debts can they pay?

148. If the firm referred to in the preceding problem, owe a certain creditor $784.60, what sum would he receive in the settlement?

CHAPTER X

DENOMINATE NUMBERS

223. Quantity is the property by virtue of which a thing is measurable and the measurement expressible by numbers. The term, *quantity*, applies to time, distance, space, velocity, weight, pressure, and many other things that are definitely measureable.

224. Denominate numbers are used to express the measurement of quantities. Thus, 5 pounds, 3 feet, 7 days, are denominate numbers.

225. A **compound number** is a combination of two or more denominate numbers which express the measurement of a single quantity, as 5 ft. 2 in., 7 lb. 2 oz., 2 d. 5 hr. 15 min.

226. A **standard** is a unit of measurement established by law or custom. The yard for measuring distances, the dollar for measuring value, the hand for measuring the height of horses, the day for measuring time, and the volt for measuring the intensity of an electric current are examples of standards.

NOTE.—All standards have been derived from some common, natural objects of measurements, which were used as measures by the people before the standards derived from them were accurately determined and established by law. The following list includes those of special interest:

INCH.—From the *digit* or width of the thumb, used by the Egyptians and other Eastern peoples. In the Roman duodecimal system (counting by 12's) the inch (*uncia*, a twelfth) was the twelfth part of the foot.

Later, in England, the inch was by law determined to be of the length of three grains of dry barley taken from the middle of the ear. The "barley corn" ($\frac{1}{3}$ in.) is still used in determining the "sizes" of shoes.

FOOT.—The length of the human foot.

CUBIT.—The length of the forearm from the elbow to the tip of the middle finger.

YARD.—(From the Norse word *gyrden*, a girdle). Distance around the waist. Later, as prescribed by King Henry III of England, the distance from the nose to the tip of the extended arm.

FATHOM.—(Faethmian, to embrace) the extent of the arms from tip to tip.

GRAIN.—The weight of a plump grain of dry and ripened wheat.

STONE.—The weight of a cobble-stone approximately the size of the human skull. The weight was that of 14 Roman pounds. The stone determined the bushel (4 stone = 56 lbs.) and the gallon which was one-eighth of a bushel. Eight stone made the English "quarter" = 112 lb., and 20 quarters, or 160 stone, the English "long" ton of 2240 lb., which is still used in England, also in America for weighing coal.

CARAT.—The weight of a small Arabian bean.

227. Reduction of denominate numbers is the process of changing a denominate quantity of one or more terms to an equivalent quantity expressed in different terms. Thus, 1 ft. 6 in., may be changed or "reduced" to the equivalent expression, 18 in. or .5 yd. or $\frac{1}{2}$ yd.

Denominate reduction involves no principles other than those of simple and fractional numbers that have already been considered.

228. The tables given in this chapter show the various units, with their relative values, that are employed in commerce and the arts for the measurement of various kinds of quantities. The more important of these tables should be committed to memory. The others are merely for reference. The teacher may designate the ones to be committed.

MEASURES OF EXTENSION

LENGTH

229. In mathematics a **line** is that which has *one dimension*, or *length*. A *straight line* is a line whose direction does not change. It is described as "the shortest distance between two given points."

TABLE OF LINEAR MEASURE

Standard: The Imperial or English yard = 3 feet or 36 inches.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

1 statute mile = 320 rd. = 1760 yd. = 5280 ft.

The inch is diveded into fourths, eights, and sixteenths; also into tenths and hundredths.

MARINE MEASURES

230. Mariners and sea-faring men use certain special units for measuring distances at sea.

The **fathom** = 6 ft., is used for measuring the depth of water.

The **cable length** is used for measuring short distances on the surface of the sea. It may be 120 fathoms (long cable) or 100 fathoms (short cable).

The **knot,** or nautical mile = 1.15 common or statute miles. It is used in coast-survey work, and to indicate the speed of ships.

The **league** = 3 knots, is a popular term used by sailors to indicate surface distances at sea. The league is also used in England to indicate land distances. In this use, it is equal to 3 common or statute miles.

OTHER MEASURES OF LENGTH

231. The following units of measure are also frequently used:

The **hand** = 4 in., is used in measuring the height of horses.

The **cubit** = 18 in., is an old Egyptian measure often referred to in the Bible.

The **pace** or **step** is used by soldiers, and in many rural districts, to indicate short distances. Five paces are considered equal to a rod.

The **furlong** equals 40 rods, or $\frac{1}{8}$ of a mile.

In England and in some parts of America, the terms, *perch, pole,* or *rood,* are used as equivalents for both the linear and the square rod.

ORAL EXERCISE

1. How many inches in 8 ft. 9 in.?

2. How many yards in 6 rods? In $3\frac{1}{2}$ rods? In 33 ft.?

3. How many inches in 2 yd. 2 ft.? In $2\frac{1}{2}$ yds.? In $7\frac{1}{3}$ ft.? In 5.25 ft.? In .125 yd.?

4. How many rods in $1\frac{1}{2}$ mi.? In $\frac{5}{8}$ mi.? In 1.25 mi.?

5. How many inches in 1 yd. 2 ft. 5 in.?

6. How many yards in 72 in.? In 51 ft.? In 390 ft.?

7. What part of a yard is $\frac{5}{6}$ ft.? $\frac{2}{3}$ ft.? .2 ft.? .75 ft.?

8. What part of a foot is $\frac{2}{3}$ in.? $\frac{5}{6}$ in.? .8 in.? .48 in.?

9. How many inches in $\frac{1}{6}$ yd. + $\frac{5}{6}$ ft. + 7 in.?

10. What part of a yard is 15 in.? 24 in.? 16 in.? 9 in.? 21 in.? $\frac{1}{2}$ in.? $\frac{2}{3}$ ft.?

11. A horse that is 16 hands high is how high in feet and inches?

12. A ship is in 14 fathoms of water. What is the depth in feet?

13. The height of a Jewish tabernacle is given as "forty and eight cubits." What was its height in feet?

14. A six-furlong track is how many rods long?

WRITTEN EXERCISE

1. A mountain is 15586 ft. above the sea. What is the elevation in miles, correct to three decimals?

2. If a surveyor's chain is 66 ft. long, how many chains are there to the mile?

3. The battle-ship Connecticut, on her trial run, made a record of $21\frac{3}{4}$ knots in one hour. What was the speed per hour in statute miles?

4. A train running at the rate of 75 miles an hour, runs how many rods a minute? How many feet does it run in a second?

5. Some soldiers fired at a mark 250 paces distant. What was the distance in feet?

6. A floor is 26 ft. 8 in. long and 19 ft. 4 in. wide. What is the distance in feet and inches around the outside of the floor?

7. How many statute miles in one degree, or 60 nautical miles?

8. A farmer fences a field $\frac{1}{4}$ mi. long and $\frac{1}{8}$ mi. wide at a cost of $1.35 per rod. What is the cost of the fencing?

9. How many rods in .277 mi.?

10. The Eiffel tower at Paris is 984 ft. high. What is the height in rods? What is the height expressed as a fraction of a mile?

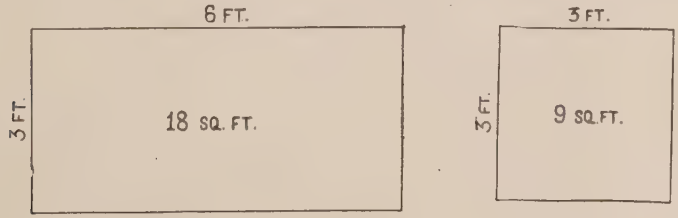
AREA OR SURFACE MEASURES

232. A **surface** in mathematics is that which has length and breadth, but not thickness.

The **area** of a surface is its extent as measured by surface units.

233. The commonest surfaces requiring measurement, are *rectangles*. A rectangle is a four-sided figure, having its opposite sides equal and all its angles right angles.

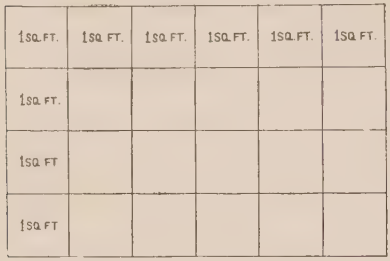
If the four sides of a rectangle are equal, the figure is called a *square*.



234. The units used in the measurement of surfaces, are the square of the corresponding linear units. Thus, the *square foot*, is the square of 1 foot, (12 × 12 in. or 144 sq. in.); the square yard is the square of 3 ft. or 9 sq. ft., and so on.

TO FIND THE AREA OF A RECTANGLE

What is the area in square feet of a surface 6 ft. long and 4 ft. wide?



We may consider the figure as consisting of 4 rows, each containing 6 sq. ft or of 6 rows, each containing 4 sq. ft.

6 sq. ft. × 4 = 24 sq. ft.
4 sq. ft. × 6 = 24 sq. ft.

235. In strictly logical analysis, the distinction as to the unit values of the factors must be observed as in the preceding operation. In practical operations however, the distinction is disregarded, the formula being as follows:

The area of a rectangle is equal to the product of its length and breadth.

236. In describing a rectangle, it is usual to write the dimensions with the sign of multiplication between. Thus, a field 25 rd. long and 15 rd. wide, would be described, "a field 25 × 15 rd."

237. In writing dimensions in feet and inches, carpenters and other artisans, also draftsmen, use the prime (′) to indicate feet and the second (″) to indicate inches. Thus, 6 ft. 8 in. is written 6′ 8″, and a surface 4 ft. long and 11 in. wide, would be indicated 4′x11″.

TABLE OF SURFACE OR SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
30¼	square yards	= 1 square rod (sq. rd. or P.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)

EXAMPLE: How may acres in a field 120x75 rd.?

OPERATION

$$\frac{\overset{3}{\cancel{120}} \times 75}{\underset{4}{\cancel{160}}} = \frac{225}{4} = 56\tfrac{1}{4}$$

EXPLANATION.—The area in acres is equal to the area in rods ÷ 160. The operation is shortened by cancellation.

WRITTEN EXERCISE

11. How many square inches in the surface of a table 2′ 9″ by 2′ 4″?

SUGGESTION.—2′ 9″ = 33″.

12. Find the number of acres in a tract of land $7\frac{1}{2}$ miles long and $3\frac{1}{4}$ miles wide.

SUGGESTION.—The area $= 7\frac{1}{2} \times 3\frac{1}{4} \times 640$.

13. At 75¢ per sq. yd., what will be the cost of a concrete walk 120x6 ft.?

14. A farmer harvested $36\frac{1}{2}$ tons of hay from a field 65x46 rd. What was the yield per acre, correct to two decimals?

NOTE.—Hereafter, compute all inexact results, correct to two decimal places, unless otherwise directed.

15. How many sq. ft. in a floor 12′ 3″x15′ 6″?

16. Property on lower Broadway, New York, was recently sold at a price that equaled $3.50 per sq. ft. At this rate, what was the value per acre?

17. At $1.25 per sq. yd., what is the cost of a concrete cellar floor 13x16 ft.?

18. How many sq. in. in the surface of a square that is 2 ft. 5 in. along each side?

19. Frank's father is to give him $\frac{1}{8}$¢ per sq. yd. for mowing a lawn $5\frac{1}{2}$x12 rd. What sum will Frank receive?

20. A lot 125 ft. square was sold for $250; what was the price per acre?

THE MEASUREMENT OF VOLUMES, OR SOLIDS

238. The **volume** of a body is the space occupied by it.

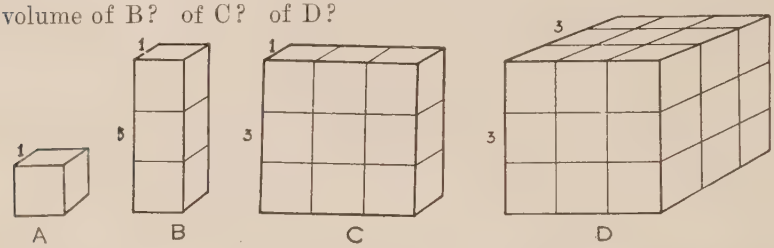
239. A **solid** is that which has the three dimensions, length, breadth, and thickness.

240. A **rectangular solid,** is a solid bounded by six rectangles.

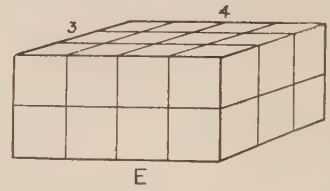
241. A **cube** is a rectangular solid whose surfaces are squares. A cubic inch (cu. in.) is a cube each of whose edges is one inch in

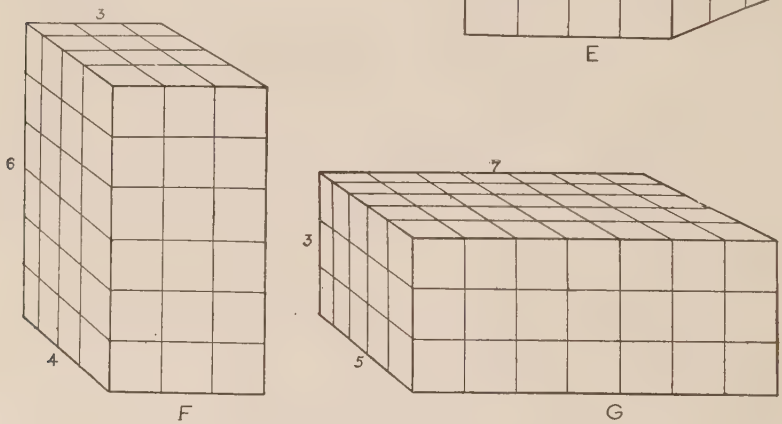
length. In a cubic foot, each edge is one foot, and so on. The volumes of solids are usually measured by the cubic units, 1 cu. in., 1 cu. ft., 1 cu. yd., etc.

If A in the accompanying figures is a cu. in., what is the volume of B? of C? of D?



If each of the small cubes in the several adjacent figures is assumed to be 1 cu. ft., what is the volume of E? of F? of G?





What is the length, width, and height of A?

What is the product of these dimensions?

What is the product of the dimensions of C? of D?

If each unit in D is a cubic foot, what is the product of these dimensions?

How can you find the volume, or solid contents of any rectangular solid ?

Find the number of cubic feet in a cubic yard.

Find the number of cubic inches in a cubic foot.

TABLE OF SOLID OR CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

WRITTEN EXERCISE

21. How many cu. ft. in a block of granite 9′ 3″x4′ 2″x3′ 6″?

22. If there are 231 cu. in. in a gallon, find the contents in gallons of a box that is 3x2x1½ ft.? (Two decimal places.)

23. At 35¢ per cu. yd., what will be the cost of excavating a cellar that is 16x18 ft. and 8 ft. deep?

24. If water weighs 62½ lb. per cu. ft., what will be the weight of the water required to fill a box tank 12′x5′ 6″x3′ 4″?

SUGGESTION.—Problems of this kind may often be greatly shortened by writing the factors in form for cancellation. Thus,

$$\frac{12 \times 11 \times 10 \times 125}{2 \times 3 \times 2} = \text{wt.}$$

25. How many cubic inches in a bar of gold 12½x2¼x2½ in., carried to the exact decimal?

26. How many cu. ft. in a wall of masonry 15 yds. long, 7 ft. high, and 16 in. thick?

27. If a person breathes 19 cu. ft. of air per hour, in how many hours (2 decimal places) will it require two persons to breathe all of the air in a room 12x10x9 ft.?

28. If water weighs 62.5 lb. to the cu. ft. and iron is 7½ times as heavy as water, what is the weight of a piece of iron 28″x12″x4″?

29. What will it cost to dig a drainage ditch 1 mile long, 16 ft. wide, 4 ft. deep, at 25 cents per cubic yard?

30. How many cubic feet in a box car measuring inside 40 ft. long, 8 ft. 6 in. wide, and 8 ft. 9 in. high?

MEASURES OF CAPACITY

242. By the **capacity** of a vessel or other containing body is meant its interior volume or contents. Measures of capacity are used for measuring liquids, seeds, grains, and other mobile products. Many of these are also measured and sold by weight.

243. There being no fixed natural standards for measures of capacity, the latter have been derived from the standards of volume or weight.

The *gallon* and *bushel* have been the standard measures of capacity from very early times, but there has been much confusion in determining them. At one time there were no less than six different "gallons" used in Great Britain. The bushel was also subject to much variation. The standard bushel of the United States is the English "Winchester bushel" of 2150.42 cubic inches.

The U. S. standard gallon is the so-called "wine-gallon," having the capacity of a cylindrical vessel 7 inches in diameter, and 6 inches high, equal to 231 cubic inches. The "beer gallon" (282 cu. in.) and the "dry gallon" (268.8 cu. in.) formerly in use, are now seldom or never used in America.

The imperial bushel and gallon, now used in England and Canada, differ from the United States standards. The Imperial bushel equals 2218.192 + cu .in., and the gallon is $\frac{1}{8}$ of a bushel, or 277.274 + cu. in.

DRY MEASURE

244. Dry measure is used for measuring grain, seeds, fruits, vegetables, and other dry products.

The dry pint and quart are now seldom used. When retailers sell cranberries or other small fruits by the quart, the liquid quart, ($\frac{1}{4}$ of a wine gallon) is generally used. Most small fruits are sold either by the box or by the pound. Of the articles sold by dry measure, quantities less than a bushel are sold by the peck or half-peck. On the Pacific coast of America, dry measure is little used, most agricultural products being sold by the pound. Apples, oranges, and other fruits are sold by the box.

TABLE OF DRY MEASURE

Standard: The Winchester bushel = 2150.42 + cu. in.

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

LIQUID MEASURE

245. Liquid measure is used in measuring all liquids except liquid medicines in compounding prescriptions, also for determining the capacity of tanks, cisterns, reservoirs, etc. The barrel as a measuring unit to indicate the capacity of tanks, etc., has the fixed capacity of 31½ gallons, but the various casks used in commerce, called barrels, hogsheads, tuns, tierces, etc., vary in size, and their capacity is determined by measuring or gauging.

TABLE OF LIQUID MEASURES

Standard: The wine gallon = 231 cu. in.

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
31½ gallons	= 1 barrel (brl.)
63 gallons } 2 barrels }	= 1 hogshead (hhd.)

WRITTEN EXERCISE

31. Find the capacity in gallons of a box 2 ft. 6 in. by 1 ft. 4 in. by 11 in. (Carry all inexact results to two decimal places.)

32. The capacity of a cask is 42.8 gal. What is its capacity in bushels?

33. A United States, or Winchester bushel is what decimal part of an imperial bushel? (243.)

34. A circular bin holds 568 bu. What is its capacity in barrels?

35. A dealer bought a 42½ gallon cask of syrup at 75¢ a gallon, and retailed it at 25¢ a quart. What was his gain, the freight charge being $2.60?

36. A grocer bought 15 brl. apples, 2¾ bu. to the barrel, at $5.25 a barrel, and retailed them at 65¢ a peck. What is his profit on the entire lot?

37. A wheat bin is 10′ 8″x5′ 3″x4′ 4″. How many bushels will it hold?

38. A cubic foot is what decimal part of a bushel, correct to four decimals?

39. How many gallons in a cubic foot?

40. How many wine gallons in a bushel?

MEASURES OF WEIGHT

246. Weight is the measure of the earth's attraction for objects at its surface. It diminishes as objects are carried below or above the earth's surface.

The **standard** of weight of the United States and England is the Troy pound of 5760 grains, but for commercial purposes, the Avoirdupois pound of 7000 grains, is commonly used.

TROY WEIGHT

247. This is the system used by jewelers, and by the government mints, for weighing coins and gold, silver, or other valuable metals.

TABLE OF TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

1 lb. Troy = 5760 grains.

Diamonds, pearls, and other jewels are weighed by carat-weight. The standard is the *carat* = $3\frac{1}{5}$ grains Troy. This is divided into eighths, sixteenths, thirty-seconds, and sixty-fourths. Fourths of a carat are called "carat-grains." A carat-grain is little more than $\frac{3}{4}$ of a Troy grain.

AVOIRDUPOIS WEIGHT

248. This system is used for weighing all ordinary articles that are sold by weight, as grains, meats, groceries, and drugs not compounded according to prescriptions.

The avoirdupois pound is equal in weight to 7000 Troy grains.

TABLE OF AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T.)

The long or British ton is 2240 lb. It is divided into 20 hundred-weight (cwt.) each 112 lb. avoir. The hundred-weight consists of 4 "quarters" each 28 lb.

In Great Britain much grain is sold by the quarter. The long ton is used in U. S. custom-house business, and in the eastern states in buying, selling, and mining coal. Coal is measured and sold by the short ton throughout the central and western states.

COMPARISON OF TROY AND AVOIRDUPOIS WEIGHTS

TROY	AVOIRDUPOIS
1 lb. = 5760 gr.	1 lb. = 7000 gr.
1 oz. = 480 gr.	1 oz. = 437.5 gr.

NOTE.—"Apothecaries Weight" is a system used by pharmacists in compounding medicine.

WRITTEN EXERCISE

41. A gold nugget weighs 1 lb. 5 oz. on an avoirdupois scales. What is its weight in Troy ounces, carried to three decimals?

42. How many penny-weights and grains in a silver spoon weighing $3\frac{1}{4}$ Troy ounces?

43. A dealer buys three cars of coal, weights, net, 41625 lb., 52156 lb., 47865 lb., at \$4.50 per long ton, pays freight at 67¢ per long ton, and retails the coal at \$5.75 per short ton. What is his profit on the three cars?

44. A New York dealer buys hard coal at \$5.60 per ton, and retails it at 80¢ per bushel of 80 lb. What is his profit per ton?

45. A grain exporter sold a shipment of 28,000 bu. of Kansas wheat in the London market at an equivalent of $68\frac{1}{2}$¢ per quarter. The weight of the wheat being 60 lb. per bu., what is the amount of the sale?

46. What decimal part of an ounce (correct to 3 places) is the weight of a piece of gold weighing 14 pwt. 16 gr.?

47. At \$16 per ounce, what is the value of a gold nugget which weighs 7 oz. 5 pwt. 12 gr.?

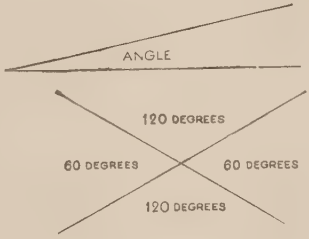
48. The union scale for mining coal in an Illinois mine is 28¢ per short ton. At that rate, what should be paid per long ton?

49. Find the duty at \$2.65 per ton on an importation of Portland cement, weighing 28,567 lb.

50. At the mint price of \$15.25 per ounce, what is the value of a sack of Klondike gold weighing 7 lb. 8 oz. 13 pwt. 18 gr.?

ANGLES, TRIANGLES, AND CIRCLES

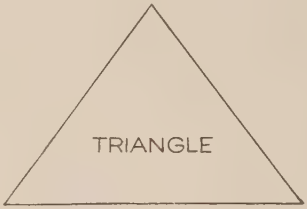
249. An **angle** is the difference in direction between two straight lines that meet.



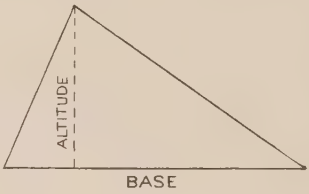
250. Angles are measured in degrees, it being assumed that the entire angular quantity at the point of two intersecting straight lines is equal to 360 degrees. (360°.)

251. A **right angle** is the angle formed by the intersection of two perpendicular lines; it is equal to 90°.

252. A **triangle** is a plain figure bounded by three straight lines. The *base* of a triangle or other figure is the side upon which it is assumed to rest.

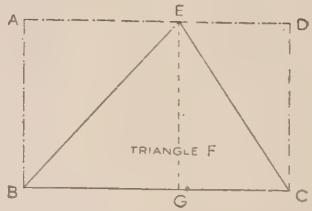


253. The **altitude** of a triangle is the perpendicular distance from its base to its apex or highest point.



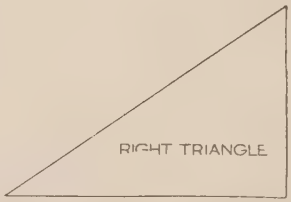
MEASUREMENT OF THE TRIANGLE

254. It may be shown by geometry that the area of any triangle is equal to one-half the area of the rectangle having the same base and altitude. Thus, in the figure, the triangle F is equal to one-half the rectangle A, B, C, D.



It is therefore evident that the area of any triangle is equal to one-half the product of its base and altitude. Thus, if in the figure the line A D is assumed to be 6 ft. and the line C D (altitude of both triangle and rectangle) is 4 ft., the area of the triangle F is equal to $\frac{4 \text{ ft.} \times 6 \text{ ft.}}{2} = 12 \text{ sq. ft.}$

A **right-triangle** or a **right angled triangle** is a triangle having one of its angles a right angle.



Either of the perpendicular sides of a right triangle may be considered the base in which case the other perpendicular side will be the altitude. Hence, *the area of any right triangle is equal to one-half the product of its perpendicular sides.*

ORAL EXERCISE

1. Find the area of a triangle whose base is 15 in. and whose altitude is 1 ft.

2. An angle of 15° is what part of a right angle?

3. The two perpendicular sides of a right triangle are respectively 21 and 18 inches. What is its area?

4. A triangle having a base of 8 ft. must be how high to contain 40 sq. ft.?

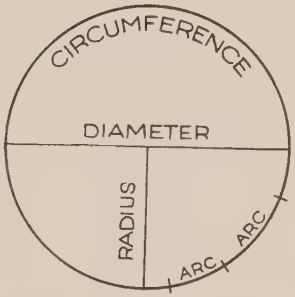
5. The base of a triangular field is 32 rd., and its altitude is 20 rd. How many acres in the field?

6. If a wheel has 8 spokes, what is the magnitude of the angle between any two adjacent spokes?

7. A pie is cut into six equal pieces. What is the angular measurement of each piece?

8. How many degrees in the angle included by the hands of a clock at eleven o'clock? at two o'clock? at five o'clock? at nine o'clock?

THE CIRCLE



255. A **circle** is a plane figure bounded by a uniformly curved line, every part of which is equally distant from a point within called the center.

256. The **circumference** of a circle is the curved line that bounds it.

257. The **radius** of a circle is the distance from the center to the circumference.

258. The **diameter** of a circle is a straight line passing through the center and terminated by two opposite points in the circumference.

An **arc** is any part of a circumference.

259. A **degree** is the one three-hundred and sixtieth part of a circumference.

260. For the purpose of calculations involved in surveying, navigation and other sciences and arts, circles and angles are measured in accordance with the following:

TABLE OF CIRCULAR OR ANGULAR MEASURE

60 seconds of arc (″)	= 1 minute of arc (′)
60 minutes	= 1 degree (°)
30 degrees	= 1 sign (S.)
12 signs	= 1 circle (Cir.)
60 degrees	= 1 sextant
90 degrees	= 1 right angle or quadrant

MEASUREMENT OF THE CIRCLE

261. It may be proved by geometry:

I. *That the circumference of every circle is equal to 3.1416 + (about $3\frac{1}{7}$) times its diameter.*

II. *That the area of every circle is equal to one-fourth the product of its circumference and diameter.*

III. *That the area of a circle is equal to .7854 + (or approximately $1\frac{1}{14}$) of the square of its diameter.*

WRITTEN EXERCISE

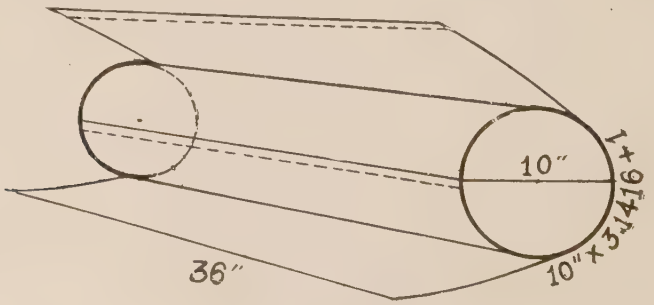
51. How many acres in a circular park 75 rods in diameter?

52. The circumference of a tree is 17 ft. 9 in. What is the diameter in feet and inches?

53. What is the circumference of a circular table whose diameter is 4 ft. 8 in.?

54. The diameter of a circular water-tank is 15 ft. 6 in. What will be the cost of the concrete floor at $1.75 per sq. yd.?

55. How many square feet of sheet iron will be required to make a pipe 10 in. in diameter and 36 in. long, allowing one inch lap for riveting?



TIME MEASUREMENTS

262. All measures of time are derived from the movements of the earth and other bodies of the Solar system. The day is the time required for the earth to make one revolution about its axis, and the year is the time for it to complete a revolution around the sun. The month is derived from the revolution of the moon around the earth.

263. Owing to imperfect means of astronomical observation, the early calendars or systems of chronology were more or less inaccurate, and required subsequent corrections and changes. The Gregorian calendar established by Pope Gregory XIII in 1582, is now used by all civilized nations except Russia. This calendar was the first to conform to the exact solar year, 365 days, 5 hours, 48 minutes, and 49.7 seconds. This period being very nearly $365\frac{1}{4}$ days, three successive years are estimated at 365 days each, and the fourth year at 366 days. This correction of one day in every four years makes a gain, in excess of the true amount, of 11 minutes and 10.3 seconds per year, or about 3 days every 400 years. In order to correct this gain, the extra day is not added to three out of every four centennial years. All years divisible by 4 are leap years, except centennial years, which must be divisible by 400.

In ordinary business calculations, the year consists of twelve months of thirty days each.

The common year contains 52 weeks and 1 day, and every year following a common year begins one day later in the week. The leap year contains 52 weeks and 2 days, and the year following leap year begins two days later in the week.

TABLE OF TIME MEASURE

Standard: One day of 24 hours.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (d.)
7 days	= 1 week (wk.)
365 days 12 months	= 1 common year (yr.)
366 days	= 1 leap year (l. yr.)
100 years	= 1 century (C.)

MONTHS IN THE YEAR

Months	Abbrev.	No.	Days	Months	Abbrev.	No.	Days
January	Jan.	1	31	July	July	7	31
February	Feb.	2	28	August	Aug.	8	31
March	Mar.	3	31	September	Sept.	9	30
April	Apr.	4	30	October	Oct.	10	31
May	May	5	31	November	Nov.	11	30
June	June	6	30	December	Dec.	12	31

264. It is convenient to remember that the 1st, 8th, 15th, 22nd, and 29th days of each month fall on the same day of the week. As a common year consists of 52 weeks and 1 day, and a leap year 52 weeks and 2 days, a date in any year falls *one day later* in the week than the same date in the previous year, unless the prvious year is a leap year, in which case, the date is *two days later*.

The successive centuries are designated by their ordinal numbers, from the beginning of the Christian Era, as "the 6th century" or the one beginning with the year 501, the 20th century or the one in which we now live and which began with the year 1901.

Notice that the years, 1701, 1801, 1901, are respectively the first years of the 18th, 19th, and 20th centuries, and that the years 1700, 1800, and 1900, are respectively the *last* years of the 17th, 18th, and 19th centuries.

ORAL EXERCISE—THE CALENDAR

1. Name the leap years between 1890 and 1900.

2. Name in their proper order the months that have 30 days. Those that have 31 days.

3. July 4, 1908, fell on Saturday. In what year will it fall on Saturday again? In what previous year did it fall on Saturday?

4. How many days elapsed between Feb. 25, 1900, and April 6, 1900?

5. The year 1908 began on Wednesday. On what day of the week did the year 1909 begin?

6. When July 1 is Wednesday, the 10th of the following August will be what day of the week?

7. If March begins on Friday, what days of the month will fall on Sunday?

8. Under what conditions will February have five Sundays?

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

265. The adding of different orders of denominate numbers is rarely required in business, but it is advisable for the student to know the process.

EXAMPLE: Add, 2 lb. 5 oz. 15 pwt. 9 gr.; 5 lb. 14 pwt. 15 gr.; 7 oz. 11 gr.; 1 lb. 12 pwt. 10 gr.

OPERATION

lb.	oz.	pwt.	gr.
2	5	15	9
5	0	14	15
	7	0	11
1	0	12	10
9	2	2	21

EXPLANATION.—Arrange the numbers so that like denominations shall be in the same column. Adding the grains, we get 45. Dividing this by 24 gives 1 pwt. and 21 gr. The 1 pwt. is carried to the next column, the sum of which is 42 pwt. =2 oz. 2 pwt. Carrying the 2 oz. to the next column and adding we have 14 oz.=1 lb. 2 oz. Carrying the 1 lb. to the next column we have 9 lb.

WRITTEN EXERCISE

56. Add, 15 gal. 2 qt. 1 pt.; 4 gal. 3 qt. 1 pt.; 7 gal. 1 pt.; 13 gal. 1 qt. 1 pt.; 5 gal. 3 qt.; 14 gal. 3 qt. 1 pt.

57. Add, 7 T. 175 lb.; 16 T. 1826 lb.; 12 T. 1427 lb.; 9 T. 1356 lb.; 18 T. 748 lb.

58. Add, 48 yd. 1 ft. 5 in.; 21 yd. 2 ft. 11 in.; 9 yd. 2 ft. 9 in.; 14 yd. 1 ft. 7 in.

EXAMPLE: From 15 lb. 10 pwt. 11 gr., subtract 9 lb. 7 oz. 14 pwt. 19 gr.

OPERATION

lb.	oz.	pwt.	gr.
15	0	10	11
9	7	14	19
5	4	15	16

EXPLANATION.—The numbers are written in columns of like orders. If any term of the minuend is less than the corresponding term in the subtrahend, the number of units in the next higher order is added to it. Thus, we have, add 24 gr. to 11 gr. making 35 gr. from which we subtract 19 gr. We then either "carry" one to the next subtrahend term, or diminish by one the next minuend term.

WRITTEN EXERCISE

59. From 20 gal. 1 qt. 1 pt. 1 gi., take 14 gal. 3 qt. 1 pt. 3 gi.

60. From 51 rd. 3 yd. 1 ft. 7 in., take 38 rd. 5 yd. 2 ft. 10 in.

SUBTRACTION OF DATES

266. This is the most important application of compound subtraction. It is used to find the difference between two dates, where a considerable period of time has elapsed.

EXAMPLE: A note was given on Sept. 16, 1903, and paid Mar. 5, 1907. How long did the note run?

OPERATION

yr.	mo.	d.
1907	3	5
1903	9	16
3	5	19

EXPLANATION.—The later date is written so as to express the entire time that had elapsed since the beginning of the Christian Era, or 1907 yr. 3 mo. (March being the 3rd month) 5 days. The earlier date is then expressed in the same way and subtracted as in any other compound subtraction, 30 days being considered a month.

WRITTEN PROBLEMS

61. Abraham Lincoln was born Feb. 12, 1809. What was his exact age when he became president, Mar. 4, 1861? He died April 15, 1865. What was his age on that date, and how long had he served as president?

62. Mr. A had been employed at a salary of $85 per month from Oct. 21, 1899, to Apr. 27, 1905. How much money did he earn during the entire period?

63. California was admitted into the Union on Sept. 9, 1850. How long had California been a state on July 4, 1910?

Find the time between the following dates:

64. Mar. 1, 1900, and Jan. 4, 1904.

65. Apr. 4, 1904, and Feb. 20, 1908.

66. July 15, 1900, and Mar. 24, 1905.

67. Aug. 20, 1906, and July 6, 1911.

68. Sept. 14, 1905, and Mar. 7, 1910.

69. What is the length of time from 20 min. past 9 o'clock, a. m., Aug. 4, 1895, to 36 min. past 11 o'clock, a. m., Nov. 24, 1897?

70. Find the length of time from 26 min. before 10 o'clock, a. m., Oct. 15, 1893, to 26 min. past 9 o'clock, p. m., July 4, 1906.

71. What is the length of time from 25 min. before 10 o'clock p. m., July 13, 1903, to 8 min. past 8 o'clock, a. m., Sept. 24, 1907?

267. When the time is short (less than one year) the difference in time is usually found by counting the exact days.

EXAMPLE: Find the exact time from April 16, 1908, to Nov. 13, 1908.

SOLUTION

14 da. left in Apr.	31 in Aug.
31 in May.	30 in Sept.
30 in June.	31 in Oct.
31 in July	13 in Nov.
	211 days.

WRITTEN EXERCISE

Find the exact time:

72. From Nov. 15, 1909, to Mar. 9, 1910.

73. From Jan. 17, 1910, to Dec. 29, 1910.

74. From Aug. 18, 1909, to May 22, 1910.

75. From Dec. 25, 1908, to Mar. 27, 1909.

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

EXAMPLE: Multiply 36 cu. yd. 8 cu. ft. 750 cu. in. by 6.

OPERATION

	27	1728
cu. yd.	cu. ft.	cu. in.
36	8	750
		6
216	48	4500
217	23	1044

EXPLANATION.—Write the multiplier under the right hand denomination, and multiply as in whole numbers.

If a product is greater than the number of units in the next higher denomination, it is reduced and the number of these units carried to the next product. Thus, the first product, 4500 cu. in. equals 2 cu. ft. and 1044 cu. in. The 2 cu. ft. is added to the next product, 48, which gives 50 cu. ft.; this equals 1 cu. yd. and 23 cu. ft. The 1 cu. yd. is then carried to the next product, giving 217 cu. yd.

WRITTEN EXERCISE

76. Find the weight of 7 bars of gold, each weighing 8 oz. 7 pwt. 15 gr.

77. One side of a square field is 67 rd. 4 yd. 5 ft. 6 in. What is the exact distance around the field?

78. A package of paper is found to weigh 7 lb. $5\frac{1}{2}$ oz. What is the weight of 150 similar packages?

79. A grocer drew off a cask of vinegar, measuring it with a tin vessel that was found to hold 1 gal. 2 qt. $1\frac{1}{2}$ pt. What was the amount of vinegar, if the measure was filled 15 times?

EXAMPLE: Divide 34 bu. 3 pk. 5 qt. 1 pt. by 6.

OPERATION

	4	8	2
6)34 bu.	3 pk.	5 qt.	1 pt.
5	3	2	$\frac{1}{2}$

EXPLANATION.—Divide as in whole numbers, reducing the remainder, if any, to the next lower denomination and continue the division.

WRITTEN PRACTICE

80. Divide 2 mi. 82 rd. 2 ft. 2 in. by 10.

81. Divide $12\frac{4}{5}$ bu. $1\frac{3}{5}$ qt. 2 pt. by 8.

82. Divide 24 yr. 14 da. 25 hr. 30 sec. by 30.

83. Divide $4\frac{3}{4}$ hhd. $4\frac{3}{4}$ gal. by $6\frac{1}{4}$.

84. The total weight of 25 hhd. of sugar is 10 T. 14 cwt. 30 lb. What is the average weight of each hhd.?

85. How many rings each weighing 8 pwt. 10 gr. can be made from a bar of silver weighing 9 oz. 14 pwt. 16 gr.?

NOTE.—To divide one denominate number by another, reduce both divisor and dividend to the same denomination.

86. How many bottles each holding 1 qt. $1\frac{1}{4}$ pt. can be filled from a cask of wine containing 18 gal. 5 qt. 1 pt.?

WRITTEN TEST PROBLEMS—REVIEW

87. A man walked 1 mile in 17 minutes. What was the distance (expressed in lower denominations) that he walked in one minute?

88. A train ran 227 miles between 10:45 a. m., and 2:20 p. m. What was the running rate per hour?

89. A vessel is aground $5\frac{1}{2}$ cable-lengths (long) from shore. What is the distance in rods?

90. A farmer is awarded damages at the rate of $50 per acre, for a 60 foot railway right of way, 96 rd. 10 ft. long, which passes through his farm. What is the amount of the damages?

91. Find the area in square yards of a room 14′ 6″x16′ 8″.

92. How many motto-ribbons, each 1 ft. 4 in. long, can be cut from a bolt containing 36 yd.?

93. Lots 125 ft. deep, fronting on Main St., sold at $12.50 per front foot. What is the price per acre?

94. At $18\frac{1}{2}¢$ per cu. yd., what is the cost of excavating a sewer ditch 128 ft. long, 4 ft. wide, and 5 ft. 6 in. deep?

95. Find the cubical contents of a vessel that is 8 in. in diameter and 11 in. deep.

SUGGESTION.—The contents is equal to the area of the bottom multiplied by the depth.

CHAPTER XI

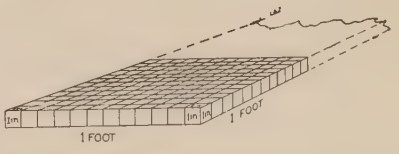
COMPUTATIONS IN THE TRADES AND INDUSTRIES

CONTRACTING, BUILDING, AND FURNISHING

268. Computations under this head involve the measurement of various forms of plane surfaces and solids, and calculations relating to such building materials, as lumber, stone, brick, cement, roofing, tiling, etc., also calculations required in painting, plastering, paperhanging, and carpeting.

LUMBER

269. The measurement unit is the *board foot,* which is the equivalent of a board 12x12 in. and 1 in. thick. The board foot therefore contains 144 cu. in., or $1\frac{1}{2}$ of a cubic foot, and its largest surface is 1 sq. ft.



270. Lumber less than an inch thick is usually counted the same as lumber an inch thick. The lumber-feet in planking an inch or less in thickness, is therefore equal to the square feet of the surface. Thus, the lumber-feet of inch lumber for a floor 12x15 ft. would be $12 \times 15 = 180$ ft. For lumber more than an inch thick, multiply the surface in square feet by the thickness. Thus, the lumber-feet for a walk 40 ft. long and 6 ft. wide to be made of 3-inch planking, would be $40 \times 6 \times 3 = 720$ ft.

271. For lumber more than an inch thick, as it is ordinarily sold in the lumber yards, the following rule offers best method of computation:

Multiply the length in feet by the width and thickness in inches and divide by 12.

EXAMPLE: How many board feet in 20 pieces of 4x3 (that is 4 in. by 3 in) joists, each 18 ft. long?

OPERATION

pc. l. w. th.

$$\frac{20 \times 18 \times 4 \times 3}{12} = 360 \text{ ft.}$$

SUGGESTION.—Lumber calculations can usually be abbreviated by cancellation.

272. Lumber as it comes from the saw mill and as sold in the yards, is in even lengths, that is, 10 ft., 12 ft., 14 ft., 16 ft., etc.

ORAL EXERCISE

1. How many feet of lumber in a block 6 ft. long, 3 ft. wide, and 1 ft. thick?

2. How many feet of 2-inch lumber will be required for a plank walk 20 ft. long and 3 ft. wide?

3. How many board feet in 5 planks of inch lumber, each 12 ft. long and 8 in. wide.?

SUGGESTION.—In inch lumber, *12 feet in length,* the board feet in each plank equals the width in inches.

4. How many board feet in 24 pieces of 2x3 joists, each 10 ft. in length?

5. Find the board feet in 18 planks each 1 ft. wide, 20 ft. long, and 1 in. thick.

SUGGESTION.—In any plank of inch lumber, 1 ft. wide, the length equals the board feet.

6. How much lumber in 15 pieces of 2-inch plank each 12 in. wide and 10 ft. long?

WRITTEN EXERCISE

1. How much lumber in 20 two-inch planks 18 ft. long, and 10 in. wide.? (Described briefly, 20 planks 2x10—18).

2. What will be the cost of 12 planks 2x12—16, 10 planks 3x12—14, 24 joists 6x2—20, 30 scantling 2x4—18, at $22 per M?

3. The following illustrates the method of billing; except that the quantities are not extended in the dimension varieties. The student will copy this bill, entering the quantities in each dimension variety and extending the product (quantity × price) of each variety in the first amount column and the total of the bill in the second column. The fourth item in the completed bill would be: "694 pc. 2x6—16, 11104 ft. @ $25, $277.60."

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Sold to J. J. Melick, Linn Junction, Iowa.

14260	ft. Sheeting	@ 22.		?	
6400	" Flooring #1	36.		?	
8496	" " #2	32.			
694	pcs 2×6–16	25.			
428	" 4×4–12	26.			
88	" 2×4–16.	25.			
160	" 2×8–14	30			
420	" 1×12–14	30.			
950	" 1×16–8	35.			
188	" 6×6–18	28.			
628	" 6×8–24	32.			
20	M. Shingles #2	3.50			
20	" " #1	4.			?

4. What will it cost to enclose a quarter-section of land with a fence 5 boards high, of 1 inch material, the bottom board being 8 inches wide, and the other boards each being 6 inches wide, posts to be placed 8 feet apart; the lumber costing $15 per M, and the posts $21 per C?

5. What will be one-half of the cost of a line fence one mile long, between two farms, the fence to be 5 boards high, of boards 16 ft. long, 8 in. wide, and 1 in. in thickness, the posts to be placed 8 feet apart, at a cost of $16 per M for lumber and $25 per C for posts?

6. At $18 per M for lumber and $22 per C for posts, what will be the cost of lumber and posts for enclosing a lot 40x160 ft. with a picket fence; the pickets being 4 ft. long, 3 in. wide, and 1 in. thick, allowing 3 in. space between pickets, the posts being placed 8 ft. apart, two 2x4's being used as stringers, and a base board 10 in. wide extending below the pickets?

ROOFING

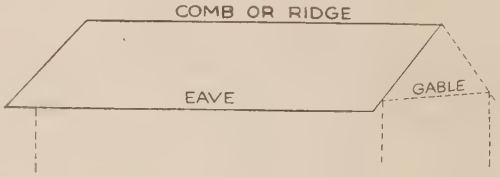
273. Shingles, slates, and tiles are used in roofing, also sheets of tin, zinc, sheet iron, etc.

The *square* (= 100 sq. ft.) is the unit of measurement for roofing.

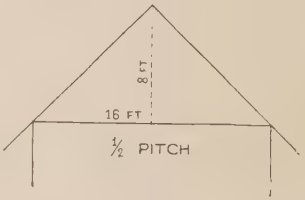
Shingles are put in bundles or "bunches" of 250 each. Shingles vary in width, but the average width is 4 in. Two lengths are used, 16 in. and 18 in. The former are laid $4\frac{1}{2}$ in. to the weather, and the latter $5\frac{1}{2}$ in.

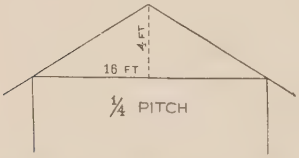
274 In ordinary roofing, each square or surface 10x10 ft., requires 800 16-in., or 700 18-in. shingles. Shingles are sold by the 1000, and parts of a bunch are not sold. Slates, tiles, metal-roofing, sheets, etc., are of various sizes, and the number required per square is given in the special tables used by builders.

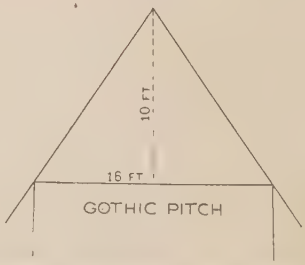
275. The dimensions of a roof are the comb or ridge length, and its width, or distance from comb to the eave, or lower edge of the roof.



276. The *gable* of a roof is the triangle formed by the end edges of the roof and the horizontal line joining the sidewalls. The *pitch* of the roof is its slant as indicated by its height above the gable base. Thus, if the gable of a roof is 16 ft. wide at its base, and the roof rises one-half of this distance, or 8 ft., above the base of the gable, the roof is said to be "half pitch." If the roof rises one-fourth the gable length, the roof is "quarter pitch," etc. A "gothic pitch" roof is one that rises five-eighths of the gable length.







WRITTEN EXERCISE

7. At $3.75 per M 16-in. shingles, and $1.15 per square for labor, what is the cost of two sides of a roof, each 26′x10′?

8. The two gables of a gothic pitch roof for a building 24 ft. wide, are to be covered with fancy shingles costing $6.75 per M, and laying 1150 shingles to the square. Two windows, each 7x3 ft. are to be deducted. Find the cost of the shingles.

9. Find the cost of both sides of a tile-roof 34 ft. 6 in. long, and 15 ft. 3 in. from comb to eaves, the tiles used, laying 875 to the square, and costing $9.25 per M and $1.45 per square for laying.

FLOORING

277. Floors are measured either by the square (100 sq. ft.) or by the 1000 sq. ft. They may be of soft wood (white pine, spruce, etc.), or of hard wood (oak, ash, yellow pine, hard maple, etc), or of tiles, concrete, etc., The lumber for soft wood floors is usually in 5 or 6 in. widths. Hardwood flooring is usually 2 in. or $2\frac{1}{2}$ in. in width.

Flooring is "tongue-and-grooved" for matching, and the matching takes up one inch of the width of each board. Hence, in buying flooring, there must be added to the floor space an allowance of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ for flooring of 2 in., 3 in., 4 in., and 5 in. width, respectively. Builders usually allow $\frac{1}{5}$ also for 6 in. flooring. Thus, if a floor of 180 sq. ft. is to be of 3 inch stuff, $180 \times 1\frac{1}{3}$ or 240 ft. of lumber will be required.

Unless otherwise specified, lumber is used for flooring.

WRITTEN PROBLEMS

10. What will be the cost of a hard-wood floor of 2 inch stuff, for a dining-room 18′ 6″x14′ 4″, the lumber costing $27.50 per M, the laying 65¢ per square?

11. A contractor engaged to floor a hall 92x75 ft. with Georgia pine, of 4 in. width, at a rate of $34.50 for each 1000 sq. ft. of floor space. If the lumber costs $19.50 per M and the cost of laying is 70¢ per square, what is the contractor's profit on the job?

12. A business school occupies one floor of a building 86x110 ft., at a monthly rental of $150. What is the yearly cost of floor space per foot?

13. A bath-room 8x10 ft. is floored with 3x4 in. tiles, costing $9 per M. If the laying costs 11¢ per sq. yd., what is the cost of the floor?

CARPETING

278. Carpets, mattings, linoleums, and other floor coverings (except rugs) are usually sold by the linear yard, but sometimes estimates are made by the square yard.

The cheaper grades of carpetings and mattings are a yard in width, but the width of Brussels and other higher grades is usually 27 in. or $\frac{3}{4}$ yd.

The price of carpetings usually includes cost of laying.

In preparing carpet for laying, the material is cut into strips, and there is usually more or less waste on account of the necessity of matching the figures. Also, as is usually the case, when the width of the room is not divisible by the width of the carpet, it may be necessary to cut off or turn under one of the strips. For these reasons, it is not practical for the ordinary purchaser to make accurate calculations as to the amount of carpeting required for a room even though the dimensions of the room may be known. But as the amount to be added for waste in matching, etc., is about 1 yd. for every 20 yds. actually used, approximate estimates of amount and cost may be made which are close enough for ordinary purposes.

WRITTEN PROBLEMS

14. What will be the cost of carpeting a parlor $17\frac{1}{2}$x$16\frac{1}{4}$ ft. with Brussels $\frac{3}{4}$ yd. in width, at \$2.75 per yd.? (In this and similar problems add $\frac{1}{20}$ or 5% for matching.)

15. At \$1.15 per sq. yd., what is the cost of covering with linoleum, a kitchen floor 15′ 4″x12′ 6″?

16. The floors of a lodge room 75x48$\frac{1}{2}$ ft. and two ante-rooms each 16 ft. 6in.x15 ft. are to be covered with body Brussels at \$2.15 per yd. Find the cost.

17. A stair-way of 22 steps having a tread of 10 in. and a riser of 8 in., also a hall-way 28 ft. long, are to be centered with strips of ingrain carpet costing \$1.25 per yard. Find the cost. (By the "tread" of a step is meant the width and the "riser" means the distance one step is above the one next below.)

18. A parlor 14′x16′ may be covered with $\frac{3}{4}$ yd. width American Brussels at \$1.45 a yd., or with full width Axminster at \$1.90 a yd. Which is the cheaper and how much?

PLASTERING AND PAINTING

279. Plastering and painting, calcimining, etc., are generally estimated by the square yard. Allowances for doors and windows are not usually made unless so specified in the contract. In some

localities it is customary to deduct one-half of the area of the opening and in some places the full allowance is made. Painters however, usually make no allowance, as the extra care required in painting sills, sashes, etc., is deemed a sufficient offset for the diminishing painting surface occasioned by the presence of doors and windows.

280. In giving the dimensions of a room, artisans write first the length, then the width, and height. Thus, a room 12′ 6″x12′x 10′ means a room 12 ft. 6 in. long 12 ft. wide, and 10 ft. high. The distance around a room or building is called the *perimeter*.

WRITTEN PROBLEMS

19. A contractor filed a bid to plaster and calcimine the walls and ceiling of a school room 54′x28′x18′, at 35¢ per sq. yd., allowing $\frac{1}{2}$ the area of 8 windows each $6\frac{1}{2}$x$4\frac{1}{2}$, and 2 doors each 7′x4′ 6″. Another bidder offered to do the work at 33¢ per sq. yd., with no allowance for openings. Which is the lower bid and how much?

SUGGESTION.—To find the area of the walls, multiply the perimeter by the height.

20. Find the cost at 27¢ per sq. yd. of painting the outside walls of a freight ware-house 96′x56′x24′, the gables extending 14 ft. above the walls.

21. At 38¢ per sq. yd., what is the cost of plastering three bed-rooms, the first 12′x15′, the second 10′x12′, and the third 9′x 12′, the ceilings of all being 13 feet high, there being an allowance of 50 sq. ft. in each room for openings?

22. The walls and ceilings of an office 18′x15′x12′ are to be plastered at 35¢ per sq. yd., and then calcimined at 7¢ per sq. yd. Find the cost of each job, an allowance in each case to be made of 15 sq. ft. for each of 5 openings.

23. The perimeter of Mr. A's house is 175 ft. 6 in., and the height of the walls is 25 ft. There are two gables, each with a base of 12 ft. and a height of 6 ft., and a third with a base of 16 ft. and a height of 8 ft. A painter agrees to apply two coats of paint, the first at 18¢ and the second at 15¢ per sq. yd. Find the cost of each coat, no allowance for openings.

PAPER HANGING

281. Wall paper is sold and hung by the roll.

A single roll of wall paper is 24 ft. long and 18 in. wide, and contains 36 sq. ft. To reduce waste in matching, paper usually comes in "double rolls," 48 ft. in length. Unfigured or ingrain papers are 30 in. wide.

There are also various styles of imported and other special papers that vary in width and length of roll. These are usually hung by the square foot.

282. The general method of determining the amount of paper required for a room is to divide the area of the walls and ceiling in square feet by 36, the number of square feet in a roll. A deduction is usually made of one roll for each door or window in the room.

As in the case of carpeting, there is a waste for matching, the amount depending on the figure of the paper. Paper hangers allow for this by taking the *full height* of the walls, instead of allowing for the base board at the bottom and the border at the top.

Fractions of a roll are counted a whole roll.

EXAMPLE: At 40¢ per roll, find the cost of the paper for a room 14′ 6″x12′x10′ 6″, the room having 5 openings.

OPERATION

ft. ft.

1. $(14\frac{1}{2} + 12) \times 2 = 53$ ft. perimeter of room.
2. $53 \times \frac{2}{2}^{1} \times \frac{1}{3}^{1}_{6} = 15+$ or 16 rolls for walls.
3. $\frac{14\frac{1}{2} \text{ ft.} \times 12 \text{ ft.}}{36} = 4+$ or 5 rolls for ceiling.
4. $16 + 5 = 21$ rolls for entire area of room.
5. $21 - 5$ (deduction for windows) $= 16$ rolls required.
6. $16 \times 40¢ = \$6.40$ cost.

WRITTEN PROBLEMS

24. A dining room is 18′x15′ 6″x12′, and there are 2 doors and 5 windows. Find the cost of papering with paper at 75¢ a roll, and 18¢ per roll for hanging.

25. A drawing room 45′x28′ 4″x15′ is to be papered with German leather paper, 24 in. wide, and 36 ft. to the roll, costing $1.65 per roll. There are 9 openings in the room. Find the cost of the paper including a border at 20¢ per ft. and the cost of hanging at $2\frac{1}{2}$¢ per sq. ft. of paper, exclusive of border.

26. Find the cost of papering the bed rooms described in problem 21 under plastering, page 163, with paper costing 35¢ a roll, and 15¢ a roll for hanging, allowing for 9 openings.

STONE AND BRICK WORK

283. Materials. Stone is sold either by the perch of 22 cu. ft. or the cord of 100 cu. ft. A perch of *masonry,* equals $24\frac{3}{4}$ cu. ft. or 22 cu. ft. of stone + $2\frac{3}{4}$ cu. ft. of mortar.

Brick are sold by the 1000. A common brick is 8 in. long, 4 in. wide, and 2 in. thick, but there are several other sizes used in various localities, thus a Milwaukee brick is $8\frac{1}{2}\times4\frac{1}{2}\times2\frac{3}{8}$ in. The number of brick required for a given volume of wall varies, of course, with the size of the brick or the thickness of the mortar, but for ordinary estimates, $22\frac{1}{2}$ bricks are allowed for each cubic foot of wall.

284. Work. In labor estimates, the length is found by measuring around the outside of the wall. This is called "girt measurement." This measures the corners twice, but it is considered a just offset for the extra work required in building the corners. In some places the girt measurements are not used, the actual length of the wall being taken.

Brick work is estimated by the 1000 laid brick (44.4 cu. ft.) and stone work by the perch, 24.75 cu. ft.

The question of allowance for openings is usually determined by the contract. In some places it is the custom to allow one-half the volume of all openings, in other localities no allowance is made unless the openings are very large.

WRITTEN PROBLEMS

27. How many cu. ft. of stone in a wall 40 ft. long, 8 ft. high, and 3 ft. thick? No allowance being made for mortar.

28. What will be the cost of a pile of stone 45 ft. long, 12 ft. wide, and 6 ft. high, at $7.50 per cord?

29. How many perches of stone in a wall 75 ft. long, 8 ft. high, and 3 ft. thick?

30. How many cords of stone in a cellar wall, the outside dimensions of which are: Length 30 ft., width 20 ft., and height of wall 8 ft., the wall being 2 ft. thick?

31. How much will it cost to lay the stone in problem 30 at $1.25 per perch?

32. What will is cost to lay the wall for a cellar 60 ft. long, 30 ft. wide, 8 ft. high, and $1\frac{1}{2}$ ft. in thickness, at $1.25 per perch?

33. What will it cost for the material for the above wall at $5.25 per perch?

34. A contractor bid on the following work: To excavate a cellar 36x24x6 ft. at 30¢ per cubic yard; to erect the wall to project 2 feet above ground, 1½ ft. thick, at $1.50 per perch; and to furnish the stone and lime at $5.25 per perch. What was the amount of his bid?

35. At $15.50 per 1000 brick, what is the cost of the walls of a building 65′x48′x36′, if the walls are 18 in. thick, and an allowance of 150 cu. ft. is made for openings?

36. Find the cost of the brick-work construction for the walls of a flat building 72′x50′x42′ at $17.50 per 1000, the walls to be 14 in. thick.

COAL AND WOOD

285. Coal is mined and usually sold by the ton except in some of the eastern states where it is retailed by the bushel. To dealers it is sold in car-load lots, by the long ton (2240 lb.) in the eastern states, and the short ton (2000 lb.) in the western and southern states.

286. In wholesale coal billing, the weight is given at the nearest even hundred. Thus, 25632 lb. would appear in the bill 25600 and 25675 would be billed at 25700.

In computing the value of coal in short tons, point off three decimal places, divide by 2, and multiply this result by the price per T.

EXAMPLE: What is the value of 28200 lb. coal at $6 per T.?

OPERATION

2)28.200
14.1
6
$84.60

287. Wood is sold by the cord. A standard cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and contains 128 cu. ft. An ordinary market cord, however, is any pile of wood 8 ft. long and 4 ft. high, the price depending on the length of the sticks.

WRITTEN PROBLEMS

37. At $7.50 per short ton, what is the value of 3 cars of coal, net weights, 27800 lb., 26500 lb., 28100 lb., respectively?

38. A car of coal, wt. 32200 lb. was bought at $5.20 per long ton and sold at $7.25 per short ton. If the freight was 75¢ per

long ton, and the cartage 60¢ per short ton, what was the profit on the carload?

39. At $6.75 per standard cord, what is the value of a pile of wood 78 ft. long, 5 ft. wide, and 6 ft. high?

40. What is the value at $1.75 per market cord, of a pile of wood 56 ft. long, and averaging 7½ ft. high?

41. A man bought a pile of 4-foot wood, 85 ft. long, and 8 ft. high, at $5.25 per standard cord. He had it sawed into 16 inch lengths, and then sold it at $2.25 per market cord. If the sawing cost 65¢ per standard cord, what was the profit?

42. Mr. A bought the wood on a tract of 5 acres of timber land for $17.50 per acre. The cutting cost $175, and the hauling and piling, $37.50. He then found that he had 3 piles of 4 ft. wood, each 240 ft. long, and 8 ft. high, which he sold at $4.50 per standard cord. What did he gain from the speculation?

FARM PRODUCTS

288. Grain in cars or wagons, in the United States, is measured by weight but sold by the standard bushel. The number of pounds per bushel is fixed by law in the different states but these weights are not uniform, and changes are frequently made. The weight of a bushel of wheat is fixed in most, if not all the states, at 60 lb., that of a bushel of shelled corn at 56 lb., and a bushel of potatoes at 60 lb.

289. In retailing potatoes, apples, corn in the ear, and other coarse products, the "heaped bushel" is used. This is estimated at 2747.7 cu. in. The standard bushel, 2150.42 cu. in. is known as the "stricken bushel." It is used for grains and other finer products.

Potatoes are sold in the wholesale trade by the standard bushel of 60 pounds. Apples, by the barrel, or by the bushel of 48 pounds. Sugar beets are sold by the ton.

290. In weighing grains and other farm products, for the wholesale trade, the weights are given in the nearest even tens; thus if a weight is 2747 lb. or 2745 lb. it would be entered 2750, and if it were 2744, or less, it would be entered 2740.

MEASUREMENT OF BINS, CRIBS, ETC.

291. It is often desirable to measure grain and other products that have been stored in bins, cribs, etc. For the purpose of such measurements, one cubic foot may be considered to equal $\frac{4}{5}$ of a standard bushel. Hence, to find the contents in bushels of any bin

or other receptacle, take $\frac{4}{5}$ of the number of cubic feet. For greater accuracy, add 1 bu. for every 200 bu. in the result, or 5 bu. for every 1000 bu.

EXAMPLE: Find the contents in bushels of a bin 10′ 8″x6′ 3″x7′ 6″.

OPERATION

$$\frac{32 \times 25 \times 15 \times 4}{3 \quad 4 \quad 2 \quad 5} = 400 \text{ bu.}$$

$400 \text{ bu.} + 2 \text{ bu.} = 402 \text{ bu.}$

EXPLANATION.—Multiplying the dimensions expressed as fractions of a foot, gives cubic feet, this $\times \frac{4}{5}$ gives the bushels approximately. For a close result we add 2 bushels.

To find the capacity in heaped bushels, multiply the cubic feet by .64.

292. Corn in the ear well settled in crib averages about $2\frac{4}{4}$ cu. ft. to the stricken bushel of shelled corn. Loose corn in the ear averages $2\frac{1}{2}$ cu. ft. to the stricken bushel, or twice the volume of shelled corn. Hence, to reduce settled cribbed corn to stricken bushels of shelled corn, multiply the cubic feet by $\frac{4}{9}$ and for loose or unsettled corn multiply by $\frac{2}{5}$.

RECAPITULATION

293. Standard or stricken bushels $= \frac{4}{5} \times$ cu. ft.

Heaped bushels $= .64 \times$ cu. ft.

Standard bushels shelled corn $= \frac{4}{9} \times$ cu. ft. of settled corn in ear.

Standard bushels shelled corn $= \frac{2}{5} \times$ cu. ft. of loose corn in ear.

294. The inside dimensions of the standard farm wagon are, length 10 ft., width 3 ft., depth (one board) 14 in., (two boards) 26 in., (three boards) 34 in. Such wagons hold approximately two stricken bushels for every inch of depth, or one bushel of ear corn for every inch of depth. Some wagons, however, do not have the standard dimensions, as to length or depth. The standard wagon weighs about 1000 lb.

WRITTEN PROBLEMS

43. Find the capacity in standard bushels of a wheat bin 25′x 9′ 4″x10′ 3″. (Make necessary correction for close measure.)

44. A crib 32′x6′ 3″x16′ filled with settled corn in the ear, is bought at 40¢ per bu. of shelled corn. What is the cost?

45. Find the capacity in heaped bushels of a wagon-box 10′ 6″ x3′x26″.

46. A farmer delivered to a neighbor 15 loads of corn in the ear, at 35¢ a bu. The wagon-box is of standard dimensions and is 34 in. in depth. What is the value of the corn?

47. A farmer sold to a dealer, three wagon-loads of potatoes, gross weights: 3150 lb., 2940 lb., 3020 lb. If the tare (weight of wagon) for each load is 1040 lb., what is the value of the potatoes at 45¢ per standard bushel?

48. A grain elevator has 50 compartments, each 20x20x16 ft. What is its capacity in bushels?

49. The Jackson Grain Co., of Cedar Rapids, Ia., shipped to Bell & Co., Adrian, Mich., 4 cars shelled corn, with weights as follows:

	Gross	Tare
1st car	63500	25600
2nd car	61700	26500
3rd car	64800	27100
4th car	63200	26300

This Grain Co. prepays the freight at 18½¢ per 100 lb. and receives 62¢ per bu. for the corn. If the market price of corn at Cedar Rapids is 50¢, do they lose or gain in the shipment, and how much?

CISTERNS, TANKS, ETC.

295. There are approximately 7½ (accurately 7.4805 +) standard gallons in a cubic foot. Hence, to change cubic feet to gallons, multiply by 7½. For close measure, deduct 3 gal. for each 1000 gal. of the result. There are approximately 4⅕ cu. ft. in a standard barrel (31½ gal.) Hence, to change cubic feet to barrels, divide by 4⅕ (multiply by $\frac{5}{21}$). For close measure deduct 3 brl. for each 1000 brl. of the result. The volume of a tank, cistern or other cylindrical vessel is equal to the area of the bottom multiplied by the depth.

As the bottom of a cylindrical vessel is a circle, its area is equal (approximately) to $\frac{11}{14}$ of the square of its diameter. (See 261.) The whole computation may always be reduced to one equation and the operation may often be greatly shortened by cancellation.

EXAMPLE: What is the capacity in gallons of a circular tank, 10 ft. 6 in. in diameter and 16 ft. deep?

OPERATION

$$\frac{21}{2}\times\frac{21}{2}\times\frac{11}{14}\times 16\times\frac{15}{2}=10395 \text{ gal.}$$

deduct for correction 31 gal.

10364 gal.

EXAMPLE: Find the capacity in barrels of a stand pipe, 60 ft. high and 28 ft. in diameter.

OPERATION

$28 \times 28 \times \frac{11}{14} \times 60 \times \frac{5}{21} = 8800$ brl.

deduct for correction 26 brl.

8774 brl.

WRITTEN PROBLEMS

50. The concrete water reservoir at Springville is 120 ft. square and 14 ft. deep. What is its capacity in barrels?

51. Mr. A has installed on his farm a motor pump, having a capacity of 80 gal. per minute. It discharges into a tank 35 ft. high and 16 ft. in diameter. What is the capacity of the tank in barrels, and how long will it take the pump to fill it?

52. The village of Fairview has a fire stand pipe, 105 ft. high and 42 ft. in diameter. If the connected fire hose discharges at full capacity 240 brl. per minute, how long would the water last in case of a general fire?

53. Maynard & Sons have erected an iron tank grain elevator, 63 ft. high and 40 ft. in diameter. What is its capacity in bushels?

54. A cistern is 12 ft. deep and averages 7 ft. in diameter. What is its capacity in barrels?

55. A tin milk can is 26 in. in diameter and 3 ft. 8 in. deep. How many quarts does it hold?

56. Find the capacity in barrels of a tank car, 42 ft. long, and $6\frac{1}{2}$ ft. in diameter inside dimensions.

TRANSPORTATION

296. Transportation is the business of carrying commodities or passengers from one part of the country to another.

297. Commodity transportation is carried on by steam and electric railway companies, ocean, lake, and river vessels, various express companies, transfer companies, stage lines, etc., and the U. S. government through its post office department.

RAILWAY FREIGHT TRAFFIC

298. This is divided into *inter-state traffic* or transportation from one state into or through another, and *intra-state traffic* or traffic between points in the same state. The former is regulated

by U. S. laws and by the rules of the Inter-state Commerce Commission. The latter is subject to the laws and regulations of the state in which the traffic occurs.

299. The railways charge by the 100 pounds for the transportation of most commodities.

For coal, sand, pig iron, and other heavy commodities, the rate is by the ton, and for oils and other liquids shipped in tank cars, the rate is by the barrel.

The amount of the charge depends:

1. On the distance of the carriage.
2. On the "classification" or value of the article.
3. On the location of the places between which the goods are shipped.

300. A **freight tariff** is a schedule, showing the freight rates charged for carrying the different classes of articles from one specified point to another.

All freight rates are required to be made public, and the charging of either a less or a greater amount than the published rate is illegal.

301. In inter-state traffic, the freight charge on packages of less than 100 lb. is the same as for packages weighing 100 lb., but this rule does not hold in intra-state traffic.

302. F. O. B. (free on board) is an abbreviation used in business to indicate the place to or from which a transportation charge is to be paid. Thus, if a Chicago firm sells goods to a firm in St. Paul, "F. O. B. St. Paul," it is understood that the Chicago seller pays the freight; but if the terms of sale are "F. O. B. Chicago," the seller delivers the goods on the cars at Chicago, and the freight charge is paid by the buyer in St. Paul.

303. C. O. D. (collect on delivery). This is an abbreviation used in business to indicate that the goods are to be paid for by the purchaser when they are delivered.

304. In entering weights where the rate is by the 100 lb., railway clerks record the weights in the nearest even tens. If the rate is by the ton, the weight is entered at the nearest even hundred. (See 286.) In transcontinental traffic or shipments to the Pacific coast, this rule does not hold, the actual weights being entered.

305. Passenger tariffs are the rates fixed by law in the several states. These are usually either 2¢, 2½¢, or 3¢ per mile for adults,

and one-half these rates for children over 5 and under 12 years of age. 150 lb. baggage is allowed for each full-fare ticket, and 75 lb. for each half-fare.

WRITTEN PROBLEMS

In the following computations, follow carefully the preceding instructions. The rates given are by the hundred unless otherwise specified.

57. What is the freight at 45¢ on a shipment of household goods from Quincy, Ill., to Chicago, wt. 1156 lb.? (See 304.)

58. At a rate of 65¢, what is the freight on a package of books, wt. 65 lb., from St. Louis, Mo., to Omaha, Nebr.?

59. Find the freight on 3 cars of sand, gross weights 68756, 71249, 69568, at 75¢ per ton. The cars weigh 25000 lb. each. Find the net weight of each car-load.

60. The distance from Mt. Vernon to Stapleton is 67 mi. and the passenger tariff is $2\frac{1}{2}$¢ per mile. Find the cost of two full-fare and three half-fare tickets with three seats in the Pullman at 35¢ each.

61. A lady, with four children, aged 6, 8, 10, and 15 years, buys transportation to Denver, Colo., the full fare being $9.75. Her baggage weighs 625 lb., and the excess charge is 35¢ per 100 lb. She must also pay for two Pullman sections at $4.50 each. What is the entire cost of the transportation?

EXPRESSAGE

306. A number of Express Companies conduct a system of rapid and more expensive transportation for smaller or more valuable packages than those usually shipped by railway freight.

Express cars are attached to the regular passenger trains, but on certain main or trunk railway lines they are made up into special fast trains.

307. Express rates are by the hundred pounds, but proportionally higher rates are charged for lighter packages.

Ordinary or "merchandise" rates are charged for packages not exceeding a valuation of $50. An additional amount is charged for carrying packages of greater value. The rate varies according to distance.

Special rates per $1000 are charged for the transportation of money in the form of currency or gold coin.

For live animals the rate is one and one-half that of an equal weight of merchandise, with an additional charge for excess of valuation over $50.

308. The following table represents a portion of an express tariff sheet for the 100 lb. rates given at the heads of the several columns, and for the weights as indicated in the columns. For weights not given, the rate for the next higher weight is taken. Thus, the rate for 16 lb. under the $2 rate, is the same as the rate for 20 lb. or 85¢.

GRADUATED CHARGES FOR PACKAGES WEIGHING FIFTY POUNDS OR LESS AT THE GIVEN RATE PER HUNDRED POUNDS

$2.00	$2.50	$3.00	$3.50	$4.00	$4.50
1 lb. $ .25	1 lb. $ .25	1 lb. $ .25	1 lb. $ .25	1 lb. $ .25	1 lb. $ .30
2 " .35	2 " .35	2 " .35	2 " .35	2 " .35	2 " .35
3 " .45	3 " .45	3 " .45	3 " .45	3 " .45	3 " .45
4 " .50	4 " .55	4 " .60	4 " .60	4 " .60	4 " .60
5 " .55	5 " .60	5 " .65	5 " .70	5 " .70	5 " .75
7 " .60	7 " .70	7 " .75	7 " .80	7 " .85	7 " .90
10 " .70	10 " .75	10 " .80	10 " .90	10 " 1.00	10 " 1.00
15 " .75	15 " .85	15 " .90	15 " 1.00	15 " 1.10	15 " 1.15
20 " .85	20 " 1.00	20 " 1.10	20 " 1.20	20 " 1.25	20 " 1.30
25 " 1.00	25 " 1.10	25 " 1.20	25 " 1.30	25 " 1.40	25 " 1.50
30 " 1.00	30 " 1.15	30 " 1.30	30 " 1.50	30 " 1.60	30 " 1.70
35 " 1.00	35 " 1.25	35 " 1.40	35 " 1.60	35 " 1.70	35 " 1.90
40 " 1.00	40 " 1.25	40 " 1.50	40 " 1.75	40 " 1.85	40 " 2.00
45 " 1.00	45 " 1.25	45 " 1.50	45 " 1.75	45 " 2.00	45 " 2.25
50 " 1.00	50 " 1.25	50 " 1.50	50 " 1.75	50 " 2.00	50 " 2.25

WRITTEN PROBLEMS

Use the table for computation where the rate is not given, also the data included in the previous instructions.

62. A package of silks weighing 26 lb. and valued at $375 was expressed to a point where the rate is $3.50 per 100. With an additional charge of 5% of the valuation in excess of $50, what is the amount of the expressage?

63. To a point where the 100 lb. rate is $2.50, a merchant expressed to different customers 5 packages weighing 28, 46, 27,

15 and 32 lb. None of the packages exceeding the valuation limit of $50, what is the amount of the expressage?

64. A dealer in pets expressed a Scotch Collie dog valued at $250, and weighing 37 lb. to a point where the rate is $2 per 100. If the valuation charge is 5% of the excess over $50, what is the cost of expressage?

65. A package of gold bullion weighing 41 lb. avoir. is expressed to a point where the rate is $4.50. The value of the package is $10,965, and the excess valuation charge is $1 per $1000. Find the amount of the expressage.

66. A package of diamonds valued at $15,000 is expressed from New York to Chicago. The package with wrappings weighs 1¼ lb., and the 100 pound rate is $2.50. The excess valuation charge being 10¢ per $100, what is the cost of transportation?

67. A publishing company in Michigan wishes to ship a package of books weighing 16 lb. to a point in Nebraska where the railway freight is 85¢ per 100, and the express rate is $2.50. Which is the cheaper transportation, by freight or by express, and how much?

POSTAL TRANSPORTATION

309. Matter transmittable through the U. S. mails is of four classes:

First Class. All pen or type-written matter, as letters, post cards, etc., or circulars partly written and partly printed. Also, all matter sealed or otherwise closed against inspection.

Rate: 2 cents for each ounce or fraction thereof: limit of weight 4 lb.

Second Class (unsealed). All newspapers or other periodicals that the postal authorites have authorized to be enterd as second class matter.

Rate: 1 cent for each 4 ounces, or fraction thereof. No limit as to weight.

Publishers' Rates. Publishers of newspapers and periodicals are entitled to send their own publications through the mails at a rate of 1 cent for each pound or fraction.

Third Class (unsealed). This includes books weighing less than eight ounces, circulars and general printed matter, other than newspapers and periodicals.

Rate: 1 cent for each 2 ounces or fraction thereof. Limit of weight 4 lb.

Fourth Class (Parcel Post). Merchandise and all mailable articles not included in the first three classes, and books weighing over eight ounces (unsealed).

The rates vary with the distance, the country being divided into eight zones to facilitate rate adjustments. Within the first two zones, the limit of weight is fifty pounds and the rate is uniform, viz., five cents for the first pound and one cent for each additional pound or fraction thereof. For shipments beyond the first two zones, the weight limit is twenty pounds, the rate being determined by the number of zones through which the package is transmitted.

A package of Fourth Class matter, weighing four ounces or less, is mailable to all zones at a rate of one cent for each ounce or fraction.

Matter of a higher class enclosed with matter of a lower class, subjects the whole package to the higher rate. Thus, if a letter were mailed in a package of merchandise, the whole package would be subject to first class rates.

The postal authorities will supply, on application, printed lists, showing the mailing classification of articles likely to be mailed, also a schedule of parcel post rates for the different zones.

WRITTEN PROBLEMS

Find the cost of mailing the following:

68. A package of printed circulars weighing 3 lb. 2 oz.

69. A bundle of newspapers and magazines weighing 2 lb. 6½ oz.

70. A package weighing 18¼ lb. for shipment to second parcel post zone.

71. A package of books weighing 14¼ lb. for shipment to sixth zone; rate: 9 cts. for first pound and 8 cts. for each additional pound or fraction.

72. A personal letter weighing 3¾ oz.

73. A case of eggs for shipment to second zone, weight 37¾ lb.

74. 175 sealed letters each weighing 1¾ oz.

COMMUNICATIONS BY TELEGRAPH

310. Telegraph messages are charged for at so much for ten words, the charge varying with the distance, number of "repeats" or re-transmissions, etc.

Messages filed between the hours of 6 p. m. and 12 midnight are known as "night messages" and the rate for these is from 20% to 25% lower than the rate for day messages, provided the night message is not required to be delivered until the following day. Night rates, however, are not quoted for all points.

311. Telegraph rates are expressed so as to show the rate for the first ten words, and the cost of each additional word. Thus, a rate of 75¢ for ten words, and 5¢ for each additional word is written "75-5." In ordinary commercial messages, the cost for each additional word is $\frac{1}{5}$ the cost of the first ten words, or the nearest amount thereto. Thus, where the ten-word rate is 60¢, the complete rate is 60-4, and for a ten-word rate of 40¢, the complete rate is 40-3.

312. No charge is made for place, date, and name and address of person to whom the message is sent, and signature of writer, but if special address of sender is given, it is charged for. Numbers

and most abbreviations are charged for at full word rates. Thus "C. A. Brown" would count as three words, and "$1525" would be "fifteen twenty-five dollars," four words. Hyphenated words, as half-morocco, twenty-five, ten-pound, are counted as two words. A few common abbreviations, as f. o. b., C. O. D., A. M., P. M., etc., go as one word.

ORAL PROBLEMS

1. Find the cost of a message of 17 words at a rate of 25-2.

2. How much will be saved in sending a 25 word message from Marion, Iowa, to Chicago, by night rate, instead of by day, the rate being 40-3 day, and 30-2 night?

3. The day rate from Iowa to Texas is 75-5 and the night rate is 60-4. How much cheaper will the night rate be for the following message?

"Left Boston Tuesday, arriving here today. Prospects for sale to Hammond Company excellent. Shall I close deal on original basis? Wire today."

4. At 40-3, what is the cost of the following?

"Jury returned verdict at 8:30 P. M., awarding us $1165.75, with attorney's fees $250. Unless you wire me by ten-thirty, shall leave for home."

5. At 35-2, what is the cost of the following:

"Express C. O. D. twelve copies Marshall's Business Speller, ten copies Inductive Commercial Arithmetic."

LAND SURVEYING AND MEASUREMENT

313. In the early settlement of America, land was parceled out by the king among the proprietors or leaders of the colonies, in large irregular tracts or "grants," bounded by rivers, mountain ranges, ocean coasts, etc. These tracts were in turn divided among the settlers into smaller, and often still more irregular tracts, whose boundaries were often marked by trees, stones, banks of streams, established roads, or other fixed objects. For this reason, the farm lands of the older eastern states are still found in irregularly shaped lots. But in the newer western states, most of the lands were surveyed and laid out by the government in accordance with a system of rectangular measurements adopted in 1802 and still retained.

314. The standard unit for the measurement of land is the acre, which is equal in area to 160 sq. rd. or 43560 sq. ft., or a square whose side is 208.7 ft.

315. A **base line** is a starting line running east and west, from which parallel lines 6 miles apart are run.

316. A **principal meridian** is a starting line running north and south, from which other meridians 6 miles apart are run.

THE GOVERNMENT SURVEY

317. Government townships are the square tracts formed by the intersection of the parallels and meridians. As these are 6 miles apart, the townships are 6 miles square and contain thirty-six square miles.

318. A **range** is a tier of townships extending north or south from the base line. Ranges are designated by number east or west of a given meridian, as "Range 2 E," "Range 5 W," etc.

319. Townships are numbered consecutively, north or south of a given base line, as "T. 6 S." (Township 6 South), "T. 5 N." (Township 5 North).

320. Townships are surveyed into tracts of 1 mile square, called *sections*. Each section contains 640 acres, and there are 36 sections in a township. The sections of a township are uniformly numbered as shown in the diagram on page 178, the northeast corner section of each township being "No. 1," and the southeast corner section "No. 36." Sections are further subdivided into "half-sections," and "quarter-sections." Quarter-sections are further divided into halves, quarters, etc. Parts of a section are described by giving location and fractional part, thus, "N ½ S. 19," (North half Section 19). S. ½ S. E. ¼ S. 21. (South half, southeast quarter, Section 21.)

321. Correction lines. As all north and south lines (meridians) tend to converge at the North Pole, it follows that the north line of a township or other square tract is slightly shorter than the south line. To prevent townships and sections from growing materially narrow as the survey extends north, secondary base lines called *correction lines,* are run at frequent intervals. Upon these lines the township lines are again started at the standard width. From the foregoing it is evident that all sections do not contain *exactly* 640 acres, also, that the sections in the north part of a tonwship are slightly smaller than those in the south part.

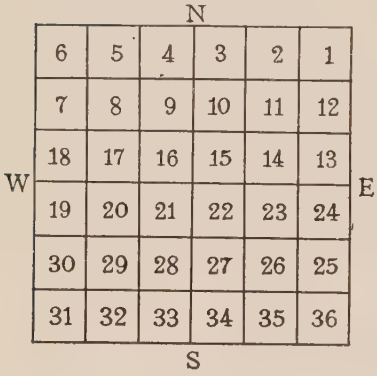
322. The accompanying diagrams illustrate the methods used and the terms employed in government surveys.

Diagram No. 1 shows a township and its division into sections, and the method of numbering them.

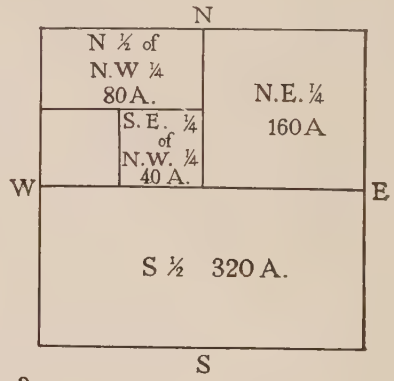
Diagram No. 2 shows the division of a section and the method of describing the divisions.

Diagram No. 3 shows the base line and principal meridian, ranges of townships, and set off of meridian on correction line.

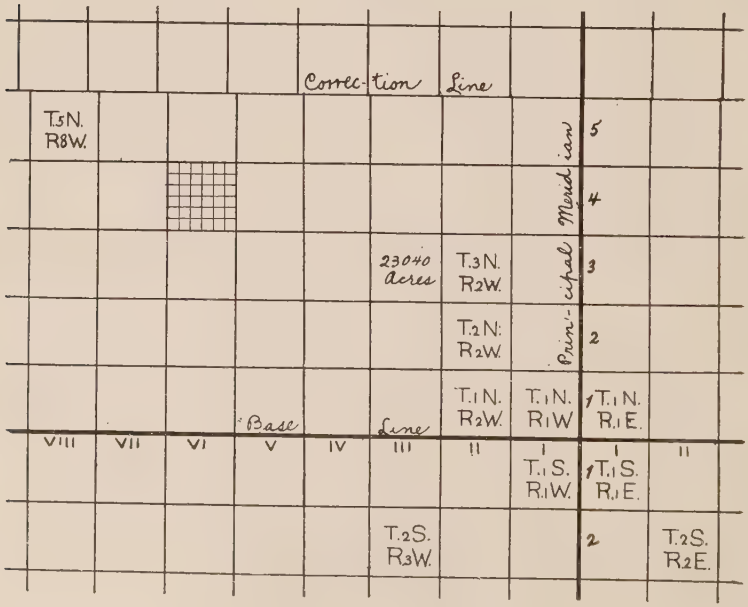
No. 1.
Township, 6 miles.



No. 2.
1 mile.



No. 3.



WRITTEN PROBLEMS

75. How many acres in a township of land?

76. How many acres in three half-sections of land?

77. A speculator sold a half-section of land at $25 per A., a quarter-section at $15 per A., a half-quarter-section at $12.50 per A., a quarter-quarter-section at $14.75 per A., and a lot containing $\frac{3}{4}$ of a quarter-quarter-section at $8.50 per A. How much did he receive in all?

78. A man bought the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$, the W. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$, and the N. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$. How many acres of land did he buy? (Let the student make a diagram.)

79. How many acres of land in the following described property: The N. E. $\frac{1}{4}$; the S. E. $\frac{1}{4}$; the N. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$; the N. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$; and the E. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$; all of Section 14, Township 12 North, Range 3 East of the 3d Principal Meridian?

80. Bought 5 alternate sections of land of the Union Pacific railway, at $5 per acre. I sold as follows: The first section at $6.25 per A., the second at $4.75 per A., the third as follows: The E. $\frac{1}{2}$ at $7.50 per A.; the N. W. $\frac{1}{4}$ and the W. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ at $10.50 per A.; the balance of the section at $4 per A.; the fourth section at cost. What did the land cost, and how much did I gain on the part sold?

LAND MEASUREMENT

323. The **surveyor's tape.** Modern surveyors use the surveyor's tape in measuring land. This is a flexible steel band, 100 feet in length, the foot sub-divisions being scaled to tenths and hundredths.

When land is measured by the surveyor's tape, the dimensions are expressed in feet and decimals of a foot. The product of these dimensions is the area of the land in square feet, and this result divided by 43560 gives the area in acres.

324. Surveyors greatly shorten this process by multiplying the area in square feet by .000023 which is the near equivalent of $\frac{1}{43560}$. To secure approximately accurate results in using this decimal, deduct $\frac{1}{5}$ of an acre for every 100 acres in the answer.

EXAMPLE: How many acres in a tract of land 11697.5 feet long and 2000 feet wide?

OPERATION

$11697.5 \times 2000 = 23395000$ sq. ft.

$23395000 \times .000023 = 538.085$ acres (uncorrected).

$538.085 - 1 = 537.085$ acres (corrected).

WRITTEN PROBLEMS

81. A tract of land is 320 ft. long and 245.5 wide. What is it worth at $100 per acre?

Note.—Use method given in Art. 324.

82. The length of a railroad right of way which crosses Mr. B's land is 1426.8 feet and the width of the right of way is 100 ft. If the appraiser allow Mr. B $40 an acre for the land occupied by the railroad, what is the amount of his claim?

83. At $275 per front foot, what is the value of a piece of city real estate whose frontage is 69.58 ft.?

84. If the lot referred to in the preceding problem is 120 feet deep, what is its valuation per acre? (Use method given in Art. 324.)

THE SURVEYOR'S CHAIN

325. Formerly, in locating and measuring land, surveyors used a measuring implement called a *chain* (ch.) This chain was four rods in length and was divided into 100 equal parts called links (l.) A square chain (sq. ch.) therefore contains $(4 \times 4 = 16)$ 16 sq. rd., and 10 sq. ch. equal one acre.

The area in acres of a tract of land when the dimensions are given in chains and links (hundredths of a chain) is therefore found by dividing the product of the dimensions by 10.

EXAMPLE: How many acres in a lot that is 15 ch. 20 l. long and 12 ch. 8 l. wide?

OPERATION

15 ch. 20. l. = 15.2 ch.
12 ch. 8 l. = 12.08 ch.

$$\frac{15.2 \times 12.08}{10} = 18.3616 \text{ A.}$$

EXPLANATION.—The links are expressed in hundredths of a chain. The product gives square chains. Moving the decimal point one place to the right divides the square chains by 10, and reduces the result to acres.

Although the surveyor's chain is now rarely used in measuring land, it is often necessary to compute areas from the field notes of former surveys in which the dimensions have been recorded in chains and links.

WRITTEN PROBLEMS

85. How many acres in a tract of land which according to the records is 126 ch. 5 l. long and 41 ch. and 60 l. wide?

86. How many acres are occupied by a roadway 2 rd. wide and 71 ch. 40 l. long?

87. The laws of a certain western state call for public road reservations one chain wide along all section lines. Under this law, how many acres in each government township would be devoted to roads, allowing 50 links for one-half the roadway on township lines?

SCIENTIFIC AND SPECIAL MEASUREMENTS

WEIGHT AND PRESSURE OF WATER

326. A cubic foot of distilled water at its greatest density (40 Fahr.) weighs 1000 ounces (avoir.), or 62.5 lb.

327. Pressure. Water and other liquids exert pressure upon the bottom and sides of the containing vessel. Pressure is proportional to the depth and for a given horizontal surface, as the bottom of a tank, is equal to the weight of a column of the liquid extending from that surface to the surface of the liquid. For instance, the pressure on each square foot of the bottom of a tank in which the water is ten feet deep, is equal to the weight of a column of water one foot square and ten feet high, or 10×62.5 lb. $= 625$ lb.

The average pressure on the side of a filled vessel is equal to the actual pressure at a point half way between the surface and the bottom, or, what is the same thing, to one-half the pressure at the bottom. Thus, in the tank instanced above, the average pressure per square foot on the sides is equal to 5×62.5 lb. (or, $\frac{1}{2}$ of 625 lb.) $= 312\frac{1}{2}$ lb.

Since the pressure at a depth of 1 ft. is 62.5 lb., the pressure per inch for each foot of depth is $\frac{1}{144}$ of 62.5, or .434+. Hence, to find the inch pressure for any depth below the "head" or surface, multiply the distance in feet below the head by .434.

WRITTEN PROBLEMS

88. In the town of Fremont the fire hydrants are 65 ft. below the head of the reservoir. What is the inch pressure at the hydrants? (All answers correct to 2 decimal places.)

89. When a tank 10 ft. in diameter and 14 ft. deep is full of water, what is the pressure per inch on the bottom? What is the pressure per foot? What is the total pressure? What is the average pressure per inch on the sides?

90. A dam is 50 ft. long and 12 ft. high. What is the total pressure on the dam when the water is level with the top? What is the pressure per inch at the bottom?

91. A tank street sprinkler is 10 ft. 6 in. long, and $3\frac{1}{2}$ ft. in diameter. What is the weight of the water when the tank is full?

92. A water-power electric plant in Colorado is 350 ft. below the head. What is the pressure of the water as delivered to the motors?

93. The outside surface of a jar is equal to 116 sq. in. If the jar is sunk in fresh water to a depth of 100 ft., what is the total force tending to crush the jar?

SPECIFIC GRAVITY

328. The weight of a substance as compared with that of an equal bulk of water is called its *specific gravity* (sp. gr.) Thus, a cubic foot of cast iron weighs 450.6 +, or, 7.21+ times the weight of a cubic foot of water, hence the specific gravity of cast iron is given as 7.21 +.

Tables giving the specific gravity of all ordinary substances may be found in cyclopedias and technical works.

329. If the volume and specific gravity of a body is known, its weight may be easily found by multiplying the weight of an equal volume of water by the specific gravity. If the weight and volume are known, the specific gravity is found by dividing the weight by the weight of an equal volume of water.

Again, if the weight and specific gravity are known, the volume in cubic feet may be found by dividing the weight of the body by the specific gravity, (thus giving the weight of an equal bulk of water) and this result by 62.5, the weight of a cubic foot of water.

EXAMPLE: A block of granite, sp. gr. 2.7, weighs 1600 lb. How many cubic feet in the mass?

OPERATION

$1600 \div 2.7 = 592.59$ lb., wt. of equal bulk of water.
$592.59 \div 62.5 = 9.48$ cu. ft. of granite.

WRITTEN PROBLEMS

94. Find the weight of a barrel of water, ($31\frac{1}{2}$ gal.)

SUGGESTION.—$62\frac{1}{2}$ lb. $\div 7\frac{1}{2} =$ the weight of one gallon of water. Why?

95. The sp. gr. of cast iron being 7.21, what is the weight of a bar of cast iron 16 ft. long, 8 in. wide, and 3 in. thick?

96. The sp. gr. of gold is 19.35. What is the weight of a cube of gold 6 inches square?

97. What is the weight of one quart of quicksilver, the sp. gr. being 13.59?

98. A mass of stone 8 ft. long, 3 ft. wide, and 2 ft. thick, weighs 6856 lb. What is its sp. gr.?

99. A tube 30 inches high is filled with quick silver. What is the pressure per square inch at the base?

SUGGESTION.—The pressure is 13.59 times the water pressure.

NOTE.—The answer to this problem gives the normal pressure of the atmosphere, which, in the barometer sustains a column of mercury 30 inches high. The same pressure will sustain a column of water about 34 ft. high. This amount of pressure is often called "an atmosphere."

CIRCULAR MEASURE AND THE COMPARISON OF TIME

330. For the purpose of calculations in astronomy, navigation, etc., the circumference of the circle is measured as follows:

A circle equals 360 degrees (°).

A degree equals 60 minutes (′).

A minute equals 60 seconds (″).

331. The *equator* is a circumference of the earth midway between the poles.

332. *Meridian circles* are circumferences of the earth which pass through both poles, and intersect the equator at right angles.

333. *Parallels* are circles parallel to the equator.

334. *Latitude* is a circular distance north or south of the equator, measured on the meridians. A degree of latitude equals 69.16 statute miles.

335. *Longitude* is a circular distance east or west of any fixed meridian (called the prime meridian) and measured on the equator or the parallels.

336. The prime meridian from which longitude is ordinarily measured, is the meridian which passes through the observatory at Greenwich in England near London.

LONGITUDE AND TIME

337. The revolution of the earth about its axis causes the sun to make an apparent circuit of the heavens every twenty-four hours.

In one hour the sun passes over $\frac{1}{24}$ of the 360° of this circuit, or 15°.

In one minute of time is passes over $\frac{1}{60}$ of 15° or 15′ of the circle.

In one second it passes over $\frac{1}{60}$ of 15′ or 15″ of the circle.

It follows, therefore, that if there is a difference in longitude between any two places on the earth's surface, there will be a corresponding difference in time. The amount of this difference is shown by the following tables:

FIRST TABLE

15° of longitude makes a difference of 1 hour of time.

15′ of longitude makes a difference of 1 minute of time.

15″ of longitude makes a difference of 1 second of time.

SECOND TABLE

1° of longitude makes a difference of 4 minutes of time.

1′ of longitude makes a difference of 4 seconds of time.

1″ of longitude makes a difference of $\frac{1}{15}$ seconds of time.

Either of these tables may be used in making computations, but the most convenient method in most cases is to divide the difference in longitude by 15, or to multiply the difference in time by 15 to find the difference in longitude.

ORAL PROBLEMS

1. If the difference in longitude between Denver and Philadelphia is 30°, what is their difference in time? When it is noon at Philadelphia, what is the time at Denver? When it is noon at Denver, what is the time at Philadelphia? What is the time at Denver when it is 9:15 P. M. at Philadelphia? If a person journeys from Philadelphia to Denver without resetting the hands of his watch, would his time-piece be too fast or too slow, and how much, when he reaches Denver? If he were to journey from Denver to Philadelphia?

2. If Jackson is 4° 15′ east of Springdale, what is the difference in local time?

Suggestion.—Use second table.

3. London, England, is 77° east of Washington, D. C. What is the difference in time? When it is six o'clock in the evening at Washington, what is the hour in London? When it is noon in London, what is the hour in Washington?

4. A ship sailed from Lat. 17° 30′ N. to Lat. 25° 15′ N. Through how many degrees and minutes of latitude did she sail?

5. A star is half way between the horizon and the zenith (point directly overhead). The star is how many degrees above the horizon?

6. Buenos Ayres is in Lat. 35° 10′ S., and Havana is in Lat. 23° 5′ N. What is their difference in latitude?

7. The longitude of Buffalo, N. Y., is 79° 5′ W., and that of Jefferson City, Mo., is 92° 30′ W. What is their difference in longitude?

WRITTEN PROBLEMS

100. Cape Town, South Africa, is in Lat. 30° 20′ S. and in Lon. 18° 10′ E. Constantinople, Turkey, is in Lat. 41° 15′ N., and Lon. 27° 5′ E. Find their difference in latitude and longitude.

101. A Havana steamer sailed in one day from Lat. 25° 16′ to Lat 29° 12′. How many miles north did she sail during the day?

102. The Gulf Coast of the U. S. runs along the 30th parallel, and the northern boundary of the U. S. is the 49th parallel. What is the width of the country in statute miles?

103. Scranton, Pa., is in Lon. 75° 40′, and Wichita, Kans., is in Lon. 97° 20′. What is the difference in the local time of the two cities?

STANDARD TIME

338. In 1883 the railroads of the United States and Canada, for their own convenience and that of the traveling public, as well as to secure greater safety in the running of trains, adopted a uniform system of time for each of the four time belts into which the continent was divided.

The system is known as *standard time,* and in nearly all parts of the country it is used instead of "sun" or local time.

Each of the time belts is governed by the local time of the specified meridian that passes through the belt. The standard time belts are as follows:

Eastern time. This belt has the local time of the 75th meridian of longitude, which passes very near to Ottawa, Ont., Utica, N. Y. and Philadelphia, Pa. It covers the Atlantic seaboard states generally, and most of Pennsylvania and West Virginia.

Central time. This belt is governed by the local time of the 90th meridan, which passes near St. Louis, and along the general course of the Mississippi river. This is the broadest time belt and covers nearly all the territory lying in the great central plain of the United States and Canada.

Mountain time. This belt embraces Colorado, Utah, and the other states of the Rocky Mountain region. It has the local time of the 105th meridian.

Pacific or western time. This belt includes the states bordering on the Pacific coast, also Alaska and parts of Nevada, New Mexico, and Arizona. It has the local time of the 120th meridian.

339. By the *sun time* or *local time* of a city is meant the time as determined by the rising or setting of the sun at that place in distinction from the standard time, that prevails all over the belt in which the city is located.

Since the place has earlier or "faster" local time than any place west of it, it is evident that if any town in a standard time belt is east of the governing meridian for that belt, its local time will be faster than the standard time, and if the place is west of the governing meridian, its local time will be slower than the standard time.

340. There are no regular boundaries between the different time belts; the railroads change the time schedule at terminal points or important stations, without reference to the longitude of the place where the change is made. Local time (except for railway schedules) is observed in a few cities, that are a considerable distance from the governing meridian.

ORAL EXERCISE

With a map before you, study the following problems:

NOTE.—Unless otherwise specified, all questions refer to standard time.

1. Is the local time at New York slower or faster than its standard time?

2. When it is noon at Boston, what is the hour at Kansas City? At San Francisco? At Salt Lake? At New Orleans?

3. The first shock of the San Francisco earthquake occurred at 5:15 A. M. What was the hour at New York? At Chicago? At Denver?

4. In going from Boston to Seattle, how many times would a man need to reset the hands of his watch in order to have his watch conform to the time of the states through which he was journeying? Would he turn the hands forward or back? If he did not reset the hands at all, what discrepancy would his watch show when he reaches Seattle?

5. Is the local time at Chicago slower or faster than standard time? At Omaha? At Boston? At Cincinnati? At Buffalo? At San Francisco?

6. If the longitude of a town in Pennsylvania is 76° 30′, how does its local time compare with the standard time?

7. Pittsburg is on the 80th meridian. How does the local time compare with eastern time? With central time?

CHAPTER XII

PERCENTAGE

341. Percentage is a general term relating to computations involving per cents or hundredths. (See 217.)

Percentage calculations have many special applications in business and financial affairs, and they include the most important division of commercial arithmetic.

No principles are involved in percentage that have not already been fully illustrated in the chapters on Fractions and Decimals. A per cent, or *rate,* is merely a convenient means of indicating a fraction or decimal having a denominator of 100. Thus, 5% means $\frac{5}{100}$, or .05, and $2\frac{1}{2}\%$ means $\frac{2\frac{1}{2}}{100}$, or .025, etc.

In dealing with various per cent rates, the student should become familiar with their equivalents expressed as common fractions in their lowest terms.

342. The following table gives the more important of these. It is the equivalent expressed in per cent terms, of the table of aliquot parts of 100 as given in (151).

PERCENTAGE EQUIVALENTS

$1\% = \frac{1}{100}$ or .01	$33\frac{1}{3}\% = \frac{1}{3}$ or .333 +
$2\% = \frac{1}{50}$ or .02	$37\frac{1}{2}\% = \frac{3}{8}$ or .375
$2\frac{1}{2}\% = \frac{1}{40}$ or .025	$40\% = \frac{2}{5}$ or .4
$4\% = \frac{1}{25}$ or .04	$50\% = \frac{1}{2}$ or .5
$5\% = \frac{1}{20}$ or .05	$60\% = \frac{3}{5}$ or .6
$6\frac{1}{4}\% = \frac{1}{16}$ or .0625	$62\frac{1}{2}\% = \frac{5}{8}$ or .625
$10\% = \frac{1}{10}$ or .1	$66\frac{2}{3}\% = \frac{2}{3}$ or .666 +
$12\frac{1}{2}\% = \frac{1}{8}$ or .125	$75\% = \frac{3}{4}$ or .75
$16\frac{2}{3}\% = \frac{1}{6}$ or .166 +	$80\% = \frac{4}{5}$ or .8
$20\% = \frac{1}{5}$ or .2	$87\frac{1}{2}\% = \frac{7}{8}$ or .875
$25\% = \frac{1}{4}$ or .25	

Also it is well to keep in mind that,

$2\frac{1}{2}\% = \frac{1}{4}$ of 10% or $\frac{1}{10}$ of 25%.

$5\% = \frac{1}{2}$ of 10% or $\frac{1}{5}$ of 25%.

$7\frac{1}{2}\% = \frac{3}{4}$ of 10%, and $15\% = 1\frac{1}{2}$ times 10%.

$\frac{1}{4}\% = \frac{1}{400}$.

$\frac{1}{8}\% = \frac{1}{800}$.

100% = 1 or the *whole* of anything upon which a percentage is computed.

$120\% = 1\frac{1}{5}$ $(\frac{6}{5})$

$125\% = 1\frac{1}{4}$ $(\frac{5}{4})$

$250\% = 2\frac{1}{2}$ $(\frac{5}{2})$

and so on.

TO FIND THE PERCENTAGE

ORAL EXERCISE

Announce as rapidly as you can the results of the following:

1. What is $12\frac{1}{2}\%$ of 40? 80? 120? 240?
2. What is 10% of \$60? \$45? \$75? \$95?
3. What is 20% of 20? 40? 2000? 85?
4. What is $16\frac{2}{3}\%$ of \$120? \$24? \$72? \$96? \$144?
5. What part of a thing or quantity is $14\frac{2}{7}\%$ of it?
6. What is $66\frac{2}{3}\%$ of 120 men? 240 men? 36 men?
7. What is 75% of \$200? \$240? \$280? \$300?
8. What is 250% of \$60? \$150? \$900? \$1200?
9. What is 125% of 120 oxen? 560 oxen? 640 oxen?
10. What is $\frac{1}{4}\%$ of 2000 bu.? 3600 bu.? 144 bu.?

WRITTEN PROBLEMS

1. A man invests \$17280 as follows: 25% in real estate, $37\frac{1}{2}\%$ in bank stock, and the remainder in city lots. How much did he invest in each?

2. What will $37\frac{1}{2}\%$ of 480 bushels of wheat cost at \$1.25 per bushel?

3. A man buying a house and lot, paid \$1500 for the lot, and $37\frac{1}{2}\%$ more than that for the house. What did both cost?

4. A farm contained a half section; 25% of it was sold at \$45 per acre, 40% at \$37.50 per acre, and the balance at \$30 per acre. How much was received for the farm?

5. A man has a yearly salary of \$2400, and spends $33\frac{1}{3}\%$ of it the first year, 45% the second, and 64% the third. How much does he save in the three years?

6. A man deposited in a bank \$1875; he withdrew 40% of the deposit, and with $9\frac{1}{3}\%$ of the amount withdrawn purchased a gun. What was the cost of the gun?

7. A man having a salary of $3600, spends 20% of it for board, 12½% for clothing, 10% for books and lecture fees, 5% for incidentals, and deposits the remainder in the bank. How much does he deposit?

8. A man had a bank account of $6000; he withdrew 33⅓% of it, and expended 75% of the money thus withdrawn in the purchase of a piano, depositing the unexpended balance. What is his present bank account?

9. A farmer having 896 bushels of wheat, sold at one time 25% of it, and at another time 37½% of what remained. How much did he sell?

10. A speculator bought a section of land at $25 per acre: he sold 25% of the land at $40 per acre, 40% at $22.50 per acre, and the remainder at cost. How much did he gain?

11. A man started in business with a cash capital of $7200, and remained in business 3 years. The first year he gained 25% of his capital, the second year he gained 33⅓% of his increased capital, and the third year he lost 12½% of his increased capital. How much had he at the end of 3 years?

TO FIND THE RATE

Solve the following, making your computations as rapidly as possible:

ORAL EXERCISE

1. What per cent of $50 is $10? $20? $30? $40? $50?

2. What per cent of 20 pk. is 5 pk.? 10 pk.? 15 pk.? 20 pk.? 30 pk.? 40 pk.?

3. What per cent of 60 ft. is 10 ft.? 20 ft.? 30 ft.? 40 ft.? 50 ft.? 60 ft.?

4. What per cent of $500 is $250? $200? $300? $400? $100?

5. What per cent of 20 qt. is 20 qt.? 30 qt.? 40 qt.? 50 qt.? 60 qt.? 25 qt.?

6. What per cent of ¾ is ¼? ½? 1½?

7. What per cent of 480 is 240? is 120? is 360? is 960?

8. What per cent is 360 min. of 360 min.? of 720 min.? of 180 min.?

9. What per cent is $33⅓ of $100? of $16⅔? of $8⅓?

10. What per cent of 200 is $66\frac{2}{3}$? is $33\frac{1}{3}$? is $16\frac{2}{3}$? is $8\frac{1}{3}$?

11. What per cent of 120 is $16\frac{2}{3}\%$ of 180?

12. What per cent of 200 is 25% of 400?

13. What per cent of .48 is .24? is .12? is .36? is .96?

14. $33\frac{1}{3}\%$ of 180 is what per cent of 25% of 180?

15. $16\frac{2}{3}\%$ of \$90 is what per cent of $12\frac{1}{2}\%$ of \$240?

16. What per cent of an hour is 30 min.? is 20 min.? is 15 min.? is 12 min.?

17. How much more is 300 than 200? than 150?

18. What part more is 300 than 200? than 150?

19. What per cent more is 300 than 200? than 150?

20. What per cent more than 200 is 210? is 230? is 250?

21. What per cent more than \$450 is \$500? is \$540? is \$630?

22. What per cent more than 500 bu. is 700 bu.? is 800 bu.? is 900 bu.?

23. What per cent less than 90 is 80? is 70? is 60? is 40?

24. What per cent less than 200 is 190? is 180? is 150? is 120? is 100? is 75? is 50?

25. What per cent less than $12\frac{1}{2}$ is \$10? is $7\frac{1}{2}$? is \$5? is $2\frac{1}{2}$?

26. What per cent of 25 is $\frac{3}{4}$ of 100? $\frac{7}{8}$ of 200?

27. What per cent of 50 is 75 per cent of 200?

28. Two-thirds of 75 is what per cent of three-fourths of 80?

WRITTEN PROBLEMS

NOTE.—Most problems of percentage involve the relation of two or more numbers (see 169-170), the relation being expressed by a per cent rate. The first step in the problem is to see which term or quantity is used as the basis of the comparison. This may be determined either by the language of the problem or by common usage in the matter to which the problem relates. Thus, in the problems relating to loss and gain, it is the custom to regard the cost or investment as the basis of the comparison. The student should decide which of the given quantities is the base before attempting to solve the problem.

12. In a brigade of 7500 men, there were 1500 Germans, 2400 Russians, and the remainder Austrians. What is the per cent of each?

13. A man traveled 24 miles the first day, 18 miles the second and 30 miles the third. What per cent of the whole journey did he travel each day?

14. A merchant bought 500 barrels of flour, selling at one time 25 per cent of it, and at another $33\frac{1}{3}$ per cent of the remainder. How many barrels had he left, and what per cent did he sell?

15. An army during an engagement lost 2400 men, and now numbers 9600 men. What was the per cent loss?

16. A merchant having 544 barrels of flour, sold $\frac{1}{4}$ of it at one time, and $\frac{1}{2}$ of the remainder at another. What per cent remained unsold? How many barrels were left?

17. Forming a partnership, A invests $6400 and B invests $9600. What per cent of the firm's capital does each invest? What per cent is each one's money of the other's?

18. An army being reinforced by a regiment of 1200 men, now numbers 6000. What was the per cent of increase?

19. A firm's real estate is valued at $6000, personal property at $4800; the former increases 25% in value, and the latter decreases $16\frac{2}{3}\%$. What per cent is each property of the other at present valuation?

20. J. V. Farwell & Co. imported goods invoiced at $6125, and paid $1531.25 tariff. What per cent was the tariff of the invoice price?

TO FIND THE BASE

Solve rapidly as in the previous oral exercise:

ORAL EXERCISE

1. 200 is 20% of what number? 25%? $33\frac{1}{3}\%$?

2. $1.20 is 6% of what number? 40%? 50%? 1%?

3. 400 is 20% less than what number? 25%?

4. 250 pounds is 25% more than what number? 50%?

5. What number increased by $16\frac{2}{3}\%$ of itself equals 350? 875?

6. What number diminished by $33\frac{1}{3}\%$ of itself equals 72? 420?

7. What number diminished by 25% of itself equals .48? .72? .96? 9.6? 24? 2.4? 240?

8. What number increased by 25% of itself equals $\frac{5}{8}$?

9. $33\frac{1}{3}\%$ of $240 is 25% of what number?

10. $66\frac{2}{3}\%$ of 360 is 20% more than what number?

11. 25% of 144 oxen is $14\frac{2}{7}\%$ less than what number?

12. If $16\frac{2}{3}\%$ of a number is 84, what is 50% of the same number?

13. If $133\frac{1}{3}\%$ of a number is 248, what is $16\frac{2}{3}\%$ of the same number?

14. If $\frac{1}{4}\%$ of a number is 24, what is 2% of the same number?

15. If $87\frac{1}{2}\%$ of a number is 490, what is $62\frac{1}{2}\%$ of the same number?

16. If a number increased by $16\frac{2}{3}\%$ of itself equals 840, what will it equal if increased by 25% of itself?

17. If a number increased by 25% of itself equals 450, what will it equal if decreased by $37\frac{1}{2}\%$ of itself?

18. If a number decreased by $33\frac{1}{3}\%$ of itself equals 720, what will it equal if decreased by 25% of itself?

WRITTEN PROBLEMS

21. A paid me \$324.25, which is $12\frac{1}{2}\%$ of the sum he still owes me. How much did he owe me?

22. A man having a two-thirds interest in a piece of property, sells 40% of his share for \$1600. What is the value of the property?

23. I bought two farms for \$9408. I paid 24% more for one than I did for the other. How much did I pay for each?

24. The expenses of a manufacturing plant for two years were \$10125. The expenses this year were $12\frac{1}{2}\%$ greater than last year. Find the expenses for last year.

25. A merchant's sales for two years were \$8800, the sales for last year being $16\frac{2}{3}\%$ less than the sales for this year. Find the sales for each year.

26. The building, machinery, and stock of a manufacturing plant were valued at \$72000. If the building is worth 40% less than the machinery, and the stock 25% more than the other two, what is the value of each?

27. During the first year in business I gained 25%, the second year I gained 20% of my increased capital, and the third year I lost 8% of my increased capital, quitting the business with \$65136. How much was my capital stock?

28. A man withdraws 60% of the money he has deposited in a bank, and spends $16\frac{2}{3}\%$ of the money thus withdrawn in the

purchase of a house, paying $2700. How much money has he left in the bank?

29. A merchant increases his bank account 50%; he then withdraws 80% of the whole, and invests the same in real estate; his bank account is now $3600. How much did he have in bank at first?

ORAL TEST PROBLEMS

1. 37½% of a flock of sheep were killed. What per cent remained?

2. A farmer sold 12½% of a flock of 200 sheep. How many sheep had he remaining, and what per cent?

3. If 16⅔% of a man's bank account is $500, what is 66⅔% of his bank account?

4. A man sold 160 acres from a section of land. What per cent of a section had he remaining?

5. What per cent is 5 sq. mi. of 5 miles square? of 10 miles square?

6. What per cent of 25 sq. rd. is 25 rods square?

7. What is the sum of 10%, 12%, and 15% of $200?

8. What is the sum of 20% of $80, and 33⅓% of 75?

9. 33⅓% of a hogshead of molasses (63 gallons) was sold at one time, and 50% of the remainder at another. How many gallons remained?

10. What is the cost of 25% of 160 bushels of oats at 37½¢ per bushel?

11. A farmer sold a horse for $90, which was 12½% more than it cost. What did it cost him?

12. Wheat was sold at 77¢ per bushel, which is 12½% less than was paid for it. How much was paid for it?

13. I deposited $1200 in gold and silver, and 16⅔% of that amount in bank bills. How much did I deposit in all?

14. A man paid $6 for a pair of shoes and $4.50 for a hat, which was 25% of his money left. How much money had he at first?

15. From a cask containing 45 gallons of kerosene, 15 gallons were sold. What per cent was sold?

16. A gentleman having $1200, lost 25% of it. How much had he left?

17. If 10% of a journey is 25 miles, what is 20% of the same journey? 30%? 40%? 50%? 75%? 100%?

18. 25% of a man's salary is $80. What is $6\frac{1}{4}\%$ of it? $12\frac{1}{2}\%$? 50%? 75%?

19. In a box of berries $\frac{1}{4}$ were damaged. What per cent was good?

20. If a pint of water is added to every gallon of milk, what per cent of the whole is pure milk?

21. A man, owning $\frac{2}{3}$ of a factory, sells $\frac{2}{3}$ of his share. What per cent of the factory has he left?

22. A grocer, after selling $\frac{7}{8}$ of a barrel of sugar, has 35 pounds left. What per cent did he sell?

23. A man's property invested at 5% yields him an income of $120 per month. How much has he invested?

24. Three sections of land are what per cent of a government township?

25. The population of a city is 17280, showing an average gain of 20% per year for three years. What was its population three years ago?

26. A city of 12500 has gained 25% on its population of five years ago, which was 25% gain on its population of ten years ago. What was its population ten years ago?

27. A father is 60% older than his son, and the difference between their ages is 30 years. How old is the father?

28. A horse and carriage cost $300, the horse costing 50% more than the carriage. How much did each cost?

30. A man owning 60% of a store sells $33\frac{1}{3}\%$ of his share for $2000. What is the remainder of his interest worth? What is the entire store worth?

31. A man's income is $1600 per year, from which he saves $400. What per cent does he spend?

32. A man had a half interest in a foundry and sold one-half of his share for $2500. What did the foundry cost, the sale being made at the rate of 25% more than the cost?

33. A store rents for $600 per year, which is 10% of its value. What was its cost, provided its value is 20% more than its cost?

34. If a pupil attends school 15 days in March, 18 days in April, and 17 days in May, what is his percentage of attendance, the term of school lasting 12 weeks of 5 days per week?

35. If a man's debts amount to $1800 and his property to $1200, what per cent can he pay on his debts?

36. If an agent sells merchandise for his principal to the amount of $4000 and remits to him cash $3950, what per cent of the sales does he retain?

37. If a farm yields 300 bushels of wheat one year and 400 bushels in the next, what is the per cent of increase?

38. A man withdraws 40% of his bank account, spending 25% of the amount thus withdrawn in the purchase of a piano, paying $240. How much money had he in the bank?

39. If I own 75% of a mill and sell 33⅓% of my share for $1800, what would be the value of the mill at the same rate?

WRITTEN TEST PROBLEMS

30. A drover lost 15% of a flock of sheep and had 340 head left. How many did he lose?

31. In a city of 25000, the deaths for a certain year were 1050. What was the per cent of deaths?

32. If there are 30 scholars enrolled in a winter school, whose term is 16 weeks, and there are altogether 500 days lost by different scholars, what is the average per cent of attendance?

33. A man whose income is $2000, spends $1850 of it. What per cent does he save?

34. A man having bought a section of land gave a half-quarter to each of his five children. What per cent of the section had he left, and how many acres?

35. A farmer sold 25% of 72% of 3000 head of sheep, at $2.75 per head. How much did he receive?

36. A man owning 50% of a foundry, exchanged 50% of his share for a business lot 40 by 140 feet, at $1.25 per sq. ft. At that rate, what was the foundry worth?

37. A merchant pays $600 for a year's rent of his store room, which amount is 20% of his income, and his income is 10% of his sales. How much are his yearly sales?

38. A man owes $6500, which is 25% more than his property is worth. What per cent can he pay?

39. A man drew 20% of his money from the bank, and paid 20% of the money thus withdrawn for three months' rent at the rate of $960 per year. How much has he left in the bank?

40. A man planted an orchard of 2880 apple and pear trees; there were 120% more apple trees than pear trees. How many of each?

41. A has $360, B has $540. What per cent is A's money of B's? What per cent is B's money of A's?

42. The profits of a business were 25% less the second year than the first year, and during the two years the profits were $13125. What were the profits for the first year? What per cent greater were the first year's profits than the second?

43. A man left $28750 to his two sons, giving to the older 30% more than to the younger; at the end of five years the younger has gained 30% on what he received, and the older has lost 30%. How much has each one now?

44. A and B each desire to sell a house and lot to C. A asks 50% more than B; A then lowers his price 20%; B lowers his in the same ratio. C now takes both properties, paying $14780. What was each one's asking price?

45. A paid $55000 for a bill of merchandise. He sold part for $60000, of which $\frac{1}{3}$ was at a profit of 25%, and $\frac{2}{3}$ at a profit of $33\frac{1}{3}\%$. Find the cost price of the remaining merchandise.

46. Two brothers receive by bequest $3600 each. One's money increases at the rate of 12% per annum for 5 years, while the other's increases at the rate of 10% per annum for the same time. How much more has the one than the other?

47. After I have spent 50% of my money and 25% of the remainder, I find that I have left $3375. How much money had I at first?

48. A merchant invested $42500 in dry goods. The first year he gained 20%; the second year he gained $12\frac{1}{2}\%$; the third year he lost $16\frac{2}{3}\%$; and the fourth year he gained 5%. Find the amount invested in business at the end of the fourth year, the gain being added or the loss subtracted each year.

49. A's farm is 1 mile long and 120 rods wide. He sold 40% of it at $36 per acre at one time, and $37\frac{1}{2}\%$ of the remainder at $40.50 per acre at another time. What was the total amount received, if the balance was sold at $25 per acre?

50. On commencing business a merchant had $5000. The first year he gained 25%, the second year he gained $37\frac{1}{2}\%$, and the third year he gained 20%. What is his present capital, if he left 80% of each year's gain in the business?

CHAPTER XIII

COMMERCIAL DISCOUNTS

JUDGMENT AND COMMON SENSE IN ARITHMETIC

343. The student is now to enter upon the very important matter of applying the principles of percentage to business affairs. At this stage of his progress, a few general suggestions may prove helpful.

The student should endeavor to use his judgment, as well as his memory in solving problems. Anyone who understands the simplest processes of arithmetic can obtain answers to the most intricate problems, if they come to him labeled with topic, case, section, and rule.

The man of business does not meet his problems thus carefully labeled. He must use judgment as to what principle of arithmetic to apply. He cannot keep a thousand rules stored up in his crowded brain, or if he succeeds in memorizing them, he cannot keep them all polished for immediate use. He must make himself master of principles and use common sense in their application. Realizing this, it has been our aim to present in the following chapters, only such problems as the student will be likely to meet with in practical affairs. We have not assorted and grouped all these under specified cases, nor have we laid down formal rules and methods to guide the student in his solutions, especially where, with a reasonable exercise of judgment, he should be able to apply the principles for himself.

Before attempting to solve a problem, the student is advised to study its statements in all their relations to each other and, reasoning from the nature of the case, obtain an approximate answer. A moment's thought will frequently save the student a half hour's extra calculation by starting him right and, what is still more important, will often aid him in detecting an absurd answer obtained from a proper application of the rule, but incorrect on account of inaccuracies in calculation.

Let us use the high endowments of reason and judgment whenever it is possible, and not allow ourselves to degenerate into mere mathematical machines.

344. A **commercial** or **trade discount** is a specified percentage deducted from the cost of goods as computed at list, or catalog prices.

Manufacturers and wholesalers usually issue printed lists with prices of the goods they sell. These prices are usually higher than those at which the goods are actually sold. This plan is adopted in order that it may not be necessary to print new price-lists every time the market price of the goods rises or falls. The actual selling price is found by deducting a certain per cent from the list price, the amount of this percentage varying with the state of the market. In billing commodities, the list discount would not be deducted from the price per item, but from the amount computed at the list price.

A buyer may order an unusually large quantity of an article and receive a second discount for quantity.

Price lists are usually prepared on a basis of *credit sales*, and the cash buyer is usually allowed an additional discount which is given to encourage cash purchases.

It is the general business custom to allow as cash terms, any purchase that is paid for within 30 days from date of bill.

345. **"Spot cash"** is a term implying the payment for goods on delivery.

A discount for cash is usually allowed if the goods are paid for within 30 days, and sometimes an additional discount is given for spot cash or for payment within 10 days.

The term *net*, as used in prices, means "not subject to discount." It also implies the amount of a bill of items after discounts have been deducted.

346. By **the terms of a bill,** is meant the conditions of sale, as to time and discounts allowed. The terms of a bill are recorded on its face, certain understood abbreviations and symbols being used. Thus, Terms: "5 30 n/60" means "5% 30 days, net 60 days," that is, that a discount of 5% is to be allowed if payment is made within 30 days and that the face or "net" amount of the bill is to be paid at the end of 60 days.

347. **Series discount.** When several consecutive discounts are allowed, as, for instance, a discount of 20% from list, a further

discount of 10% for quantity and a third discount of 5% for cash, the several discounts are called a *series discount*, and are indicated "20-10-5," and read, "twenty, ten, and five off." Discounts from list prices are sometimes expressed in series. Series discounts are not to be *added* and computed as one discount, but are computed consecutively.

EXAMPLE: Find the discount and net amount of a bill of $280, which is subject to a series discount of 25-20-5.

FIRST OPERATION

4)280
70 1st dis.
5)210
42 2nd dis.
20)168
8.40 3rd dis.
159.60 Net amount.

$ 70.
42.
8.40
$120.40 discount

SECOND OPERATION

1.00
.25
5) .75
.15
20) .60
.03
.57

$ 1.00 — $ .57 = $ .43
280. × .43 = 120.40 dis.
280. — 120.40 = 159.60 net amt.

EXPLANATION.—In the first operation, we compute and subtract the discounts separately, the last subtraction yielding the net amount of the bill. We then add the several discounts.

In the second operation, we first reduce the series to a single rate, then compute the discount, and subtract it from the face of the bill to obtain the net amount. Accountants use both methods, the choice depending on the numbers involved.

Series discounts may also be reduced to a single discount by subtracting the rates separately from 100%, multiplying together the several remainders, and subtracting this result from 100%. Thus, $1.00 - .25 = .75$; $1.00 - .20 = .80$; $1.00 - .05 = .95$. $.75 \times .80 \times .95$ $(= \frac{3}{4} \times \frac{4}{5} \times \frac{95}{100} = \frac{57}{100}) = 57\%$. $100\% - 57\% = 43\%$.

In some cases this method is most convenient. Thus, the computation for the series $33\frac{1}{3}$-25-20, would be $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$ or $\frac{2}{5} = 40\%$. Thus, $100\% - 40\% = 60\%$.

The result of a series computation is, of course, the same regardless of the order in which the rates are taken.

It is very important for accountants to be able to compute discounts mentally as much as practicable, taking advantage of the table given in paragraph 342.

ORAL EXERCISE

1. Find the discount at 20% on each of the following amounts: Increase the resulting discount by any remainder of a half cent or more.

$ 7.25	$135.50	$ 9.36	$ 8.19
15.50	65.40	12.25	18.20
21.20	32.50	14.26	135.15
25.40	17.27	29.50	48.46

2. Name at sight the discount at 15% of each of the following. ($15\% = 1\frac{1}{2}$ times 10%):

$ 12.00	$ 16.20	$ 22.00	$520.00
6.40	24.50	10.50	75.00
140.00	54.00	72.00	51.00
8.60	150.00	250.00	28.50

3. Reduce the following series rates to single rates:

20% and 10%	20% and 15%	30% and 20%
10% and 10%	25% and 20%	60%, 40% and 10%
25% and 10%	10% and 5%	50%, 30% and 10%
50% and 20%	10% and 6%	25%, 20% and 2%

4. Find the following net amounts:

$45.00 less 10%	$ 12.40 less 25%	$ 80.00 less 25% and 20%
85.00 less 20%	80.00 less 5%	150.00 less 20% and 10%
60.00 less 15%	120.00 less 10%	50.00 less 10% and 5%
7.50 less 20%	160.00 less 15%	75.00 less 20% and 2%

WRITTEN EXERCISE

At twenty, ten, and five off, find the net amounts of the following, reducing series to single rates:

1.	$67.50	*3.*	$39.50	*5.*	$ 55.70	*7.*	$17.50	*9.*	$75.25
2.	45.90	*4.*	8.97	*6.*	128.00	*8.*	16.80	*10.*	41.60

Reduce the following to single rates:

11.	60%, 25%, 10%	*14.*	50%, 40%, 20%
12.	40%, 20%, 5%	*15.*	40%, 25%, 10%
13.	25%, 15%, 2%	*16.*	30%, 10%, 5%

Find the discount and net amounts of the following:

	Amt. of Bill	Rate of Discount
17.	\$625	10 % and 10 %
18.	\$324	25 % and 20 %
19.	\$600	$33\frac{1}{3}$% and 25 %
20.	\$720	$16\frac{2}{3}$% and $33\frac{1}{3}$%
21.	\$327.85	50 % and $12\frac{1}{2}$%
22.	\$348.20	20 %, 25 % and 10 %
23.	\$462	10 %, 10 % and 10 %
24.	\$850	30 %, 20 % and 10 %
25.	\$325.50	25 %, $12\frac{1}{2}$% and 10 %
26.	\$144	20 %, 5 % and $2\frac{1}{2}$%
27.	\$360	50 %, 20 % and 5 %
28.	\$ 45.50	60 %, 20 % and $2\frac{1}{2}$%
29.	\$120	65 %, 10 % and 10 %
30.	\$324	$33\frac{1}{3}$%, 10 % and 20 %
31.	\$127	$66\frac{2}{3}$%, 10 % and 5 %
32.	\$316.80	40 %, 25 % and 10 %
33.	\$415	10 %, 10 % and 5 %
34.	\$243.50	5 %, $2\frac{1}{2}$% and $2\frac{1}{2}$%
35.	\$142	$12\frac{1}{2}$%, $2\frac{1}{2}$% and $2\frac{1}{2}$%
36.	\$421	$37\frac{1}{2}$%, 5 % and $2\frac{1}{2}$%

37. What is the net value of a bill of goods amounting to \$120 per list price, the rate of discount being $33\frac{1}{3}$% and 5%.

38. Find the net of a bill of wire screens amounting to \$1265.70, discount off 25%, 20%, and 10%.

39. Bought a bill of hardware amounting to \$1560; of this \$125 less 10% and 10%; \$268 less 40% and 10%; \$600 less 25% and 20%; the balance of the bill $33\frac{1}{3}$% and 10%. What is the net price?

40. The list price of a bill of hose, pipes, couplings, etc., was \$760.50. What was the net price, discounts of 25% and 20% being allowed?

41. What is the cost of 15 governors at \$15.50 each, the rates of discount being $33\frac{1}{3}$% and 10%?

42. The gross amount of a bill of boots and shoes was \$1260; of which \$200 was subject to discounts of 25% and 10%; \$350 to 20%, 10%, and 10%; and the balance to $33\frac{1}{3}$%. What was the net of the bill?

43. Bought the following bill of hardware of H. B. & S., Aug. 24, 6% for cash, 5% 10 da., 4% 30 da.:

$ 72	less a	discount of	3-10's (10, 10, and 10%)
$124	"	"	25%, 20%, and 10%
$324.60	"	"	60% and 10%
$125.53	"	"	40% and 25%
$163.82	"	"	60%, 10%, and 5%
$224	"	"	20%, 12½%, and 2½%
$216.30	"	"	10%, 5%, 2½%, and 2½%
$125	"	"	50% and 50%
$178.60	"	"	25%
$194.50	"	"	75% and 10%
$175	"	"	50%
$142	"	"	60% and 25%
$ 36	"	"	62½% and 10%
$120		billed	Net
$142		"	Net

What was the net on the date of sale? How much would pay the bill Sept. 2? How much would pay the bill any time between Sept 14 and Sept. 23 inclusive?

44. The net amount of a bill is $417.60. If a series discount of 20% and 10% is allowed, what is the face of the bill?

Suggestion.—What was the single rate of discount? After this rate had been deducted, what per cent of the bill remained?

45. On May 13, I bought a bill of stationery at 10/30, 5/60, n 90, amounting to $165.35. What sum will pay the bill on June 1? On July 6? On July 20?

46. A publishing house allows a discount of 15% from its list prices, and a further discount of 5% for cash. A customer, Mr. A, who paid cash for all his purchases, remitted to the publishing company, during the year 1907, $565.25. What was the amount of his orders estimated at list prices?

47. On Jan. 15, 1907, a retail grocer bought of a wholesaler, a bill of goods amounting to $756.15, a discount of 10% to be allowed on any part of the bill that should be paid within 30 days. On Feb. 10 the grocer sent to the wholesaler a check for $500. What credit should he have for this payment?

48. On a purchase of goods where a discount of 25, 20 and 5 was allowed, the discount amounted to $16.40. What was the amount of the bill?

49. Made a cash payment of $75 within a 15% cash discount period. What is the amount of the credit?

NOTE.—Strictly speaking, a part payment of a bill is not subject to a discount allowance, nevertheless, such allowances are very commonly made. Accountants often find the credit by adding the discount to the payment. Thus, if $50 is paid on a bill subject to a 5% discount, 5% of $50 or $2.50 is added to the payment, making a credit of $52.50. This method is not accurate, however, since the payment is taken as the *base* of the calculation, whereas, the base is the payment plus the discount.

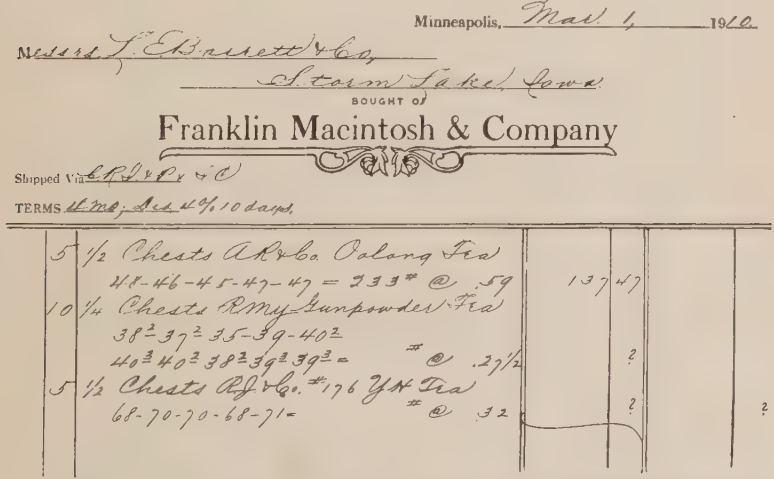
50. A buyer made a payment of $250 within a 10% discount period. The seller added the discount to the payment, instead of treating the payment as 90% of the required credit. What was the amount of the error, and was it in favor of the buyer, or the seller?

51. Abbott & Co., wholesalers, owe a retailer the sum of $428.60, cash on demand. The retailer agrees to receive the amount in goods at regular prices, less a cash discount of 5%. What is the amount of the bill?

52. I buy goods at 25%, 20%, and 10% off. What is the net price of the goods, the total discount being $211.14?

53. A bill for 25 stoves amounted to $567 net, after a cash discount of 10% and a list discount of 30% had been allowed. What was the list price per stove?

54.



Minneapolis, Mar. 1, 1910.

Messrs. L. E. Barrett & Co.,
Storm Lake, Iowa.

BOUGHT OF

Franklin Macintosh & Company

Shipped Via C.R.I.&P.& C.

TERMS 4 mo; Dis. 4% 10 days.

5	1/2 Chests A.R.&Co. Oolong Tea 48-46-45-47-47 = 233# @ .59	137	47		
10	1/4 Chests R.M.y Gunpowder Tea 38^2-37^2-35-39-40^2 40^3-40^2-38^2-39^3-39^3 = # @ .27 1/2		?		
5	1/2 Chests R.J.&Co. #176 Y.H. Tea 68-70-70-68-71 = # @ .32		?		?

What is the net, and how much would settle the bill March 10, after date of bill?

55.

St. Louis, Mo., Mar. 1, 1910

Messrs Johnson & Garver,
Columbia, Mo.

Bought of **SMYTHE FURNITURE CO.**

TERMS: Cash 5%, 30 days 2%.

2	doz Kitchen Tables ea	4.20	10.80	
	Less 10%		1.08	9.72
4	doz Common Lounges ea	9.00		
	Less 15%			
15	Walnut Ext Tables	16.00		
	Less 12½%			
6	doz Dining Chairs pr doz	1.25		
¾	" Cottage Bedsteads ea	4.75		
7	Walnut M.T. Center Tables	12.50		
	Less 20%			
3	doz M O Bedroom Sets ea	25.00		
	Less 15%			
1	doz Cherry Bedroom Sets ea	32.00		
	Less 25%			
½	doz Birds Eye M. Sets ea	75.00		
	Less 33⅓%			
7	Office Desks, Oak ea	24.00		
	Less 10 & 10%			
¾	doz Rockers Uph pr doz	36.00		
	Less 20%			
3	doz Fancy Baby Cabs ea	11.50		
	Less 25%			

What is the net of the bill? How much would settle the bill on date of sale? How much would settle the bill March 11, following the date of the bill?

56. Bought of J. V. Farwell & Co., wholesale merchants: 15 brl. A sugar, each 327-36, at 6¢; 10 sk. Rio coffee, each 155-3, at 12¢; 9 ch. Young Hyson tea, each 95-8, at 37½¢; 12 ch. Japan tea, each 76-12, at 40¢; 14 bx. laundry soap, each 74-14, at 4½¢; 12 brl. kerosene, each 45 gal., at 14¢. I am allowed a list discount of 20%, and a second discount of 5% on the first three items. What is the net amount of the bill?

57.

Cincinnati, Apl 25, 1910.

Messrs Gregory & Co,
Minneapolis, Minn.

BOUGHT OF

Hibbard, Bartlett, Spencer & Co.

TERMS 30 DAYS { 1% deduction if paid within 10 days
TERMS 60 DAYS { 2% deduction if paid within 10 days

		Extension	30 days		60 days	
5	doz. 3½" to 4½" Axes 10—				50	
2	" 1½" Hammers 7—					
2	C ⅜ x 6½ Bolts 2 26					
2	C ½ x 3 " 3 60					
	Less 40-10-10%					
1	C ½ x 4 Log Screws	5 25				
2	C ½ x 5 " " 5 25					
3	C ⅝ x 2½ " " 2 25					
	Less 60-5%					
2	doz #10 Hand Saws 18 00					
3	" #111 Mortiser 4 25					
1	" #00 Wash Basins					80
3	" 30 qt Pans 12—					
2	" #106 O+B Shears 3 75					
2	" #107 " " 4 50					
2	" #108 " " 4 85					
2	" 6" Wilson's Butchers 2 25					
2	" 7" " " 3 15					
2	" 8" " " 4 10					
2	" #50 Razor Strops 2 25					
2	grs. #900 Knives & Forks 4—					
2	doz. Putz Pomade 65					
			15			

What is the net? How much would settle the bill on May 1 after date of purchase?

58.

Omaha, Nebr., July 15, 1910.

Mr. George Hunter

Fremont, Nebr.

BOUGHT OF

FOSKETT & COMPANY

Shipped Via C & N W.

TERMS: 2% 10 days

#5048	348 9 ft in	Pipe 1"	11¾	40	98		
		Less 50-10-5%		23	46	17	52
	109 4 ft in	Pipe 5"	1.42		?		
		Less 60-7½%			?		?
	16	Globe Valves 1"	2.90		?		
		Less 50-10-5%			?		?
	60	Nipples 1x8	47		?		
		Less 80-10-5%			?		?
	75	Couplings 2"	60		?		
		Less 65-5%			?		?
	25	Flanges 1x6	.95		?		
		Less 80-10%			?		?
	125	Ells	.35		?		
		Less 75-10%			?		?
	55	Tees 2½x2x1	1.75		?		
		Less 60-10-5%			?		?
	6	Water Gauges	12		?		
		Less 50-10-2½%			?		?
	10	Oil Cups 2½	3.25		?		
		Less 40-5-2½%			?		?
	6	Flue Cleaners	4.25		?		
		Less 50-12½-5%			?		?
							?

What is the net of the bill?

59.

CHICAGO, Apr. 28, 1910

Messrs Grayson & Co.,

Clinton, Iowa

BOUGHT OF

H. C. JURGENSON & CO.

VIA C & N W Ry.

TERMS 30 d; 1% 10 d.

5	brl Empire A Sugar 362/361/364/356/358 = 1801# @ 5.04	9077		
10	brl Fine Gran Sugar 354/353/318/325/304/314 324/319/317/304/=3232# @ 5.60		?	
25	bx Morgan Evap Raspberries 25 x 50# = @ .20½		?	
30	bx Harlem Evap Blackberries 30 x 25# = @ .07⅜		?	
85	cs. Pacific 1# Salmon 85 x 4 doz = per doz 1.45		?	
39	bx Apricots 39 x 50# = @ .06½		?	

What is the net, and how much would settle the bill on May 1, following the date of the bill?

60. A merchant bought goods at 30% and 10% off and sold them at list cost without discount. What per cent profit did he make?

61. What should be the list selling price of wagons costing $80 net, in order to sell at a profit of 25% after allowing a list discount of 25%?

62. A merchant bought three special invoices of goods as follows:

$320.50 at 20% and 10%.
152.00 at 10% and 10%.
217.50 at 20% and 20%.

He sold the entire lot at list buying prices less 10% and 5%. What was his net profit after allowing $50 for freight and selling expenses?

CHAPTER XIV

LOSS AND GAIN

348. Gains, or profits and losses, in business, are commonly computed as percentages of the cost or (in case of a single transaction) of the investment.

Several classes of problems arise, as, to find the rate of gain when the cost and amount of gain are given, or to find the cost when the sales and rate of gain are given, etc. There are no principles involved other than those applied in Chapter XII. That is, we may have to find any one of the three general percentage terms, rate, base, or percentage, and the student should be sure that he understands the conditions and requirements of a problem before attempting to solve it. Do not experiment in a mere attempt "to get the answer," but reason everything out and be sure you are right before proceeding to the solution. Keep in mind the fact that the *cost* or *investment* is always the basis of the percentage computation.

ORAL EXERCISE

Announce the amount gained for each of the following costs and rates:

	Cost	Rate Gain		Cost	Rate Gain		Cost	Rate Gain
1.	\$ 75.00	5 %	*7.*	\$560.00	25 %	*13.*	\$2500.00	3 %
2.	480.00	$12\frac{1}{2}$%	*8.*	48.00	$2\frac{1}{2}$%	*14.*	960.00	$6\frac{1}{4}$%
3.	71.45	10 %	*9.*	64.00	$7\frac{1}{2}$%	*15.*	75.00	$22\frac{1}{2}$%
4.	19.25	2 %	*10.*	19.15	20 %	*16.*	148.00	5 %
5.	50.00	$33\frac{1}{3}$%	*11.*	250.00	40 %	*17.*	9000.00	12 %
6.	85.65	20 %	*12.*	84.00	15 %	*18.*	1600.00	$7\frac{1}{2}$%

Announce the sales, at the following costs and rates of gain:

	Cost	Rate Gain		Cost	Rate Gain		Cost	Rate Gain
19.	\$200.00	30 %	*24.*	\$250.00	5 %	*29.*	\$900.00	25 %
20.	85.00	20 %	*25.*	660.00	$16\frac{2}{3}$%	*30.*	19.25	20 %
21.	570.00	$33\frac{1}{3}$%	*26.*	2.40	$12\frac{1}{2}$%	*31.*	18.00	2 %
22.	12.40	25 %	*27.*	40.50	20 %	*32.*	12.50	5 %
23.	8.50	10 %	*28.*	32.00	$7\frac{1}{2}$%	*33.*	24.40	$12\frac{1}{2}$%

Announce the rate of gain or loss in the following:

	Cost	Gain		Cost	Loss
34.	$ 75.00	$15.00	*39.*	$200.00	$17.00
35.	2.50	1.25	*40.*	600.00	18.00
36.	48.00	.96	*41.*	450.00	22.50
37.	150.00	25.00	*42.*	500.00	12.50
38.	.28	.03½	*43.*	125.00	5.00
	Cost	Sales		Cost	Sales
44.	$ 60.00	$ 50.00	*49.*	$ 4.50	$ 6.00
45.	125.00	100.00	*50.*	25.00	23.00
46.	40.00	56.00	*51.*	12.00	11.00
47.	.75	.90	*52.*	1.20	1.08
48.	66.00	60.00	*53.*	5.00	5.25

ORAL PROBLEMS

54. I sold 20 lb. coffee for $4, gaining 25%. How much did it cost me per lb.?

55. What was the per cent of loss on oranges bought for $5 per hundred, and sold at 4¢ apiece?

56. Mr. A bought a horse for $90, and sold him for $105. What per cent did he gain?

57. Seven gallons of oil leaked from a barrel holding 42 gallons. What per cent was there left?

58. A boy bought apples at the rate of 3 for 2¢, and sold them at the rate of 2 for 3¢. What per cent did he gain?

59. A merchant sold a carpet for $35, gaining 25%. What did it cost?

60. A jeweler sold a watch for $27, losing 10%. How much did he lose?

61. When any article is sold for $\frac{6}{5}$ of its cost, what is the per cent gain?

62. When any article is sold for $\frac{5}{6}$ of its cost, what is the per cent loss?

63. What is the greatest per cent loss that any article may have? The greatest per cent gain?

64. Paid $10 for a dog, and sold him so as to gain 100%. What did I receive for him?

65. By selling peaches at 3 for 5¢, I gain 25%. What must I sell them for to gain 50%?

66. Paid $1 for 8 qt. of berries, 2 qt. of which soured and became unsalable. What was my per cent loss?

67. If I buy apples at the rate of $1\frac{1}{3}$ for 1¢, and sell them at the rate of $1\frac{1}{2}$¢ apiece, what is my per cent gain?

68. One-half of my property is what per cent of $\frac{2}{3}$ of it?

69. Sold a farm for $500 less than it cost, thereby losing 8%. What did it cost?

70. A newsboy gains $\frac{1}{3}$ of the cost of his papers by selling them at 4¢. What per cent would he gain by selling them at 5¢?

71. Coal costs $4.50 per ton, and sells for $5.25. What is the per cent gain?

72. If the materials in a glass of soda water cost $1\frac{1}{4}$¢ and it sells for 5¢, what is the per cent gain?

73. I paid $200 for a horse. How much shall I ask for him that I may gain 20%?

74. I paid $800 for a house, and after spending $200 for repairs, I sold it for $1250. What per cent did I gain?

75. Sold a quantity of fruit at a profit of $16\frac{2}{3}$%, and gained $150. What did the fruit cost me?

76. Sold a farm for $1200, losing 20%. For how much should I have sold it to have gained 20%?

77. I sold 2 horses at $100 each. On the first I gained 20%, on the second I lost 20%. How much did I gain or lose on the whole transaction?

WRITTEN TEST PROBLEMS

1. Bought a horse for $100, paid $25 for his keeping and sold him for $150. What per cent did I gain on the entire investment?

2. Tea sold at 75¢ per pound produces a loss of $16\frac{2}{3}$%. What would be the gain per cent if sold at $1 per pound?

3. Mr. B bought a section of land of the government, paying $2.50 per acre. He sold $\frac{3}{4}$ of it at $7.50 per acre, and the remainder at $8 per acre. What was his gain per cent?

4. A sold a farm to B and gained 20%. B sold it to C for $11000, losing 20%. What did the farm cost A?

5. A merchant's expenses are as follows: Rent $600, clerk hire $1500, sundry items $100, advertising $250. What must be the amount of his annual sales, in order that they may average $16\frac{2}{3}$% above the cost, and allow him a gain of $2000 above expenses?

6. I sold two houses at $6000 each. On one I gained 25%, and on the other I lost 25%. Did I gain or lose on the whole and how much?

7. I bought two houses at $6000 each. In the sales I cleared 25% on one, and lost 25% on the other. Did I gain or lose on the whole, and how much?

8. A house and lot were offered for sale at an advance of 50% above their cost; but the agent sold the house at 20% below the asking price, receiving $500 more than the cost. What is the owner's per cent gain, after allowing the agent 2% for his services?

9. If sugar costs $4\frac{1}{2}$¢ per pound, and loses 10% in drying out, down weights, and bad accounts, how much must a grocer ask for the sugar in order to make a net profit of $12\frac{1}{2}$%?

10. If a grocer sells $\frac{1}{2}$ of a bushel of apples for the same that he paid for $\frac{2}{3}$ of a bushel, what is his per cent gain?

11. Which merchant makes the greater average per cent gain and how much, A, who sells at an average profit of $12\frac{1}{2}$%, changing his stock every four months; or B, who sells at an average profit of $16\frac{2}{3}$%, changing his stock every six months?

12. A grocer bought 200 barrels of apples at $1.50 per barrel, and sold 75 barrels at a profit of $16\frac{2}{3}$%, and 100 barrels at a profit of 25%. At what price per barrel must he sell the remainder, to net 20% on the whole deal?

13. A speculator bought a farm for $9000. It cost him $350 a year to pay the taxes and cost of repairs. If he receives a rental of $3375 for a three years' lease, what rate per cent per annum does he make on the investment?

14. A merchant bought carpeting at $1.20 per yard, and marked it to sell at an advance of 50%, but sold at a reduction of 10% from his asking price, thereby making $39.90. How many yards did he sell?

15. I invest and sell at a loss of 25%; I invest the proceeds of sale and sell at a gain of 25%. Do I gain or lose on the two transactions, and what per cent?

16. I sell at 25% gain, and invest the proceeds and sell at an advance of 10%. I again invest the proceeds and sell at a loss of 5%, and find that I have $7440.40. What amount did I start with?

17. A merchant bought 320 yards of broadcloth at $4 per yard, and sold 25% of it at a profit of 25¢ per yard, and 50% of the remainder at $4.50 per yard. At what price per yard must he sell the remainder to net $37\frac{1}{2}$% on the whole?

18. My agent sold an invoice of flour at an advance of $16\frac{2}{3}\%$. I instructed him to invest the sum in corn, which he sold at a profit of 30%. My net profit on the two transactions was $607, the agent retaining $75 of the last sale for his work. What did he receive for each sale?

19. A man bought $\frac{3}{4}$ of a farm and sold $\frac{3}{4}$ of his share for $8100, which was at a profit of $33\frac{1}{3}\%$. What was the cost of the farm?

20. What is my per cent gain, if 20% of what I receive for an article is gain?

21. I bought land at $36 per acre, intending to ask $72 per acre for it. What abatement may I allow and still make 25% profit?

22. A grocer bought 400 barrels of apples at $2.40 per barrel. He sold 90 barrels at a gain of $16\frac{2}{3}\%$, 164 barrels at a gain of $33\frac{1}{3}\%$, 100 at a loss of 5%, and the remainder at $1.50. What was his per cent of loss or gain?

23. A merchant's loss at wholesale was 4%, his retail price being 25% in advance of the wholesale price. What was his per cent gain at retail?

24. A jeweler bought a watch for $24. How much must he ask for it, that he may offer an abatement of 25% and still net 25% profit?

25. A merchant buys a bill of goods for $7500 and sells $\frac{1}{2}$ at a profit of 25%; he sells the rest at an advance of 10¢ per yard, receiving 40¢ per yard. What is his total profit?

26. I invest and sell at a loss of 15%; I invest the proceeds again and sell at a gain of 15%. Do I gain or lose on the two speculations, and what per cent?

27. I sold at 8% gain, invested the proceeds and sold at an advance of $12\frac{1}{2}\%$. I invested the proceeds again and sold at 4% loss, and quit with $2332.80. What was my first investment?

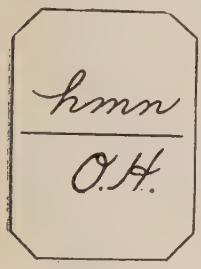
28. When beef is selling at $12.50 per barrel, I lose 20%. What would be my loss or gain if sold at $20 per barrel?

29. A real estate dealer sold two building lots for $6000, thereby gaining 20% on one, and losing 20% on the other. The selling price for the first lot was 50% more than for the other. How much was the cost for each?

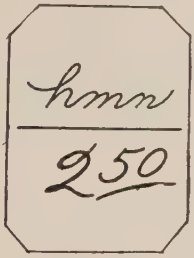
30. A merchant's sales for two years were $39600; his sales this year were 25% more than his sales last year. What were his sales last year?

PRICE MARKING

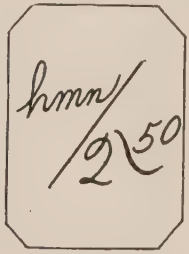
349. In merchandising, it is customary to attach tickets to the goods, showing the cost and sometimes the sale price. Merchants often wish to keep strictly private, the cost of the goods, and sometimes the regular selling price. To effect this, various "ciphers" or "keys" are used, these being known to the merchant himself or his salesmen.



No. 1.



No. 2.



No. 3.

A price key consists of ten letters, or other characters, one for each of the ten figures, with an extra character known as a "repeater" to use in case a figure occurs more than once in a price, and sometimes characters to represent the fractions ½, ¼, etc. Commonly some word or phrase containing ten different letters is used as a key, as for instance, "Now be sharp," "Cumberland," "An old tribe," "Cash profit."

For the exercises of this lesson, the following keys will be used:

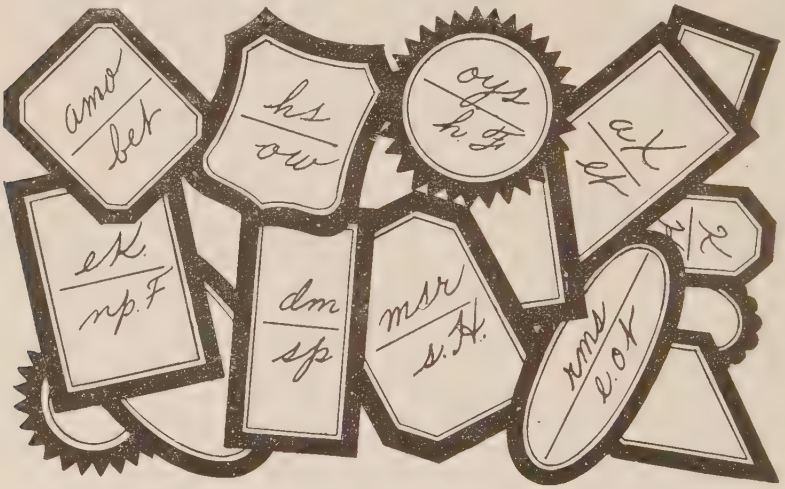
Cost	Sales
h a r d m o n e y s	n o w b e s h a r p
1 2 3 4 5 6 7 8 9 0	1 2 3 4 5 6 7 8 9 0
Repeater, X.	Repeater, t.
Fractions: ¼ K. ½ B.	Fractions: ¼ F. ½ H.

In entering prices marked on the tickets, the cost is usually written first, and the sale price below with a line between them. Thus, a hat that cost $1.57, and was to be sold for $2.50, 2½ might with the keys here given, appear as in No. 1. Sometimes the key is not used for the sale price, as in No. 2. An oblique line is often used to separate the cost and sale price mark as in No. 3.

The question as to whether the key indicates dollars, cents, or dollars and cents, is easily determined by the kind or qualiay of the goods. For the students' guidance in interpreting the price marks here given, a decimal point is used.

ORAL EXERCISE

Using the keys given in 349, find the cost and sale price from each of the following tickets:



1. Using the same keys, announce the cost and sale marks for the following as required:

	Cost	Selling		Cost	Gain
1.	$3.20	$5.25	*7.*	$3.60	25 %
2.	$1.20	$3.45	*8.*	$1.25	20 %
3.	$ .75	$1.26	*9.*	$ .80	50 %
4.	$ .60	$1.50	*10.*	$ .12	$33\frac{1}{3}$%
5.	$ .16	$ .32	*11.*	$ .06	$16\frac{2}{3}$%
6.	$ .08	$ .12	*12.*	$ .18	$33\frac{1}{3}$%

WRITTEN EXERCISE

Mark the following prices as required:

	Cost	Selling		Cost	Gain
31.	$1.20	$2.35	*35.*	$1.50	60 %
32.	$3.40	$5.25	*36.*	$1.44	$16\frac{2}{3}$%
33.	$ .56	$ .84	*37.*	$ .36	$33\frac{1}{3}$%
34.	$ .36	$ .72	*38.*	$3.50	10 %

Find marks (each single article) for the following:

39. Hats costing $18 per dozen and sold at $2.25 each.

40. Cassimere costing $2.40 per yard sold at profit of 35%.

41. Shoes costing $27 per dozen sold at advance of 33⅓%.

42. Broadcloth sold at $9.60 per yard giving a profit of 25%.

A great many articles are bought by the dozen at wholesale and sold at so much a piece. In selling or making inventories of such goods, clerks must be able to find readily the price each from the price per dozen, also to compute rapidly the price for any part of a dozen. To do this the following aliquot parts of 12 should be kept in mind:

2 articles = $\frac{1}{6}$ of a dozen.
3 articles = $\frac{1}{4}$ of a dozen.
4 articles = $\frac{1}{3}$ of a dozen.
6 articles = $\frac{1}{2}$ of a dozen.
8 articles = $\frac{2}{3}$ of a dozen.
9 articles = $\frac{3}{4}$ of a dozen.
10 articles = $\frac{5}{6}$ of a dozen.

There are several "short cuts" in using this table, as follows:

1. To find the cost of 5 articles, deduct a sixth from the cost of $\frac{1}{2}$ dozen.

2. To find the cost of 7 articles, add a sixth to the cost of $\frac{1}{2}$ dozen.

3. To find the cost of 9 articles, add a half to the cost of $\frac{1}{2}$ dozen or deduct a fourth from the cost of a dozen.

4. To find the cost of 11 articles, deduct a twelfth from the cost of a dozen.

ORAL EXERCISE

See how many of the following costs you can compute mentally:

1. At $5 a dozen, find the cost of 2 articles, of 3, of 5, of 9, of 11.

2. Find the cost of any number of pairs of gloves from 1 to 12, at $15 a dozen.

3. Find the cost of 7 articles at $1.80 per dozen; at $1.50 per dozen; at $2.00 per dozen; at $3.00 per dozen; at $8.00 per dozen.

4. Find the value of 5 articles at $7.20 a dozen; at $3.00 a dozen; at $8.40 a dozen; at $3.50 a dozen; at $18 a dozen.

5. Find the cost of 9 handkerchiefs at 50¢ a dozen; at 75¢ a dozen; at $1.40 a dozen; at $2.50 a dozen; at $7.00 a dozen.

CHAPTER XV

INTEREST

SIMPLE INTEREST

350. Interest, broadly defined, is the earning power of wealth, or capital. In a narrower sense, it is an allowance for the use of money. It may or may not involve a loan. For instance, if I invest $1000 in a town lot, keep it a year without expense, and then sell it for $1100, I apparently gain $100, but if the current rate for the use of money is 6%, the increase derived from the speculation, $60, must be considered as interest, (since I could have obtained that amount by loaning the $1000), and the actual profit on the deal is but $40.

351. The **principal** is the sum loaned or invested. Interest is computed by taking a certain percentage for each year that the principal is loaned or invested. Thus when we speak of a sum of money being invested so that it yields "6 per cent interest," we mean that each dollar invested yields returns at the rate of 6 per cent of itself, or 6 cents each year.

352. For the purpose of computing interest for fractional parts of a year, business custom has long decreed that a year shall be considered as consisting of 360 days, or of 12 months of 30 days each. Hence in all ordinary business affairs, a day's interest is $\frac{1}{360}$ of a year's interest.

353. In computing interest, three units of time are involved, (1) the year, (2) the month, (3) the day. Of these, the year is the standard of computation, since, as stated above, interest rates relate to a year's use of money.

354. There are various so-called "methods" or "short processes" for computing interest, the practicality of these depending on whether the *time*, or period upon which the interest is to be computed is expressed in years, months, or days, or in combinations of these units.

355. The **Analytic method.** The student should first become familiar with this method as all the short "methods" are based on it.

THE TIME GIVEN IN YEARS

356. This is the simplest case of interest computation.

There are two steps to the analytic process:

1. To find the interest for 1 year (principal × rate).

2. To find the interest for the given time (interest for 1 year × given time).

EXAMPLE: What is the interest on \$748 for 5 years at 5%?

FIRST OPERATION	SECOND OPERATION
\$748	$5\% \times 5 = 25\% = \frac{1}{4}$ rate for 5 yr.
.05	4)748
\$37.40 int. for 1 yr.	187. int. for 5 yr.
5	
\$187.00 int. for 5 yr.	

This problem illustrates one of the many opportunities for shortening interest computations. The student should always be on the alert to take advantage of these.

THE TIME GIVEN IN MONTHS

357. There are three steps to the analytic process:

1. To find the interest for 1 year (principal × rate).

2. To find the interest for 1 month (interest for 1 year ÷ 12).

3. To find the interest for several months (interest for 1 month × number of months).

EXAMPLE: What is the interest on \$360 for 7 months at 8%?

SOLUTION:

$\$360 \times .08 = \$28.80$ int. for 1 yr.

$\$28.80 \div 12 = \$2.40$ int. for 1 mo.

$\$2.40 \times 7 = \$16.80$ int. for 7 mo.

358. The process may often be greatly shortened by cancellation, thus,

$$\frac{\overset{30}{\cancel{\$360}} \times .08 \times 7}{\underset{2}{\cancel{12}}} = \$16.80$$

The three steps of this case may, for the application of cancellation, be resolved into the following general formula:

$$\frac{\text{Principal} \times \text{rate} \times \text{time in months}}{12} = \text{interest.}$$

In reducing the formula to practice, express the rate as a whole number, and instead of the whole principal, write one per cent of it. Thus, $\$114.65 \times .08 = 1.1465 \times 8$. By observing this method, there is less likely to be confusion in determining the decimal places in the result.

EXAMPLE: What is the interest on \$180 for 7 months at 5%?

OPERATION

$$\frac{.15}{\$1.80 \times 5 \times 7 = \$5.25}$$
$$12$$

EXPLANATION.—We multiply one per cent of the principal by the rate (expressed as a whole number) and by the time in months, and divide the result by 12, shortening the operation by cancellation.

359. When the time is an aliquot part of a year or in the case of certain rates, the process may often be still further shortened both for mental and written problems.

EXAMPLE: What is the interest on \$125.40 for 4 mo. at 10%?

OPERATION

3)12.54
4.18 ans.

EXPLANATION.—The interest for 1 year is $\frac{1}{10}$ of \$125.40 or \$12.54. As 4 mo. is $\frac{1}{3}$ of a year \$12.54 ÷ 3 = \$4.18, the required interest.

Solve the following problems, shortening the process wherever possible:

ORAL EXERCISE

Announce the interest results of the following:

1. \$500 at 8% for 5 yr.
2. \$150 at 6% for 2 yr.
3. \$75 at 10% for 4 yr.
4. \$186.50 at 5% for 2 yr
5. \$190.00 at 7½% for 4 yr.
6. \$250 at 4% for 5 yr.
7. \$179 at 2½% for 4 yr.
8. \$86 at 7½% for 2 yr.
9. \$700 at 4% for 3 yr.
10. \$250 at 7% for 4 yr.
11. \$160 at 10% for 3 mo.
12. \$720 at 5% for 6 mo.
13. \$219 at 6% for 2 mo.

SUGGESTION.—The interest at 6% for 2 mo. is 1% of the principal.

14. \$416 at 6% for 4 mo.
15. \$512 at 12% for 5 mo.

SUGGESTION.—At 12%, the interest for 1 mo. is 1% of the principal.

16. \$618 for 3 mo. at 12%.
17. \$129 for 4 mo. at 6%.
18. \$319 for 1 yr. 6 mo. at 6%.

19. $62 for 1 yr. 8 mo. at 12%.
20. $96 for 1 yr. 2 mo. at 10%.
21. $2000 for 2 yr. 6 mo. at 6%.
22. $500 for 1 yr. 4 mo. at 12%.
23. $6000 for 8 mo. at 4%.

WRITTEN EXERCISE

Prepare solutions to the following problems, shortening the process by cancellation or other means, wherever possible:

Find the interest on:

1. $3600 for 3 yr. at 6%.
2. $960 for 5 yr. at 8%.
3. $1260 for 4 yr. 6 mo. at 5%.
4. $324 for 3 yr. 3 mo. at 4%.
5. $480 for 2 yr. 4 mo. at 6%.
6. $320.50 for 4 yr. 8 mo. at 9%.
7. $748.25 for 1 yr. 7 mo. at 10%.
8. $263.84 for 6 mo. at 12%.
9. $843.75 for 5 mo. at $4\frac{1}{2}$%.
10. $1250 for 2 yr. 6 mo. at 6%.
11. $375 for 3 yr. 4 mo. at 6%.

The sum of the principal and interest is called the *amount*.

Find the amount of:

12. $600 for 2 yr. 6 mo. at 6%.
13. $420 for 3 yr. 8 mo. at 5%.
14. $320 for 2 yr. 3 mo. at 10%.
15. $160 for 8 mo. at $4\frac{1}{2}$%
16. $36 for 9 mo. at 5%.
17. $92 for 1 yr. 6 mo. at 6%.
18. $360 for 2 yr. 4 mo. at $16\frac{2}{3}$%.
19. $120 for 2 yr. 6 mo. at $12\frac{1}{2}$%.
20. $350 for 3 yr. at 4%.

THE TIME GIVEN IN DAYS

360. In this case, there are also three steps to the analytic process, viz.:

1. To find the interest for 1 year: Principal × rate.
2. To find the interest for 1 day: Interest for 1 year ÷ 360.
3. To find the interest for several days: Interest for 1 day × number of days.

These steps are included in the following working formula:

$$\frac{\text{Principal} \times \text{rate} \times \text{time in days}}{360} = \text{interest.}$$

EXAMPLE: Find the interest on \$164 for 53 days at 9%.

OPERATION

$$\frac{\overset{.41}{\cancel{1.64}} \times \cancel{9} \times 53}{\cancel{360}\ \cancel{40}\ 10} = \frac{21.73}{10} = \$2.17$$

EXPLANATION.—We multiply 1% of the principal by the rate and the time and divide by 360.

NOTE.—To avoid confusion or oversight as to decimal points, we may, when using this formula, drop the cipher in the 360 and point off *three* instead of two places in the principal, thus,

$$\frac{\$\ .164 \times 9 \times 53}{36}$$

361. Many prefer to separate the dividend from the divisor by the vertical instead of the horizontal line.

EXAMPLE: Find the interest on \$156 for 81 days at 4%.

OPERATION

$$\cancel{4}\ \cancel{36} \mid \begin{matrix} .156 \times 9 = \$1.404 \text{ ans.} \\ \cancel{\$1} \quad 9 \\ \cancel{4} \end{matrix}$$

EXPLANATION.—After canceling we have only to multiply the remaining factors together. This form places the numbers in a more convenient position for multiplying or dividing.

Using the cancellation formula, solve the following problems. Use your preference as to the horizontal or the vertical line:

WRITTEN EXERCISE

What is the interest on:

21. \$720 for 19 days at 4%?

22. \$275 for 72 days at 10%?

23. \$340 for 50 days at 6%?

24. \$285 for 29 days at 12%?

25. \$750 for 63 days at 4½%?

SUGGESTION.—Write the rate 4½ (=4.5) as 45 and add another decimal place to the principal (.0750).

26. \$940 for 27 days at 4½%?

27. \$1200 for 54 days at 4%?

28. \$156.75 for 75 days at 3½%?

29. \$68.40 for 42 days at 6%?

30. \$91.75 for 54 days at 5%?

THE SIXTY-DAY OR "BANKERS" METHOD

362. Since 60 days is one-sixth of an interest year ($360 \div 6 = 60$) it follows that when the interest rate is 6%, the interest for 60 days is 1% of the principal. The interest on any sum for 60 days at 6% can therefore be found instantly by moving the decimal point of the principal two places to the left. Thus,

The interest for 60 days at 6% on \$175 is \$1.75.
The interest for 60 days at 6% on \$50 is \$ .50.
The interest for 60 days at 6% on \$116.25 is \$1.16.

By increasing or diminishing the interest for 60 days at 6% proportionally, we may find the interest for any given number of days.

By increasing or diminishing the interest at 6% for the given time, proportionally, we may find the interest for the given time at the given rate.

363. The banker's or sixty-day method is based on these processes. It is more often applied, as in banking, where the interest period is usually less than one year, and the time is expressed in days. To apply it effectually, it is desirable to keep in mind the following aliquot parts of the basal period, 60 days:

2	days	= $\frac{1}{30}$	of	the	60 day period.
3	"	= $\frac{1}{20}$	"	"	"
4	"	= $\frac{1}{15}$	"	"	"
5	"	= $\frac{1}{12}$	"	"	"
6	"	= $\frac{1}{10}$	"	"	"
10	"	= $\frac{1}{6}$	"	"	"
12	"	= $\frac{1}{5}$	"	"	"
15	"	= $\frac{1}{4}$	"	"	"
20	"	= $\frac{1}{3}$	"	"	"
30	"	= $\frac{1}{2}$	"	"	"

EXAMPLE: Find the interest at 6% on \$250 for 75 days.

OPERATION

\$2|50 = int. at 60 days.
 |625 = int. at 15 days.
\$3|125 = int. at 75 days.

EXPLANATION.—We point off 2 decimal places, obtaining \$2.50 as the interest for 60 days. Since 75 days is $\frac{1}{4}$ more than 60 days, we add $\frac{1}{4}$ of \$2.50 or \$ .625, obtaining \$3.125 (\$3.13) as the required interest.

In problems of this nature it is better to use a decimal line as above rather than decimal points.

EXAMPLE: Find the interest on $175 for 50 days at 6%.

OPERATION

$1.75 = int. at 60 d.
.291 = int. at 10 d.
$1.459 = int. at 50 d.

EXPLANATION.—Here we deduct $\frac{1}{6}$ of the interest at 6%, since 50 days is $\frac{1}{6}$ less than 60 days.

Carry divisions to three decimal places when using this method.

EXAMPLE: Find the interest on $1500 at 8% for 93 days.

OPERATION

15.00 int. at 6% for 60 d.
7.50 int. at 6% for 30 d.
.75 int. at 6% for 3 d.
23.25 int. at 6% for 93 d.
7.75 int. at 2% for 93 d.
31.00 int. at 8% for 93 d.

EXPLANATION.—We add $\frac{1}{2}$ for the interest at 30 days, and $\frac{1}{10}$ of this for the interest at 3 days giving $23.25 as the interest at 6% for the time. Then we add $\frac{1}{3}$ of this since 8% is $\frac{1}{3}$ more than the interest at 6%.

ORAL DRILL ON THE SIXTY-DAY METHOD

	60d.	30d.	20d.	15d.	12d.	10d.	6d.	5d.	3d.	1d.	90d.	120d.	2d.
$2400	$24	$12	$8	$6	$4.80	$4	$2.40	$2	$1.20	$.40	$36	$48	.80
$1200	?	?	?	?	?	?	?	?	?	?	?	?	?
$3600	?	?	?	?	?	?	?	?	?	?	?	?	?
$360	?	?	?	?	?	?	?	?	?	?	?	?	?
$7200	?	?	?	?	?	?	?	?	?	?	?	?	?

WRITTEN PRACTICE

Compute the interest on the following at 6%:

31. $840 for 63 da. (60 and 3).

32. $960 for 75 da. (60 and 15).

33. $320 for 96 da. (60, 30, and 6).

Moving the decimal point 3 places gives the interest for 6 days at 6%.

34. $1260 for 123 da. (2 60's and 3).

35. $2480 for 85 da. (60, 20, and 5).

36. $1244 for 54 da. (60 less 6).

37. $2360 for 59 da. (60 less 1).

38. $9200 for 36 da. (6 6's).

39. $1125 for 42 da. (7 6's).

40. $480.50 for 18 da. (3 6's).

Compute the interest on the following at 8%:

41. $720 for 73 da.

42. $460 for 63 da.

43. $920.60 for 72 da.

44. $843.50 for 87 da.

Find the interest on:

45. $675 for 125 days at 4%.

46. $850 for 85 days at 4½%.

47. $1275 for 25 days at 5%.

48. $295.60 for 45 days at 7%.

49. $168.40 for 72 days at 10%.

50. $229.60 for 150 days at 3%.

51. $778 for 54 days at 4½%.

THE TIME GIVEN IN YEARS AND MONTHS

364. In problems under this head, there are four methods of computing as follows:

1. *The time may be expressed in years.* Thus, 2 yr. 4 mo. = $2\frac{1}{3}$ yr., 4 yr. 2 mo. = $4\frac{1}{6}$ yr.

2. *The time may be reduced to months.* 2 yr. 5 mo. = 29 mo., 1 yr. 3 mo. = 15 mo., etc.

3. *The rate for the whole time may be found.* Thus, at 2 yr. 6 mo., at 10%, the rate for the whole time is $2\frac{1}{2} \times 10$ or 25%, and for 1 yr. 8 mo., at 6%, the rate for the entire time is $1\frac{2}{3} \times 6\%$ or 10%. This method often greatly shortens the computation.

4. *The interest may be computed separately for the years and months and the results added.* Thus, for 3 yr. 2 mo. at 6%, find the interest for 3 yr. and add 1% of the principal for the interest at 2 mo. An inspection of the problem will determine which is the best method to use.

Determine mentally the interest for the following:

1. $75 for 2 yr. at 10%.

2. $80 for 3 yr. at 12½%.

3. $120 for 4 yr. at 5%.

4. $240 for 2 yr. at 7%.

5. $72 for 5 yr. at 6%.

6. $84 for 3 yr. at 5%.

7. $150 for 2 yr. at 6%.

8. $240 for 3 yr. at 10%.

9. $320 for 5 yr. at 4%.

10. $325 for 5 yr. at 4%.

11. $320 for 3 yr. at 6%.

12. $620 for 2 yr. at 10%.

13. $800 for 2 yr. at 7½%.

14. $120 for 4 yr. at 5%.

15. $314 for 2 yr. 6 mo. at 10%.

16. $215 for 3 yr. at 8%.

17. $32 for 1 yr. at 5%.

18. $60 for 2 yr. at 6½%.

19. $75 for 3 yr. at 6%.

20 $1200 for 2 yr. at 9%.

Find mentally the interest for the following:

1. \$600 for 6 mo. at 6%.
2. \$420 for 4 mo. at 6%.
3. \$320 for 3 mo. at 10%.
4. \$160 for 8 mo. at $4\frac{1}{2}$%.
5. \$36 for 9 mo. at 10%.
6. \$92 for 6 mo at 6%.
7. \$360 for 6 mo. at 12%.
8. \$120 for 9 mo. at $12\frac{1}{2}$%.
9. \$350 for 7 mo. at 4%.
10. \$480 for 8 mo. at 3%.

WHEN THE TIME IS GIVEN IN DAYS

365. A day, in interest computations, is considered $\frac{1}{360}$ of a year, or $\frac{1}{30}$ of a month. When months and days are given, it is often better to reduce all to days.

Find mentally the interest for the following:

1. \$490 for 60 da. at 6%.
2. \$324 for 59 da. at 6%.
3. \$524 for 45 da. at 8%.
4. \$651 for 72 da. at 7%.
5. \$478 for 78 da. at 12%.
6. \$295 for 3 mo. 10 da. at 9%.
7. \$956 for 2 mo. 6 da. at 5%.
8. \$483 for 1 mo. 20 da. at 4%.
9. \$496 for 6 mo. 20 da. at 9%.
10. \$125 for 1 mo. 25 da. at 4%.
11. \$1466 for 2 mo. 5 da. at 3%.
12. \$427 for 1 mo. 12 da. at $4\frac{1}{2}$%.
13. \$848 for 2 mo. 15 da. at $7\frac{1}{2}$%.
14. \$164 for 7 mo. 10 da. at 8%.
15. \$800 for 1000 da. at 9%.

TIME GIVEN IN MONTHS AND DAYS

366. In this class of problems, the interest may be computed for the months and then for the days by the banker's method, or in many cases the whole time may be expressed in months and a fraction or decimal. Whenever the days *are divisible by three*, they may be expressed in tenths, since 3 days are $\frac{1}{10}$ of a month. Thus, 5 mo. 12 da. = 5.4 mo., 7 mo. 21. da. = 7.7 mo., etc.

Also, 2 mo. 5 da. may be expressed $2\frac{1}{6}$ mo. or in the cancellation form, $\frac{1}{6}\frac{3}{0}$ mo., and 7 mo. 10 da. as $7\frac{1}{3}$ or $\frac{2}{3}\frac{2}{0}$ mo.

This expression of the time in months usually shortens the process.

EXAMPLE: Find the interest on $240 for 8 mo. 18 da. at 5%.

OPERATION

$$\frac{\overset{.20}{\cancel{2.40}} \times 8.6 \times 5}{\underset{12}{\cancel{12}}} = \$8.60$$

EXPLANATION.—Express 8 mo. 18 days as 8.6 mo. and cancel.

EXAMPLE: Find the interest on $288 for 7 mo. 5 da. at 8%.

OPERATION

$$\frac{\overset{.04}{\cancel{.24}} \times 43 \times \overset{8}{\cancel{8}}}{\underset{12}{\cancel{12}}\ \underset{6}{\cancel{6}}} = \$13.76$$

EXPLANATION.—Express 7 mo. 5 d. as $7\frac{1}{3}$ or $\frac{43}{6}$ mo. and cancel.

NOTE.—If more convenient, use the vertical line with the divisors on the left.

WRITTEN EXERCISE

Find the interest on:

52. $275 for 4 mo. 6 da. at 8%.
53. $720 for 8 mo. 20 da. at 7%.
54. $42.50 for 11 mo. 6 da. at 6%.
55. $115.40 for 6 mo. 15 da. at 4%.
56. $279 for 10 mo. 12 da. at 6%.
57. $870 for 2 mo. 27 da. at 10%.
58. $356 for 17 mo. 10 da. at 5%.
59. $960 for 9 mo. 25 da. at 3%.
60. $472 for 4 mo. 21 da. at 8%.
61. $640.70 for 3 mo. 24 da. at 6%.

THE TIME GIVEN IN YEARS, MONTHS, AND DAYS

367. In this case it is usually better to compute the interest for the years separately, make a separate computation for the months and days as in the previous case, and then unite the two results. In some cases, it may be well to compute the interest for the years and months as in Art. 359, and then make a separate computation for the days, either by the cancellation method as in Art. 361, or by the banker's method, Art. 362. The method to be used, should depend on the time quantities in the problem.

In the following problems, choose the method that seems to offer the simplest form of solution.

WRITTEN EXERCISE

Find the interest on the following:

62. $1250 for 1 yr. 2 mo. 29 da. at 6%.
63. $756 for 2 yr. 5 mo. 24 da. at 8%.
64. $358 for 4 yr. 25 da. at 4½%.
65. $156 for 1 yr. 11 mo. 10 da. at 3%.
66. $865 for 3 yr. 7mo. 20 da. at 6%.
67. $640 for 1 yr. 1 mo. 16 da. at 5%.
68. $875 for 3 yr. 2 mo. 12 da. at 4%.
69. $427 for 2 yr. 6 mo. 19 da. at 7%.
70. $860 for 2 yr. 8 mo. at 7%.
71. $940 for 3 yr. 9 mo. at 8%.
72. $942.50 for 2 yr. 10 mo. at 6%.
73. $365 for 11 mo. 20 da. at 5%.
74. $284.60 for 9 mo. 18 da. at 4%.
75. $1386 for 7 mo. 25 da. at 4½%.
76. $8426.30 for 6 mo. 6 da. at 8%.
77. $1265.75 for 7 mo. 24 da. at 9%.
78. $3640 for 8 mo. 16 da. at 10%.
79. $12640 for 4 mo. 5 da. at 6%.

Find the amount due in each of the following twenty-one problems:

80. $450 for 65 da. at 6%.

The amount is found by adding the interest and principal.

81. $860 for 1 mo. 20 da. at 5%.
82. $320 for 3 mo. 15 da. at 4%.
83. $1265 for 4 mo. 18 da. at 8%.
84. $1460.25 for 1 yr. 6 mo. 23 da. at 6%.
85. $2375 for 2 yr. 3 mo. 20 da. at 5%.
86. $425 for 1 yr. 4 mo. 14 da. at 8%.
87. $960.75 for 3 mo. 21 da. at 4%.
88. $896.43 for 2 mo. 16 da. at 9%.
89. $2640 for 4 mo. 25 da. at 4½%.
90. $1230 for 7 mo. 16 da. at 8%.
91. $4261 for 2 yr. 8 mo. 21 da. at 4%.

Find the time in the next four problems by compound subtraction.

92. $800 from Jan. 15, 1910, to June 18, 1911, at 8%.
93. $2400 from Feb. 11, 1909, to June 3, 1910, at 6%.
94. $650 from June 17, 1908, to Jan. 21, 1910, at 7%.
95. $565.25 from Jan. 29, 1910, to Jan. 1, 1913 at 5%.

Find the time in the next five problems by counting the exact number of days in each month—a method that is sometimes used, although a year is counted 360 days. This method is sometimes called the banker's method.

96. $1400 from May 1, 1910, to Sept. 25, 1910, at 6%.
97. $560 from June 24, 1909, to Dec. 16, 1909, at 7%.
98. $840.50 from Feb. 2, 1908, to Oct. 4, 1908, at 5%.
99. $375 from April 3, 1910, to May 1, 1910, at 9%.
100. $484.60 from June 4, 1908, to July 16, 1908, at 8%.

COMPUTING INTEREST BY MEANS OF TABLES

368. In banks and counting rooms, where there are many interest computations, tables are often employed in which the results are already worked out. By simply adding the results as given in the tables, the interest on any sum for any time may be quickly obtained.

The tables, pages 228-229, are identical with a page of an interest table book in common use. Such a book gives the interest on any sum from $1 to $9999, at 5, 6, 7, and 8 per cent. One page is required for two periods of time. On page 1 of the book may be found the interest for any amounts and rates within the above limitations for 1 and 2 days. Thus, if the interest on any sum of money and at any rate, for 1 or 2 days is to be found, turn to page 1. The tables for 79 and 80 days are here given and would be looked for on page 40 of the interest book. Separate pages are thus provided for results up to 360 days or one year.

EXAMPLE: Find the interest on $7391 for 79 days at 5%.

Referring to the page headed 79 days and to the columns headed 5% we find that

Interest on	$7000	for 79 days at 5%	is	$76.81	
Interest on	300	for 79 days at 5%	is	3.29	
Interest on	90	for 79 days at 5%	is	.99	
Interest on	1	for 79 days at 5%	is	.01	
	$7391	Total Amount		$81.10	Total Interest

INTEREST TABLE—79 DAYS

Amt.	5%	6%	7%	8%	Amt.	5%	6%	7%	8%
$1000	10.97	13.17	15.36	17.56	*$100*	1.10	1.32	1.54	1.76
2000	21.94	26.33	30.72	35.11	*200*	2.19	2.63	3.07	3.51
3000	32.92	39.50	46.08	52.67	*300*	3.29	3.95	4.61	5.27
4000	43.89	52.67	61.44	70.22	*400*	4.39	5.27	6.14	7.02
5000	54.86	65.83	76.81	87.78	*500*	5.49	6.58	7.68	8.78
6000	65.83	79.	92.17	105.33	*600*	6.58	7.90	9.22	10.53
7000	76.81	92.17	107.53	122.89	*700*	7.68	9.22	10.75	12.29
8000	87.78	105.33	122.89	140.44	*800*	8.78	10.53	12.29	14.04
9000	98.75	118.50	138.25	158.	*900*	9.88	11.85	13.83	15.80

INTEREST TABLE—80 DAYS

Amt.	5%	6%	7%	8%	Amt.	5%	6%	7%	8%
$1000	11.11	13.33	15.56	17.78	*$100*	1.11	1.33	1.56	1.78
2000	22.22	26.67	31.11	35.56	*200*	2.22	2.67	3.11	3.56
3000	33.33	40.	46.67	53.33	*300*	3.33	4.	4.67	5.33
4000	44.44	53.33	62.22	71.11	*400*	4.44	5.33	6.22	7.11
5000	55.56	66.67	77.78	88.89	*500*	5.56	6.67	7.78	8.89
6000	66.67	80.	93.33	106.67	*600*	6.67	8.	9.33	10.67
7000	77.78	93.33	108.89	124.44	*700*	7.78	9.33	10.89	12.44
8000	88.89	106.67	124.44	142.22	*800*	8.89	10.67	12.44	14.22
9000	100.	120.	140.	160.	*900*	10.	12.	14.	16.

If the sum to be computed runs into cents, consider 50 cents or more an additional dollar.

WRITTEN EXERCISE

Using the table, find the interest on:

101. $657 for 79 da. at 7%.
102. $1943 for 79 da. at 8%.
103. $5213.75 for 80 da. at 6%.
104. $999.90 for 80 da. at 5%.
105. $1434 for 80 da. at 8%.
106. $263 for 2 mo. 19 da. at 7%.
107. $548.30 for 2 mo. 20 da. at 5%.
108. $431.84 for 2 mo. 19 da. at 5%.
109. $5000 for 79 da. at 3% ($\frac{1}{2}$ of 6%).
110. $9405.75 for 80 da. at 4% ($\frac{1}{2}$ of 8%).

LAWFUL AND UNLAWFUL INTEREST

369. The rate at which interest may be collected is in many states and countries regulated by law.

OR 2 MONTHS 19 DAYS

Amt.	5%	6%	7%	8%	Amt.	5%	6%	7%	8%
$10	.11	.13	.15	.18	*$1*	.01	.01	.02	.02
20	.22	.26	.31	.35	*2*	.02	.03	.03	.04
30	.33	.40	.46	.53	*3*	.03	.04	.05	.05
40	.44	.53	.61	.70	*4*	.04	.05	.06	.07
50	.55	.66	.77	.88	*5*	.05	.07	.08	.09
60	.66	.79	.92	1.05	*6*	.07	.08	.09	.11
70	.77	.92	1.08	1.23	*7*	.08	.09	.11	.12
80	.88	1.05	1.23	1.40	*8*	.09	.11	.12	.14
90	.99	1.19	1.38	1.58	*9*	.10	.12	.14	.16

OR 2 MONTHS 20 DAYS

Amt.	5%	6%	7%	8%	Amt.	5%	6%	7%	8%
$10	.11	.13	.16	.18	*$1*	.01	.01	.02	.02
20	.22	.27	.31	.36	*2*	.02	.03	.03	.04
30	.33	.40	.47	.53	*3*	.03	.04	.05	.05
40	.44	.53	.62	.71	*4*	.04	.05	.06	.07
50	.56	.67	.78	.89	*5*	.06	.07	.08	.09
60	.67	.80	.93	1.07	*6*	.07	.08	.09	.11
70	.78	.93	1.09	1.24	*7*	.08	.09	.11	.12
80	.89	1.07	1.24	1.42	*8*	.09	.11	.12	.14
90	1.00	1.20	1.40	1.60	*9*	10.	.12	.14	.16

Usury is the collection of a higher rater than that permitted by the law. The law usually prescribes two rates:

1. The **maximum rate,** which is the highest rate that the lender is allowed to charge.

2. The **legal rate** or the rate that may be charged when there is no rate specified in the contract or in the case of an over-due debt, where there is no agreement as to interest.

The laws of the several states vary as to both the maximum and legal rate, also, as to the penalties imposed for usurious interest contracts. The student should inform himself as to the law in his own state.

PERIODIC INTEREST

370. This is an additional interest charged on installments of simple interest that are not paid when due. It is not collectible at law, in most states, unless separate notes are given for the unpaid simple interest. Periodic interest is sometimes called "annual interest" because in most cases where it is used, the interest installments are payable annually. We will illustrate the method of computation.

ILLUSTRATIVE PROBLEM

A mortgage note of $1500 was given Jan. 1, 1902, to mature 5 yr. from date, with interest at 6%, the interest to be paid annually before and after maturity of note, and to bear interest at 6% unless paid when due. The first and second interest payments were made when due, but there were no more payments until the mortgage was foreclosed on July 1, 1908. What was the amount due?

SOLUTION: $405., 4½ yr., simple interest, due at time of foreclosure.

yr.	mo.	da.	
1908	7	1	
1904	1	1	
4 yr.	6 mo.		time from last interest payment to settlement.
3 yr.	6 mo.		time first unpaid installment is overdue.
2 yr.	6 mo.		time second unpaid installment is overdue.
1 yr.	6 mo.		time third unpaid installment is overdue.
	6 mo.		time fourth unpaid installment is overdue.
8 yr.			total time overdue for installments.

$\$90 \times .06 \times 8 = \$43.20$ interest on unpaid installments.

$\$1500 + \$405 + \$43.20 = \$1948.20$, amount due.

NOTE.—The work of finding the total time for overdue interest installments, may be shortened by finding the average length of the periods. This is equal to one-half of the sum of the longest and shortest period. (In the above example $\frac{3 \text{ yr. } 6 \text{ mo.} + 6 \text{ mo.}}{2} = 2$ yr.)

This time multiplied by the number of unpaid installments equals the total time (2 yr. $\times 4 = 8$ yr.)

WRITTEN EXERCISE

Find the amount due at maturity on the following at periodic interest:

111. A note of $1200 given June 1, 1904, due 5 yr. from date, with interest payable annually at 5%, no interest or other payment having been made.

112. A note of $500, dated Jan. 15, 1901, payable six years from date, with interest payable semi-annually, at 6%, no payments having been made.

113. On July 1, 1902, a man bought a farm for $5000, agreeing to pay for it in ten semi-annual installments of $500 and accrued interest on the entire principal at the rate of 6 per cent per annum from the date of purchase, and deferred payments to draw interest at the same rate. The first three installments of

principal and interest were paid when due, the remaining installments were not paid. Six months after the maturity of the last installment, the seller foreclosed. What was the amount of his claim?

COMPOUND INTEREST

371. The parties to a note may agree that all interest accruing upon it is to be added periodically to the principal, the sum to be taken as the principal for the succeeding period; the interest for this period is again added, the sum comprising the principal for the next period, and so on. The total interest which accrues on an obligation of this kind is known as **compound interest.**

The interest is usually added to the principal or "compounded" annually, semi-annually, or quarterly; but compound interest may not be collected unless there is a specific agreement to that effect in the note. In some states the form of this "compound interest clause" is prescribed by the statutes.

EXAMPLE: Find the compound interest on $3600 for 3 years at 6%.

SOLUTION

$3600. = Principal.
216. = Int. for 1st year.
3816. = 1st amount.
228.96 = Int. for 2d year.
4044.96 = 2d amount.
242.70 = Int. for 3d year.
4287.66 = 3d amount.
3600. Subtract principal.
$687.66 = Compound interest.

WRITTEN PRACTICE

114. What is the amount of $1340 for three years at 5% compound interest?

115. Required the compound interest on $3500 for 6 years at $7\frac{1}{2}\%$.

116. Find the amount of $4000 for 1 years 6 months at $4\frac{1}{2}\%$ compound interest.

Compound interest is generally computed by means of Interest Tables, giving the amount of $1 for various times and rates. The table on the next page gives the amount of $1 at compound interest for any number of years from 1 to 50 inclusive, at the principal rates from 4% to 10%.

The student can compute the amount in the blank spaces, thereby showing how these tables are made.

COMPOUND INTEREST TABLE

Showing the amount of $1 at compound interest for any number of years from 1 to 50, inclusive, at rates given.

Year	4 per ct.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	10 per ct.
1	1.0400 0000	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	
2	1.0816 0000	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	
3	1.1248 6400	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	
4	1.1698 5856	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	
5	1.2166 5290	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	
6		1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	
7		1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	
8		1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	
9		1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	
10		1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	
11		1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	
12		1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	
13		1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	
14		1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	
15		1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	
16		2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	4.5949 730
17		2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	5.0544 703
18		2.2084 7877	2.4066 192	2.8543 382	3.3799 323	3.9960 195	5.5599 173
19		2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	6.1159 390
20		2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	6.7275 000
21		2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	
22		2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	
23		2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	
24		2.8760 1383	3.2250 999	4.0489 346	5.0723 679	6.3411 807	
25		3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	
26		3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	
27		3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	
28		3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	
29		3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	
30		3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	
31	3.3731 3341	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	
32	3.5080 5875	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	
33	3.6483 8110	4.2740 3018	5.0031 885	6.8405 899	9.3253 393	12.6760 496	
34	3.7948 1634	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	
35	3.9460 8899	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	
36		4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	30.9126 805
37		5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	34.0039 486
38		5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	37.4043 434
39		5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	41.1447 778
40		5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	45.2592 256
41		6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	
42		6.3516 1548	7.7615 876	11.5570 327	17.1442 563	25.3394 819	
43		6.6374 3818	8.1496 669	12.2504 546	18.3443 548	27.3666 404	
44		6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5559 717	
45		7.2482 4843	8.9850 078	13.7646 103	21.0024 518	31.9204 494	
46		7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	
47		7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	
48		8.2714 5557	10.4012 697	16.3938 717	25.7289 065	40.2105 731	
49		8.6436 7107	10.9213 331	17.3775 040	27.5299 300	43.4274 190	
50	7.1066 8335	9.0326 3627	11.4673 993	18.4201 543	29.4570 251	46.9016 125	117.3908 529

WRITTEN PRACTICE

Use the interest table in the following:

117. What is the compound interest on $25000 for 24 years at 5%?

118. A son, ten years of age, is left by bequest $20000, which is immediately placed at compound interest at 8%, payable annually. What will be the amount of the bequest when the son is of age?

119. What sum will amount to $6866.72 in 25 years at 6%, compounded annually?

120. What sum of money will amount to $17599.97 in 40 years at 5%, compounded annually?

121. What sum of money will produce $17191.86 in 30 years at 7%, compounded annually?

122. What sum must be invested at the age of 20 years that will amount to $25000 at the age of 50 years at $4\frac{1}{2}$%, compounded annually?

OTHER PROBLEMS IN INTEREST

374. Several kinds of interest problems may arise other than the computing of interest on a given principal.

Problem I. To find principal when time, rate, and interest are given.

EXAMPLE: What sum of money at interest for 1 yr. 6 mo. at 6% will gain $36.72?

SOLUTION

Int. on $1 for 1 yr. 6 mo. = .$ .09

$36.72 ÷ .09 = $408

OR

$$\frac{\$36.72}{1\frac{1}{2} \times .06} \text{ or } \frac{\$36.72 \times 2}{.06 \times 3} = \$408$$

EXPLANATION.—Since $ .09 is the interest on $1 for the given time, $36.72 is the interest on as many dollars as .09 is contained times in $36.72, or 408 times.

WRITTEN EXERCISE

What sum of money will yield

123. $1094.40 in 2 yr. 6 mo. 12 da. at 6%?

124. $597.60 in 1 yr. 4 mo. 18 da. at 12%?

125. $174.40 in 1 yr. 9 mo. 24 da. at 4%?

126. $284.60 in 1 yr. 2 mo. 4 da. at 9%?

127. $96.48 in 3 mo. 15 da. at 5%?

128. $120.30 in 9 mo. 12 da. at 8%?

129. $246.85 in 11 mo. 27 da. at 9%?

130. $423.60 in 3 yr. 4 mo. 21 da. at 4%?

131. $520.60 in 3 yr. 6 mo. 21 da. at 2½%?

132. $360 in 2 yr. 4 mo. 18 da. at 3%?

133. On Jan. 1, 1910, a merchant borrowed a certain sum of money at 6%; on Oct. 1, 1910, he paid the principal and interest. What sum did he borrow if the interest was $288? (Use compound subtraction.)

Problem II. To find principal when amount, time, and rate are given.

EXAMPLE: What sum of money at interest at 6% will amount to $690 in 2 yr. 6 mo.?

SOLUTION

$1.00 + .15 = $1.15
$690 ÷ 1.15 = $600

EXPLANATION.—6% interest on $1 for 2 yr. 6 mo. is $ .15, and the amount of $1 is $1.15. $690 is the amount of as many dollars at interest as $1.15 is contained times in $690, or 600 times.

WRITTEN EXERCISE

What sum of money will amount to

134. $8294.40 in 2 yr. 6 mo. 12 da. at 6%?

135. $4027.50 in 2 yr. 4 mo. 15 da. at 5%?

136. $3125.00 in 2 yr. 9 mo. 18 da. at 8%?

TRUE DISCOUNT

375. The present worth of a debt due at a specified future date, is the sum of money which, if put at interest for the specified time at the given rate of interest, will amount to the face of the debt. This is an application of Problem II, which sometimes arises in legal and other equitable settlements.

376. The difference between the present worth of a debt and the face, is called the **true discount.**

377. In operations involving true discount, the face of the debt is the *amount*, the present worth is the *principal*, and the time discount is the *interest* on the present worth, for the time involved.

WRITTEN PROBLEMS

137. What sum would equitably discharge a debt of $418 due in 2 yr. without interest, when the current interest rate is 6%? What is the true discount?

138. A and B go to law over their accounts, and a jury finds that A owes B $695.45, due 5 mo. 20 da. in the future. The legal rate being 7%, for what sum should they return their verdict?

139. I am offered a piece of real estate for $2000 cash, or for $2200 on six months' time, without interest, money being worth 6%, which is the better offer *now,* and how much?

140. A offered to sell to B a stock of goods and store fixtures for $15000 on 10 months' time, or for the present worth in cash at a 7 per cent rate. B chose the latter proposition. What did he pay and what was the amount of discount allowed?

Problem III. To find the rate when principal, time, and interest are given.

EXAMPLE: At what rate will $250 produce $37.50 in 2yr. 6 mo.?

SOLUTION

$\$250 \times .01 \times 2\frac{1}{2} = \$6.25$

$\$37.50 \div 6.25 = 6$ times, or 6%

EXPLANATION.—Since the interest on $250 for 2 yr. 6 mo. at 1% is $6.25, $37.50 is as many per cent as $6.25 is contained times in $37.50, or 6 times, or 6%.

OR

$$\frac{37.50}{250 \times 2\frac{1}{2}} = \frac{37.50 \times 2}{250 \times 5} = .06$$

WRITTEN PRACTICE

At what per cent will

141. $960 produce $88.32 in 1 yr. 6 mo. 12 da.?

142. $7200 produce $1159.20 in 2 yr. 3 mo. 18 da.?

143. $4250 produce $811.75 in 3 yr. 2 mo. 6 da.?

144. $720 produce $53.90 in 2 yr. 1 mo. 20 da.?

145. $1000 produce $192.67 in 3 yr. 2 mo. 16 da.?

146. $1260 produce $133.98 in 2 yr. 7 mo. 27 da.?

147. $380 produce $21.09 in 7 mo. 12 da.?

148. $596 produce $13.91 in 4 mo. 20 da.?

149. $430.50 produce $105.47 in 4 yr. 1 mo.?

150. $263.85 produce $31.66 in 1 yr. 4 mo.?

151. On Feb. 28, 1910, a merchant borrowed $600, and on Dec. 17, 1910, he paid the interest and principal with $643.35. What was the rate per cent? (Compound subtraction.)

Problem IV. To find the time, when the principal, rate, and interest are given.

EXAMPLE: In what time will $300 gain $52.50 at 7%?

SOLUTION

$300 \times .07 = \$21$
$\$52.50 \div 21 = 2\frac{1}{2}$
$2\frac{1}{2}$ yr. $= 2$ yr. 6 mo.

EXPLANATION.—One year's interest on $300 at 7% is $21. $52.50 is as many years' interest as $21 is contained times in $52.50, or $2\frac{1}{2}$ times, equal to 2 years 6 months.

OR

$$\frac{52.50}{300 \times .07} = 2\tfrac{1}{2}, \text{ or } 2 \text{ yr. } 6 \text{ mo.}$$

WRITTEN PRACTICE

In what time will

152. $630 produce $56.70 at 6%?

153. $7500 produce $506.25 at $4\frac{1}{2}$%?

154. $1200 produce $97.65 at $7\frac{1}{2}$%?

155. $420 produce $42 at 4%?

156. $940 produce $117.50 at 5%?

157. $3260 produce $365.80 at 4%?

158. $5000 produce $1260 at 7%?

159. $4200 produce $420 at 5%?

160. $3200 produce $48 at 6%?

161. $840 produce $216 at 8%?

162. A merchant borrows $900 on Aug. 1, 1910, at 8%, and on the date of maturity he paid $924 for the note and interest. Find the time and date of payment.

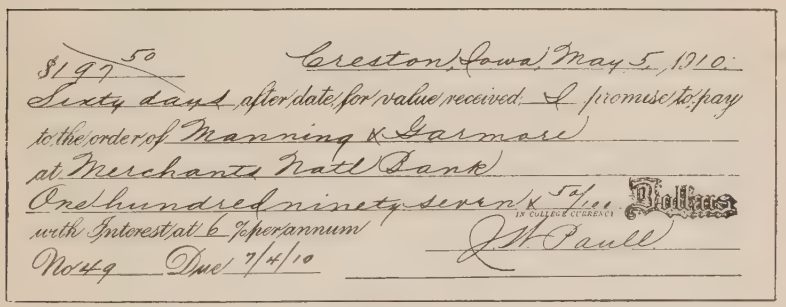
CHAPTER XVI

COMMERCIAL PAPER AND DISCOUNTS

378. Banks are private, state, or national, according to the laws under which they are organized. The general functions of banks are to receive and pay money, make loans, and aid in transmitting money from one person or place to another.

379. **Commercial paper** is a general name for notes, drafts, or other commercial obligations upon which banks loan money.

380. A **promissory note** is a written promise to pay a specified sum of money to a person or firm or corporation and at a time and place named in the instrument. Promissory notes are usually made payable at a bank. The following is an ordinary form:

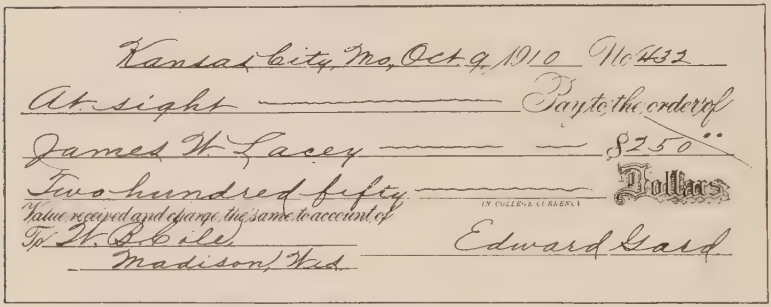

$197.50
Creston, Iowa, May 5, 1910.
Sixty days after date for value received I promise to pay
to the order of Manning & Garmare
at Merchants Natl Bank
One hundred ninety seven 50/100
Dollars
IN GOLD COIN
with Interest at 6 % per annum
No 49 Due 7/4/10
J W Paull

381. A **draft** is a written order from one person to another, ordering him to pay a stated sum of money to another person named in the draft. The person who writes or "draws" the draft is called the *drawer*. The person to whom the draft is addressed and who is directed to pay the money is called the *drawee* or *payer*, and the person who is to receive the money is called the *payee*.

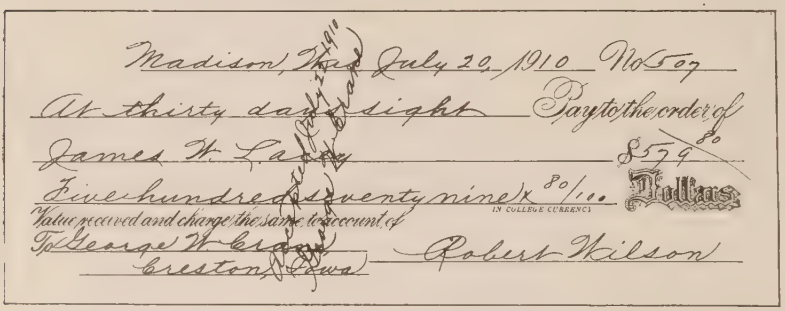
382. If the payment is to be made immediately upon presentation, the instrument is called a *sight draft*, but if it is not to be paid until a specified time after presentation or "sight," it is called a *time draft*.

383. The person to whom a time draft is presented may "accept" it (which is, in effect, agreeing to pay it at maturity) by writing, or stamping the word "accepted" across the face, with his signature. Such an accepted draft is known in business as an *acceptance.*

A SIGHT DRAFT


Kansas City, Mo, Oct 9, 1910 No 432
At sight Pay to the order of
James W Lacey $250.00
Two hundred fifty IN COLLEGE CURRENCY Dollars
Value received and charge the same to account of
To H R Cole
Madison, Wis
Edward Gard

AN ACCEPTED TIME DRAFT, OR ACCEPTANCE


Madison, Wis July 20, 1910 No 507
At thirty days sight Pay to the order of
James W Lacey $579.80
Five hundred seventy nine 80/100 IN COLLEGE CURRENCY Dollars
Value received and charge the same to account of
To George W Crane
Creston, Iowa
Robert Wilson

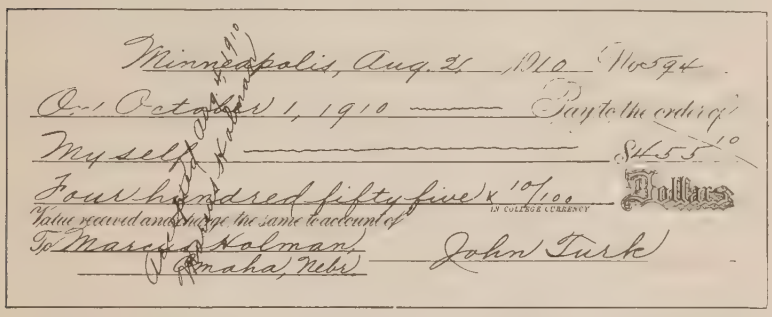
Explanation of the above drafts. In the sight draft, the *drawee,* Mr. Cole, presumably owes the *drawer,* and signer of the draft, Mr. Gard, a sum of money equal to or exceeding the amount named or "face" of the draft. Mr. Gard directs the payment of this money to a third party, Mr. Lacey, the *payee* of the draft.

In the *acceptance,* or time draft, the money is not payable on demand, but in 30 days after sight, so Mr. Crane, the drawee, *accepts* it by writing "accepted" with the date and his name across the face, and thereby becomes responsible for its payment at maturity.

384. In commercial transactions, acceptances are more often made payable to the drawer himself instead of to a third party.

In this form they have merely the purpose and function of a promissory note. The following form shows an acceptance in this form:

ACCEPTANCE PAYABLE TO DRAWER


Minneapolis, Aug. 21 1910 No 594
On October 1, 1910 Pay to the order of
Myself $455.10
Four hundred fifty five 10/100 Dollars
IN COLLEGE CURRENCY
Value received and charge the same to account of
To Marcus Holman
Omaha, Nebr.
John Turk

Explanation. In the above acceptance it may be supposed that the drawer, Mr. Turk, has sold a bill of goods on 60 days' credit to Mr. Holman the drawee. Mr. Turk then has a draft prepared which Mr. Holman signs, or "accepts." The commercial paper thus formed, may then be discounted at a bank or otherwise disposed of by Mr. Turk, the same as a promissory note.

385. The **face** of a commercial paper is the sum of money expressed in the instrument.

386. The **amount** of a commercial paper is the sum due at maturity. It may be the face, or the face plus interest, according to the conditions of the contract.

387. Bank discount is the deduction made by a bank for advancing money on commercial paper before it is due. It is computed by taking the simple interest on the amount of the paper from the date it is received or "discounted" until it falls due.

388. The **proceeds** of a commercial paper is the amount less the bank discount.

389. The **discount period** or term of discount is the exact number of days from the date of discounting to the date of maturity. Banks do not ordinarily discount commercial paper for a longer period than 90 days.

390. Days of grace. In some states the maker of a promissory obligation is allowed by law three extra days beyond maturity in which to pay the debt. This was a privilege accorded to debtors under the old English law at a period when means of communica-

tion were slow and uncertain. The reason for the concession has long ceased to exist, and most of the United States have abolished days of grace. In states where they are still retained, banks add the three days to the term of discount. In the problems of this book, days of grace will not be used unless specified in the problem.

FINDING THE PERIOD OF DISCOUNT

391. In finding the maturity of commercial paper, count the months and days according to the terms of the paper.

In finding the term of discount, count the exact days.

EXAMPLE: A note dated July 15, 1910, and maturing six months from date is discounted Oct. 21. Find the discount period.

OPERATION

July 15, 1910 + 6 mo. = Jan. 15, 1911, maturity.
31 − 21 = 10 days in Oct., 1910
30 days in Nov., 1910
31 days in Dec., 1910
15 days in Jan., 1911
86 days, dis. per.

EXAMPLE: A note dated Feb. 6, 1908, is due 90 days from date. It is discounted April 11, 1908. Find the discount period.

OPERATION

Feb. 6, 1908 (leap year) + 90 days = May 6, maturity.
30 − 11 = 29 days in April
6 days in May
25 days, discount period.

WRITTEN EXERCISE

Find the date of maturity and term of discount of the following paper:

	Date of Note	Time	Date of Discount	Date of Maturity	Term of Discount
1.	April 16, 1910	60 da.	April 16, 1910	?	?
2.	May 3, 1910	3 mo.	May 3, 1910	?	?
3.	May 16, 1910	90 da.	May 16, 1910	?	?
4.	June 14, 1910	6 mo.	June 14, 1910	?	?
5.	July 25, 1910	4 mo.	Aug. 4, 1910	?	?
6.	Aug. 20, 1910	75 da.	Sept. 1, 1910	?	?
7.	Sept. 24, 1910	90 da.	Oct. 15, 1910	?	?
8.	Oct. 29, 1909	8 mo.	Dec. 24, 1909	?	?
9.	Nov. 1, 1910	120 da.	Nov. 30, 1910	?	?
10.	Dec. 15, 1910	9 mo.	Mar. 15, 1911	?	?

DISCOUNTING NON-INTEREST BEARING PAPER

392. Find the proceeds of a note for $600 dated Jan. 12, 1909, maturing 6 mo. from date, and discounted at 7%, June 2, 1909.

OPERATION

July 12 maturity.

40 days term of discount.

$4.67 interest on note for 40 days (bank dis.)

$600 − 4.67 = $595.33 proceeds.

Find the proceeds of the following:

	Date of Note	Date of Discount	Time	Face	Rate of Disct.
11.	Mar. 4, 1911	Mar. 4, 1911	90 da.	$ 720	6 %
12.	Apr. 15, 1911	May 15, 1911	80 da.	$ 980	5 %
13.	Dec. 31, 1911	Jan. 31, 1912	60 da.	$ 925	4 %
14.	Dec. 28, 1911	Jan. 15, 1912	60 da.	$1260	4½%
15.	Jan. 14, 1910	Feb. 14, 1910	75 da.	$ 760	5 %
16.	June 3, 1911	July 15, 1911	3 mo.	$2400	6 %
17.	July 5, 1911	Sept. 15, 1911	4 mo.	$5680	8 %
18.	Sept. 3, 1911	Mar. 12, 1912	8 mo.	$ 420.75	4 %
19.	Dec. 28, 1911	Jan. 18, 1912	2 mo.	$ 860.50	7 %
20.	Dec. 30, 1911	Feb. 1, 1912	2 mo.	$ 924	5 %

DISCOUNTING INTEREST BEARING PAPER

393. A note of $500, dated Aug. 26, 1907, matures May 15, 1908, and bears interest at 6%. On Feb. 21, 1908, it is discounted at a bank at 7%. Find the proceeds.

OPERATION

yr.	mo.	d.	
1908	5	15	
1907	8	26	
	8	19	time from date to maturity.

$ 21.58 interest to maturity.
500.

$521.58 amount due at maturity.

84 days time of discount.

$8.52 bank dis. (int. on $521.58 for 84 da.)

$521.58 − $8.52 = $513.06 proceeds

WRITTEN EXERCISE

Find the proceeds of the following:

	Date of Note	Date of Discount	Time	Face	Rate of Int.	Rate of Dis.
21.	May 3, '01	June 4, '01	72 da.	$ 840	6 %	6%
22.	July 5, '02	July 16, '02	65 da.	$1300	5 %	7%
23.	June 16, '03	Aug. 1, '03	95 da.	$ 750	7 %	6%
24.	Aug. 14, '04	Sept. 3, '04	83 da.	$ 860	5 %	8%
25.	Aug. 31, '05	Sept. 30, '05	75 da.	$ 680	6 %	7%
26.	Sept. 30, '06	Oct. 15, '06	3 mo.	$ 500	7½%	9%
27.	Aug. 31, '07	Sept. 30, '07	6 mo.	$1275	6 %	10%
28.	Oct. 15, '08	Dec. 20, '08	8 mo.	$5280	5 %	7%
29.	Nov. 24, '09	Jan. 2, '10	7 mo.	$3400	4½%	6%
30.	Dec. 30, '10	Feb. 28, '11	5 mo.	$2100	5 %	12%

ILLUSTRATIVE PROBLEM

The following note is discounted Aug 1, 1910. Find the proceeds discounted at 8 per cent, less collection charges at the rate of 10 cents per $100 or part of $100 of the amount collected:

$2000. *Cincinnati, June 18, 1909.*

On December 1, 1910, for value received, I promise to pay to the order of George E. King Two Thousand Dollars with interest at the rate of six per cent per annum.

Sam'l B. Willey.

OPERATION

1 yr. 5 mo. 13 da., time of note.
122 da., discount period.

$2000.	principal
174.33	interest on pricipal to maturity
$2174.33	amount of note
58.95	discount on amount for 122 days
$2115.38	proceeds of note
2.20	collection charge at rate of 10 cts. per $100 on amt.
$2113.18	net proceeds of note

394. Collection charges. When banks discount "out-of-town paper" (that is, paper that has to be sent to other cities for collection) they usually make a small charge (about 10¢ for each $100) for collection. This is deducted from the proceeds. There is little uniformity as to the amount of collection charges. In the following problems deduct 10¢ per $100 from gross totals of amounts:

WRITTEN EXERCISE

31. I offered at a bank the following paper which was discounted and the proceeds placed to my credit Sept. 12, 1910, at 7% and collection:

Note dated Aug. 16, 1910, time 3 mo., int. 6%, face $890.50.

Note dated Sept. 4, 1910, time 90 da., int. 5½%, face $1260.00.

Draft accepted Aug. 24, 1910, time 60 da. after acceptance, face $1250.75.

Draft accepted Sept. 3, 1910, time 4 mo. after acceptance, face $564.25.

Draft maturing Nov. 15, 1910, face $1000.00.

What credit am I entitled to?

32. My bank account is overdrawn $3164.75, and I offer the following paper at discount, which is passed to my credit Jan. 4, 1910, at 6%, and collection:

Note dated Nov. 1, 1909, for $2500, time 6 mo., int. 5%.

Note dated Nov. 24, 1909, for $4800, time 3 mo., int 4%.

Draft accepted Dec. 5, 1909, for $1000, time 60 da. from acceptance.

Find my present credit balance in the bank.

33. Find the proceeds of the following draft, discounted Jan. 20, 1910, at 6 per cent and collection.

$10000. *Chicago, January 1, 1910.*

At sixty days sight pay W. M. Whigam, or order, Ten Thousand Dollars and charge to the account of

To Illinois Steel Co.,
South Chicago.

Crane Bros. & Co.

There was written across the face: "Accepted Jan. 5, 1910, Illinois Steel Co., per H. S. Lehr, Pres.

34. Find the proceeds of the following draft, discounted March 1, 1910, at 6 per cent and collection:

$25000. *New York, Feb. 1, 1910.*

At sixty days after date, pay James B. Brown Mfg. Co., or order, Twenty-five Thousand Dollars, and charge to the account of

To Metropolitan Elevated R. R. Co.,
Chicago, Ill.

Pullman Palace Car. Co.

The above was accepted in February.

35. Find the proceeds of the following draft, discounted on Aug. 1, 1910, at bank at 8 per cent, allowing 6 days for acceptance, and same time for returns, collection charges as before:

$600.00 *Chicago, Aug. 1, 1910.*

At ninety days after sight pay to A. N. Palmer, or order, Six Hundred Dollars and charge same to the account of

To Grant & Snyder,
San Francisco, Cal.

A. B. Manning & Co.

36. Sold goods at 25% above cost price, for $5500 on 3 months credit; the buyer offers to pay cash if allowed 10% discount; seller does not accept offer, but takes buyer's note and has it discounted at bank at $\frac{1}{2}$% a month. How much more did he gain by the latter than by the former transaction, and what was his total gain? (Collection charges deducted.)

Brokerage is a certain per cent paid to a broker for effecting the sale or purchase of commercial paper. It is computed upon the face of the paper.

EXAMPLE

$9000. *San Francisco, July 2, 1910.*

Eight months after date I promise to pay to Hinman & Smith, or order, Nine Thousand Dollars, value received with interest at the rate of 6 per cent per annum until paid.

A. R. Kipp.

Discounted Nov. 16, 1910, at 8 per cent, brokerage $\frac{1}{8}$ per cent.

OPERATION

8 months, time of note.
166 days, discount period.

$9000.	principal
360.	interest
$9360.	amount
220.48	discount
$9139.52	
11.25	brokerage $\frac{1}{8}$% of $9000.
$9128.27	net proceeds.

Find the proceeds of the following:

	Date of Note.	Date of Disct.	Time.	Face	Rate Int.	Rate Dis.	Rate Bro.
37.	Aug. 13, '10	Aug. 13, '10	2 mo.	$3200	4%	6%	$\frac{1}{8}$%
38.	Oct. 3, '10	Nov. 2, '10	90 da.	$1225	6%	7%	$\frac{1}{8}$%
39.	July 2, '10	July 2, '10	6 mo.	$3400	5%	6%	$\frac{1}{4}$%
40.	Apr. 21, '10	May 20, '10	100 da.	$5000	$4\frac{1}{2}$%	6%	$\frac{1}{8}$%
41.	May 15, '10	June 25, '10	7 mo.	$2100	6%	8%	$\frac{1}{4}$%

FINDING THE FACE OF THE COMMERCIAL PAPER

395. It is sometimes necessary to know the face of a note or acceptance, which, when discounted, will yield a given sum.

EXAMPLE: What is the face of a note without interest which when discounted at the bank for 90 days at 6%, will yield proceeds amounting to $147.50?

OPERATION

Suppose the note were drawn for $1.

$ .015 = discount on note for $1 at 90 days.

$1 − .015 = $ .985 proceeds of note for $1.

$147.50 ÷ .985 = $149.75 face of note.

The note must be drawn for as many dollars as the proceeds of a note of $1 is contained times in the required proceeds.

WRITTEN EXERCISE

42. The Corn Exchange Bank presents to Carter & Co., for collection, their note for $247.60, with accrued interest $12.75. To meet the obligation, Carter & Co. give the bank their own 60 day acceptance for such a sum that the proceeds at 7% will meet the note and interest. Find the face of the acceptance.

43. Brown Brothers & Co. buy a bill of goods on cash terms, amounting to $187.50, and pay for it with their 90 day acceptance drawn for such a sum as will, when discounted at 6%, yield the face of the bill in cash. What is the face of the acceptance?

44. Owing $960, I have my note discounted at a bank at 6% for 90 days, for such a sum that the proceeds will pay the debt. What was the face of the note?

45. A man owes $12000, and to pay this has a note of $7500, maturing in 45 days, discounted at 6%, and draws a second note payable in 60 days for such a sum that when discounted at the same rate the proceeds of both notes will enable him to pay the debt. Find the face of the second note.

CHAPTER XVII

COMMISSION

396. Agency is a relation created by contract, under which one person called the *agent* undertakes to do business for another person called the *principal*. Agents are variously known as *factors, commission merchants, brokers,* etc., according to the kind of business covered by their agency.

397. The compensation of an agent may consist of a salary or of a certain percentage computed upon the amount of business done. In the latter case the compensation of an agent is called *commission,* or *brokerage.*

398. The business of commission agencies includes the following general divisions:

1. Making collections.
2. Sales of goods or other property.
3. Purchases and investments.

399. Collections. Debts, taxes, premiums on insurance policies, etc., are often collected through agents, who receive as their compensation, an agreed percentage of the sum collected. The *proceeds* of a collection (or of a sale) is the amount handed over, or credited to the principal after all commissions or other charges have been deducted.

ORAL EXERCISE

1. Frank is engaged by a grocer to collect delinquent accounts at a commission of 5%. At the end of the day he had collected $126.40. What sum does he receive for his day's work, and how much does he turn over to the grocer?

2. The tax collector for the town of Brocton receives $2\frac{1}{2}\%$ of all sums collected. If he collects $7500 during the month, how much does he earn?

3. A man received \$50 for collecting a debt of \$800. What was the rate of commission?

4. A collecting agent working on a commission of 5%, earns an average income per month of \$150. How much does he collect per year?

5. A firm wishing to go out of business, has due them on book accounts, approximately \$25000. If 80% of this is collectible, and they pay 15% for collection, what sum should they realize on the accounts?

Find the amount of commission and proceeds on each of the following collections:

6. \$128 at a commission of 10%.

7. \$5600 at a commission of 5%.

8. \$8400 at a commission of $2\frac{1}{2}$%.

9. \$12.80 at a commission of $12\frac{1}{2}$%.

10. \$250 at a commission of 6%.

11. \$45.75 at a commission of 20%.

12. \$1500 at a commission of 8%.

WRITTEN PROBLEMS

1. A merchant turned over to a collecting firm accounts amounting to \$879.65, to be collected on a commission of 15%, not including legal and other charges. If 75% of the accounts were collected and there were charges amounting to \$8.15, what was the amount of the proceeds?

2. A tax collector collected for the town of Greenwood, delinquent taxes amounting to \$2897.50 on a commission of 6%. How much was turned into the treasury, and what was the collector's commission?

3. On June 16, 1910, Carter & Co. turned over for collection to the First National Bank, the following overdue notes: (a) a note for \$450, dated Oct. 16, 1909, with interest at 7%; (b) a note for \$325, dated Dec. 21, 1909, interest 8%; (c) a note for \$628.50, dated Feb. 1, 1910, interest 6%. If the bank collected each note in

full, and charged a collection fee of 2½%, what was the amount of Carter & Co.'s credit?

4. A law firm collected for a client, a debt amounting to $750, and charged for the collection, the sum of $25. To what rate of commission was this equivalent?

5. A collector finds by his cash book that his commissions for a year amounted to $1248.90. If his commission charge is 5%, what were the total collections?

400. Selling on commission. Many kinds of goods, as books, patented appliances, machinery, etc., are often sold on commission, by special agents; also grain, cotton, fruit, dairy and other products are both bought and sold on commission by agents variously known as grain brokers, factors, and commission merchants. Commissions for selling are computed on the amount of the sales.

ORAL PROBLEMS

1. A book agent took 32 subscriptions for a $5 book, on which his commission is 30%. What amount did he earn, and what sum must he remit to the publishers?

2. Mr. A is agent for a patent window cleaner which sells for $1.50. If he receives 20% commission and sells a dozen cleaners a day, what can he earn per week?

3. A grain factor's sales average $15000 per month, with commissions at 2%. What is his income per year from his sales department?

4. An implement dealer is agent for a patent harrow which sells for $18. He receives a commission of 25% and guarantees collections. In one season he sold 100 harrows, but in the case of two of the sales, was unable to collect. What were his net commissions?

5. A book dealer received a consignment of books amounting to $420, to sell on comission. He paid freight on the books $10, and has sold all but $60 worth which the publishers take back. How much shall he pay the publishers after deducting the freight payment and a commission of 25 per cent on the books sold?

401. Account sales. This is an itemized statement of sales, with commission and other charges, rendered by an agent to his principal. Following is an ordinary form:

ACCOUNT SALES

FROM **Central Commission Co.**

College City, May 4, 1910.

TO Wallace Paulson,
Marshalltown, Iowa.

FOR MERCHANDISE RECEIVED Apr 27, 1910. VIA C & N.W. Ry.

Paul Sandy
CLERK

DATE			MERCHANDISE RECEIVED		
4	27		549		
			4		
			545 lbs. Potatoes		
			MERCHANDISE SOLD		
4	27		300 lbs. Potatoes	46	138
	28		120 " "	46½	55 80
5	3		100 " "	48	48
	4		25 " "	50	12 50

CHARGES			TOTAL SALES	254	30
Freight	29	80	TOTAL CHARGES	39	10
Drayage			PROCEEDS REMITTED	215	20
Other	1	50	CREDITED		
Cash Adv.					
Storage	1	12			
Insurance		32			
Commission 2½	6	36			
Other					
Total Charges	39	10			

Consignment No. 7396

WRITTEN PROBLEMS

6. Find the total of the following commissions:

15000 bu. wheat at 96¢, commission 2%.

750 baskets peaches at $1.30, commission 3%.

380 boxes oranges at $2, commission 2½%.

16950 lb. butter at 18¢, commission 3½%.

1275 baskets grapes at 15¢, commission 4%.

7. A grain broker sold for his principal 16000 bu. No. 2 wheat at 86½¢, charging 2½% for sales and ¼% for guarantee. Find the proceeds.

8. Copy and carry out extesions in the following account sales:

DATE	MERCHANDISE RECEIVED	
Feb 10	50 bbl Cranberries	
	1 " Unsalable	
	49	
	MERCHANDISE SOLD	
Feb 11	16 bbl Cranberries @ 6.05	
	5 " " 6.	
	8 " " 5.90	
	20 " " 5.22	

CHARGES		TOTAL SALES	
Freight	13 90	TOTAL CHARGES	
Drayage	2 10	PROCEEDS ~~DISPUTED~~	
Other	4	CREDITED	2
Cash Adv.	100		
Storage	1 58		
Insurance	35		
Commission (3%)	2		
Other			
Total Charges			

Consignment No. 1007

9. A broker charged 5½% on gross sales of 784 shares of mining stock, at $5.25 per share. Find the commission and net proceeds.

10. If the net proceeds of a sale of butter amounted to $187.50 and the commission charge was 2½%, and other charges amounted to $17.20, what was the amount of the gross sales?

11. Copy and carry out extensions in the following account sales:

DATE		MERCHANDISE RECEIVED		
8	15	1 Car Watermelons		
		MERCHANDISE SOLD		
8	16	300 Melons @	.21	
	17	160. "	.22½	
	17	221 "	.21½	
	18	356 "	.20¾	
	22	140 "	.18	
	26	16 "	.11	

CHARGES				
Freight	34	10	TOTAL SALES	
Drayage			TOTAL CHARGES	
Other	1	60	PROCEEDS { REMITTED / ~~CREDITED~~	
Cash Adv.				
Storage	1	80		
Insurance		95		
Commission 5%				
Other				
Total Charges				

Consignment No. 4290

12. Rule and complete an account sales for the following, omitting dates:

SALES: 400 baskets peaches at 40¢, 20 boxes pears at 75¢.

CHARGES: Freight $44, drayage $4, insurance 45¢, commission 6% of sales.

13. Copy and carry out extensions in the following account sales:

19— DATE		MERCHANDISE RECEIVED		
Dec 7		5 bx. Dressed Chickens		
		MERCHANDISE SOLD		
Dec 8		160# D. Chickens @ .19		
9		48# " " .18½		
9		108# " " .18¾		
10		73# " " .19½		
12		24# " " .20¼		
13		52# " " .20¾		

	CHARGES			TOTAL SALES		
	Freight	8	10	TOTAL CHARGES		
	Drayage		✓	PROCEEDS { REMITTED		?
	Other		50	~~CREDITED~~		
	Cash Adv.		✓			
	Storage		✓			
	Insurance		13			
	Commission 6%					
	Other		✓			
	Total Charges					

Consignment No. 649

14. If an agent's commission at 5% on a certain sale was $62.90, and the other charges were $12.75, what were the net proceeds?

402. Purchasing and investing. Factors or brokers often buy as well as sell on commission, and brokers, banks, or other financial agents may make investments of capital for others. Such services are usually paid for by means of a commission on the amount of property purchased, or on the sum invested.

403. Account purchase. This is an itemized statement, similar in form to an account sales, setting forth purchases, commissions, and other charges. The various charges are added to the first cost making a total that is chargeable to the principal in the transaction.

WRITTEN PROBLEMS

15. A financial agent loaned for a capitalist, the sum of $38500 on farm mortgages, charging a commission of $2\frac{1}{4}\%$. What was the capitalist's total investment?

16. A broker invested for a speculator $25000 in the stock of a Nevada gold mine, at $12.50 per share, charging a commission of 6% on the amount invested. Sixty days later the stock was sold at $11.25 per share, commission 5% of sales. Allowing for interest at 7%, on the sum invested, what were the speculator's losses?

17. A man bought through a Chicago grain broker, 28000 bu. Iowa corn at $51\frac{1}{2}¢$, brokerage $1\frac{3}{4}\%$. Later, the corn was sold at $55\frac{3}{8}¢$, brokerage rate the same. If the charges amounted to $378.15, did the speculator gain or lose, and how much?

18. Charles E. Brown is a produce buyer for a commission firm, receiving 5% commission on all purchases, with an allowance for necessary charges and personal expenses. During the first week his purchases and expenses were as follows:

1264 lb. butter	@	$ .18
398 lb. butter	@	.22
746 doz. eggs	@	.16
298 lb. honey	@	.12
326 brl. apples	@	2.85

Personal expenses (per bill rendered) $21.50

Drayage and other charges (per bill) $64.20

What is the total of Brown's account purchase?

ORAL TEST PROBLEMS

1. What is an agent's commission on sales amounting to $600 at 2%?

SUGGESTION.—2% of $600 is the commission.

2. What is an agent's commission on a purchase of goods amounting to $500 at $2\frac{1}{2}\%$?

3. An agent's commission is 3%. How much does he receive on a sale of 120 bushels of oats at 40¢ per bushel?

4. What commission must be paid an agent for collecting a bill of $750 at 2%?

5. What commission does an agent earn by collecting 75% of a claim of $800 at 3% commission

6. Find the commission at $1\frac{1}{2}\%$ on a sale of grain for $640.

7. A commission merchant sold 325 barrels of apples at $4 per barrel. What is his commission at 3%?

8. My agent sold 120 bushels of corn at 50 cents per bushel, and 240 bushels of wheat at 75 cents per bushel. What is his total commission at $2\frac{1}{2}\%$?

9. What is the commission on the sale of a farm for $6400 at 3%?

10. How much do I receive from sales amounting to $300, less a commission of 2%?

11. My agent sells flour for $725, and deducts a commission of 2%. How much do I receive?

12. Find the net proceeds of a sale of flour amounting to $720, less a commission of 3%.

13. An agent sells 150 bushels of wheat at 80 cents a bushel, and deducts a commission of 2% and drayage charge of $5. What are the net proceeds?

14. An agent deducted $72 as commission at 2% for selling corn. How much did he receive for the corn?

SUGGESTION.—$72 is 2 per cent of the sales.

15. An agent's commission is $50 for selling land. What were the sales, if his rate of commission is $2\frac{1}{2}\%$?

16. An agent is paid $16 for buying corn at a commission of 2%. How many bushels does he buy at 50 cents per bushel?

17. An agent receives 5 per cent commission for collecting rents on a store room for 12 months at $50 per month. What is his commission?

18. An agent sells a farm for $5000 and remits to his principal $4850. What per cent commission does he retain?

19. A lawyer collects a note for $100 and interest $5, on which he is to receive a commission of 10%. How much shall he remit to his client?

20. A country buyer purchases produce for a firm, amounting to $240, on which he is to receive a buying commission of $7\frac{1}{2}\%$. How much does the firm owe him for the produce?

WRITTEN TEST PROBLEMS

19. What is an agent's commission for selling goods to the amount of $1250 at 3%?

20. An agent sells merchandise to the amount of $750, and retains $15 for his commission. What rate does he charge?

21. An agent charges 2½% on the sales of a consignment of cigars, receiving for his commission $100. The other expenses being $75, what are the consignor's net proceeds?

22. A consignor's net proceeds are $1500.50, after allowing $100 expenses and 3% commission. What was the amount of the sale?

23. Paid a cotton broker $50 for selling 50 bales of cotton, averaging 500 pounds each, at 8 cents per pound. What is the rate of commission?

24. My commission merchant sent me $1100 as the net proceeds of a consignment, after deducting $125 for charges and 2% commission. What was the amount of the sale?

25. The net proceeds of a consignment are $675.50, and the rate of commission 3½%. What is the amount of the sale?

26. An agent collected part of an account for me, and, after deducting his commission of 5%, he sent me $427.50. What per cent of the original debt of $900 remains unpaid?

27. My agent in Birmingham bought 100 barrels of sweet potatoes for me at $1.75 per barrel. His charges were: Commission 2½%, drayage $14.50, sundry charges $2.75. For what sum must he draw, allowing 75 cents for collection?

28. A speculator bought 2500 bu. of wheat, at 75 cents per bushel. He immediately shipped the same to Chicago, to be sold on a commission of 3%, other charges being: Freight $175, drayage $23, insurance ¼% of sale. How much did he gain, the wheat being sold at 95 cents per bushel?

29. A commission merchant sells 600 bu. of oats at 32 cents per bu. for cash, 750 bu. rye at 64 cents per bu., on account at 30 days. What is his total commission, if, besides the regular commission of 3%, he charges 2½% for guaranty?

30. A commission merchant bought for his principal 12000 bu. wheat at $1.20 per bu., and immediately sold it at $1.30, in-

vesting the proceeds in the purchase of oats in car load lots of 1000 bu. each, at 35¢ per bu. His commission being 2% for buying, and 2½% for selling. Find the following: Number of car loads; amount unexpended for oats; total commission.

31. A commission merchant received 4000 bushels of wheat and 24000 pounds of beef, with instructions to sell, and invest $2500 of the proceeds in cotton, and remit the balance after deducting the charges. He sold the wheat at 62½ cents per bushel, and the beef at 9½ cents per pound. He paid $375 for freight, $24 for drayage, and charged a commission of ⅞ cents per bushel on the wheat, 2½% on the beef, and 2¼% on the cotton. How much did he remit?

32. A cotton factor bought for his principal 200 bales of cotton weighing 102850 lb. at 13½¢, commission 1½%, and sold the same later at 15⅝¢, commission 2%. The charges other than commission amounted to $121.50. What was the net gain on the speculation?

33. Find the net proceeds of the following account sales:

Chicago, Jan. 1, 1910.

Sold for account of SAM'L B. WILLEY

By W. J. DURAND & Co., Commission Merchants.

1909.		SALES					
Nov.	1	250 bu. Beans	1.06	.....			
"	16	400 lb. Cheese	.10	.....			
Dec.	4	40 brl. Flour	3.80	.....			
"	28	1200 bu. Wheat	.64	.....		.....	
		Charges.					
Freight $84.60. Drayage $12.85. Storage $24.30.							
Inspection $4. Insurance ⅛%. Commission 3%.							
			Total charges			.....	...
			Net proceeds			.....	...

34. Rule and complete an account sales, in the form given in the preceding example, for the following:

SALES: 19—, Oct. 1, 50 baskets grapes @ 30¢, 10 brl. apples @ $3.25; Oct. 3, 75 baskets grapes @ 28¢, 15 brl. apples @ $3.20.

CHARGES: 19—, Sept. 29, freight $27.50; Oct. 4, drayage $2.75, insurance 25¢, commission 5% of sales.

CHAPTER XVIII

STOCKS, BONDS, AND MARKET SPECULATIONS

STOCKS

404. A **corporation** or **joint stock company** is an association of individuals who are authorized by law to transact business as one person.

405. The **charter** of a corporation is the legal instrument, under which the corporation is authorized to act. It specifies the name and location of the corporation, the names of the incorporators, the amount of the capital stock, and the kind of business in which the corporation is authorized to engage. The charter of a corporation is usually issued by the secretary of the state in which the corporation is established, in accordance with the terms of a general law of the state.

406. The **capital stock** of a corporation, is the amount of capital to be invested in the business, as provided by the charter. The stock of a corporation is divided into units called shares. The *face,* or *par value* of a share may be for any amount desired, as specified in the charter, but the face of a share is usually $100.

407. A **certificate of stock** is an instrument issued by the officers of a corporation to evidence the ownership by the holder, of the number of shares specified in the certificate. Certificates of stock are usually transferable at the option of the holder, but such transfer, to be valid, must be recorded on the books of the coporation. The control of a corporation is vested in a body of officers called a Board of Directors. These are elected by the stockholders, each stockholder being entitled to as many votes as he has shares in the corporation.

408. Dividends. The profits of a corporation are distributed to the several stockholders in amounts proportional to the number of shares held by them. These distributions of profits are known as *dividends.* They are computed at a certain percentage of the par or face value of the stock. Thus, if a five per cent dividend were

declared (that is, allowed) by the officers of a company, each stockholder would receive $5 for each share of $100 face value held by him. For instance, a holder of five shares would receive $25, and a holder of 12 shares would receive $60, and so on.

409. Classes of stock. Several classes of stock are used in corporation affairs. The most important are the following:

410. Preferred stock. This is stock which entitles the holder to a specified annual dividend, out of the profits, which dividend must be paid before the holders of other stock may receive profits. Thus, the holder of "seven per cent preferred" is entitled to a yearly dividend of $7 per share, before the holders of other stock are entitled to a dividend. The holders of preferred stock do not share in the general dividends, nor receive more than the per cent specified in their certificates.

411. Common stock. This is the ordinary stock of the company, which entitles its holders to an equal proportion of the profits that remain after the dividends on the preferred stock have been paid.

412. Treasury stock. This is unissued stock (either common or preferred) which is held in reserve as an asset of the company, and which may be either sold or canceled and retired as circumstances may warrant. Any proportion of the original stock of a company may be held as treasury stock, but new stock in excess of the amount prescribed by the charter may not be issued.

In some states the holding of unissued or non-paid up shares in the company treasury as treasury stock is forbidden by law. Of course, the company can acquire stock previously issued, either by purchase or donation, and such stock can be held as treasury stock.

413. Watered stock or **"water."** These are terms used in financial circles to indicate the excess of the face value of the stock over and above the actual value of the company's property or legitimate assets.

414. An **assessment** is a sum levied upon the stockholders to meet expenses or make up deficiencies of income. Assessments, like dividends, are computed at a percentage of the par or face value of the stock.

415. An **installment** is a part payment on subscribed stock. Persons who are to form a company usually begin by signing a "stock subscription agreement," in which they severally agree to take, and pay for, the number of shares set opposite their names.

This stock is not usually paid for in one sum, but in installments, of a certain per cent of the face, as called for from time to time by the company directors.

416. Par and market value. As already indicated, the par value of stock is the face or amount for which the shares are issued. Thus, if A holds five shares in a corporation, the par value of his stock would be $500 (unless the shares are issued at a face value other than $100). But if the average annual dividends of a company are in excess of current interest rates, the market value of stock may be much greater than the par value. Suppose for instance, that Mr. A owns stock in a corporation which pays 10% annual dividends, and that the current interest rate on money is 5%. It is evident that each share of his stock, having a par value of $100, will yield him an income of $10 a year, or as much as he would obtain from $200 in money loaned at 5%. If the continuance of his dividends is assured, it is evident therefore that for income producing purposes, each share of Mr. A's stock is worth $200, or double its par value. On the other hand, stock in an unprosperous corporation that pays a low rate of dividend, might have a market value much less than its par value.

417. Buying and selling stocks. As before observed, most stocks are transferable at the will of the holder, and "stock operations" or the buying and selling of railroad, and other important stocks, involve the largest transactions known to the financial world.

418. A **stock broker** is a person who makes a business of buying and selling stocks for the public, receiving as his compensation a cash percentage of the par value of the stock handled, called *brokerage* or *commission*. In some classes of mining stock, the broker receives a commission on the amount of the purchase or sale, without regard to the par value of the stock. A broker's commission, on ordinary stock transactions, is usually $\frac{1}{8}$% of the par value, or $12\frac{1}{2}$¢ per share.

419. The **Stock Exchange** and **quotations.** Most of the great stock operations are carried on in our large cities, at a place called a Stock Exchange. Stock exchanges are operated by associations of brokers, and other financiers, the members of which meet at the exchange to buy or sell stocks in the open market, in accordance with rules established by the members who hold seats in the exchange. Stocks that are commonly dealt in by the public, are listed at the exchanges, and their current prices are published daily by the

press, in lists called "stock quotations." The market fluctuations are also transmitted automatically during stock exchange hours by telegraphic instruments called "tickers." In the large cities, these tickers are installed in the offices of business men who are interested in the stock market, also in hotels and other public places. The quotations are recorded on a strip of paper called "the tape" which constantly passes from the machine.

420. Stock quotations are based on the price of a single share of stock at a par value of $100. Thus, a stock quotation of $87\frac{1}{8}$ means that a $100 share of that stock is worth in the market $87.12\frac{1}{2}$, when the quotation was issued. Stock quotations are given in even dollars, with the fractions, halves, quarters, and eighths.

421. In stock exchange parlance, an eighth is called a "point." Thus, to say that a certain stock "fell off three points," means that the price fell three-eighths of a dollar.

422. The following form shows recent quotations of the New York Stock Market, through the letter "A," complete lists being published for every business day of the year:

	Sales	Open	High	Low	Close
Allis-Chal.	2,300	$7\frac{5}{8}$	$8\frac{3}{4}$	$7\frac{5}{8}$	$8\frac{1}{4}$
Amal. Copper	40,075	$58\frac{3}{4}$	$60\frac{1}{2}$	$58\frac{3}{4}$	60
Am. C. & F.	1,400	$33\frac{1}{2}$	$34\frac{1}{2}$	$33\frac{1}{2}$	$34\frac{1}{2}$
Am. C. & F. pfd.	200	$94\frac{1}{2}$	$94\frac{7}{8}$	$94\frac{1}{2}$	$94\frac{7}{8}$
Am. Can.	200	5	5	5	5
Ah. Cotton Oil	1,800	29	$29\frac{1}{2}$	$28\frac{1}{2}$	29
Am. Ice Sec.	1,480	25	$25\frac{1}{2}$	$24\frac{1}{2}$	$24\frac{1}{2}$
Am. Loco.	3,700	$46\frac{1}{2}$	$47\frac{1}{4}$	$46\frac{1}{4}$	$46\frac{7}{8}$
Am. Loco. pfd.	200	97	97	97	97
Am. Smelt.	29,300	71	$72\frac{1}{4}$	$70\frac{1}{2}$	$71\frac{7}{8}$
Am. Smelt pfd.	1,700	$95\frac{3}{4}$	$95\frac{3}{4}$	$95\frac{1}{8}$	$95\frac{5}{8}$
Am. Sugar.	6,025	126	$127\frac{5}{8}$	126	$127\frac{5}{8}$
Am. Tob. pfd.	760	$90\frac{3}{4}$	91	$90\frac{3}{4}$	91
Am. Woolen.	1,500	$19\frac{3}{4}$	21	$19\frac{5}{8}$	21
Anaconda	9,200	39	$39\frac{1}{4}$	$37\frac{7}{8}$	$38\frac{3}{4}$
Atchison	8,500	$77\frac{1}{2}$	$79\frac{1}{2}$	$77\frac{3}{8}$	$79\frac{1}{4}$
Atchison pfd.	580	$88\frac{1}{2}$	89	$88\frac{3}{8}$	$88\frac{3}{8}$
At. Coast Line.	1,600	$75\frac{3}{8}$	$75\frac{3}{4}$	$75\frac{1}{4}$	$75\frac{3}{4}$

The list embraces various "industrials," or stocks of manufacturing concerns, also leading mining and railroad stocks, with the understood abbreviations. The abbreviation pfd. indicates "preferred stock."

The first figure column shows the numbers of shares that changed hands during the day; the second column, the price when the market opened in the morning; the third column the highest price reached; the fourth, the lowest price; the fifth, the price when the market closed. Thus, in the case of the common stock of the Atchison, Topeka, and Santa Fe railroad (Atchison) there were 8500 shares sold, the stock opened in the morning at \$77.50 per share, sold during the day as high as \$79.50 per share, as low as \$77.37½, and was selling when the exchange closed in the afternoon at \$79.25 per share.

In some quotation lists, the difference between the current closing price and the closing price for the previous day is also given.

ORAL EXERCISE ON THE ABOVE STOCK QUOTATIONS

1. Had a member of the N. Y. Stock Exchange bought 100 shares of Amal. Copper at the opening, and sold at the closing price, what would he have gained?

2. Suppose he had bought 50 shares of Am. Smelt, pfd., (American Smelter preferred) at the opening and sold at the closing price, what would he have lost?

3. What would he have gained by buying 1000 shares of common Am. Loco. (American Locomotive Works, common stock) at the opening price and selling at the highest price?

4. What would he have lost by buying 1000 shares of Anaconda at the opening price and selling at the lowest price?

5. Allowing for brokerage, $\frac{1}{8}\%$ for buying and the same rate for selling, what would a non-member of the exchange have gained by buying 100 shares of Am. Sugar at the opening price, and selling at the highest price?

6. What would he have lost by buying 100 shares of Atchison pfd. at the opening price and selling at the closing price?

7. How do the prices of preferred stocks in this list compare with the prices of corresponding common stocks? Can you suggest a reason for the difference? Can you think of conditions that would result in the price of common stock being higher than the price of preferred stock in the same company?

8. What would be the probable effect of financial depression or "hard times" on the relative prices of common and preferred stocks? Of prosperous times?

NOTE.—Spirited contests sometimes occur among different factions of a company as to which group of stockholders shall control in choosing the directors. In these contests the competition for ownership of the common stock often becomes very keen, and the market price rises temporarily to a figure far beyond the ordinary quotation.

ORAL TEST PROBLEMS

In all the following problems involving purchases or sales, it will be understood that the brokerage is $\frac{1}{8}\%$ or $12\frac{1}{2}$c per share, both for buying and for selling.

1. Mr. A bought 20 shares of Northern Pacific, quoted at $130\frac{1}{2}$. What did the stock cost him?

2. Mr. B holds 150 shares of Illinois Central common, on which a dividend of $5\frac{1}{2}\%$ is declared. What sum does he receive?

3. Thomas & Brown subscribed for 60 shares of stock in a new street railway company. The officers have called for an installment of 15 per cent of all subscriptions. For what sum must the firm draw their check?

4. Mr. Carter sold, through a broker, 100 shares of Southern Pacific common at $76\frac{1}{8}$, and invested the proceeds in Mexican Central at $18\frac{3}{8}$. How many shares of the latter stock were purchased?

5. Mr. Smith holds 46 shares of stock in an undeveloped coal mine upon which an assessment of 15% has been called for. What sum will Mr. Smith have to pay?

6. If I buy through a broker 50 shares of U. S. Steel pfd. on a market quotation of $104\frac{3}{8}$, and later sell at $99\frac{1}{8}$, what do I lose?

7. Mr. S bought through a broker, 100 shares of Rock Island at 25, received a dividend of 3%, and then sold at $26\frac{1}{4}$. What sum did he gain by the speculation?

8. A man holds 1000 shares of Northern Pacific. If the price falls six points, what does he lose?

9. An investor bought through a broker, 100 shares of 6 per cent preferred stock at $91\frac{1}{2}$, drew the annual dividend and then sold at $89\frac{1}{4}$? What were his profits on the speculation?

10. Mr. C subscribed for 40 shares of woolen mill stock, paid three assessments of 15% each, and then sold his installment certificates, without brokerage, at $51\frac{1}{2}$. What sum did he gain?

423. Stock market operations. The New York Stock Exchange, a chartered association of 1100 members, is the general market in the United States for American stocks. Any detailed account of stock market operations would be out of place in a work

of this kind, but some stock market terms are of such frequent occurrence in business as to merit explanation.

424. Margin. This is a sum of money paid in advance by a principal to his broker, to cover expenses, and secure the latter against loss in purchasing stocks which are to be held for speculative purposes. In ordinary operations in active stocks, the cash margin required is an amount equal to 10% of the par value of the stock purchased. In margin operations, the stock is not really delivered to the purchaser, but is bound or contracted for by the broker who holds it for his principal, the stock itself often being put up (hypothecated) as security for the broker's purchase. In only a small part of the speculative operations of the stock exchange, is the stock actually delivered by the seller to the buyer.

In rendering accounts to their customers, stock-brokers charge interest at the legal rate for all purchases, not taken up and settled for within three days. They also allow interest on the cash margins advanced.

425. Bulls and bears. In stock-broker phraseology, a "bull" is an operator who buys stock in the hope that it will rise in value. A "bear" is an operator who sells stock for future delivery that he does not own, in the hope that the price will fall and that he may make a profit by buying it later at a lower price than that at which he has agreed to sell it. It is evident therefore that the bulls are interested in keeping the price of stocks up, while the bears are equally interested in forcing the prices down.

426. Long and short. A bull is said to be "long" on the stock that he is holding for sale. A bear is said to be "short" or to have "sold short" on the stock that he has to buy.

427. Realizing and liquidating. When a bull sells at a profit the stock that he has bought, he is said to have "realized" on his stock. When he has to sell at a loss, he is said to have "liquidated."

428. Covering. When a bear buys stock for delivery on the sales he has made, he is said to have "covered" his deals, no matter whether he has lost or gained.

429. Call money. In stock market operations, brokers usually borrow money subject to "call," that is, the conditions of the loan are such that the lender may demand payment or "call" the money at any time. Also, the borrower may tender payment at will. Money thus loaned is known as "call money." It is usually secured by stocks, bonds, or other collateral taken at about 20% of their

Market value. There are great fluctuations in the interest charges for call money. During the money panic of 1907-8, these rates ranged from $1\frac{1}{2}\%$ and 2%, to as high as 125%.

WRITTEN PROBLEMS.

1. On Sept. 5, a broker bought for a customer, 500 shares of Am. Woolen at $19\frac{3}{4}$, borrowing the purchase money on call at 7%. On Sept. 17, he sold the stock at $21\frac{3}{8}$, rendering an account to his customer for interest and brokerage. What did his customer gain by the speculation? (As the brokerage charges are entered at time of settlement, interest is not computed on them.)

2. On Mar. 9, I sent my broker a draft for $5000 as a margin to cover the purchase of 700 shares of Am. Smelt. common, at $81\frac{5}{8}$. On Mar. 18, the stock had declined to 70 and I sent my broker an additional $2500 to cover further decline. On Mar. 24, the price had risen to $76\frac{1}{4}$ and I ordered him to sell. He sent me a statement including charges for interest at 8% and brokerage, and credits for interest on margins advanced, also a check to close my account. What was the amount of this check, and what was my net loss on the speculation?

3. A speculator on the N. Y. Exchange on Oct. 5, bought on his own account 8000 shares of Anaconda at $42\frac{1}{4}$, borrowing the money for the purchase on call at 15%. On Oct. 11, he paid the first loan and negotiated another for the same sum at 10%. On Oct. 20, he sold the stock at $46\frac{1}{2}$ and paid his loan. What was his net gain?

4. On Oct. 15, Mr. A sent a New York broker a draft for $3000 as a margin to cover the purchase of 200 shares of Am. Sugar, at $142\frac{1}{8}$. On Oct. 25, the broker wired Mr. A that the stock had declined to $131\frac{5}{8}$ and that he must either sell or send a further margin to cover possible loss. Mr. A wired him to sell. What was his loss, including brokerage and interest at 7%?

5. I send my broker $200 as a ten per cent margin, to cover a purchase of 25 shares of Atchison common, at $76\frac{1}{4}$. To what point must the stock decline to absorb the margin?

6. In one day the price of Amalgamated Copper went up seven points. What are the gains of a holder who owns 527 shares?

7. James Gordon sold his farm in Illinois for $8500, and invested the proceeds through a broker in Am. Sugar, at $126\frac{1}{2}$. Two years later he sold the stock at $131\frac{3}{4}$, having in the meantime received two dividends of 6% each. Allowing for interest on his investment at 5%, what were his net gains?

PRODUCE EXCHANGE OPERATIONS

430. As the New York Stock Exchange is the center of stock speculations and investment, the Chicago Board of Trade is the leading produce exchange of the United States. Speculation in grain and produce is carried on through sales or purchases for future delivery, called futures, and sales for immediate delivery called "cash transactions" or "cash." Most of the transactions in grain and cotton are futures. The months of May, July, September, and December are the usual periods for the delivery of futures. Thus, "September wheat" means wheat that may be delivered anytime during the month of September, and "July corn" means corn that is sold for delivery in July.

The brokerage charges or commissions on the Chicago Board of Trade are in accordance with a fixed schedule. For grain it is $\frac{1}{8}¢$ per bushel, and for meats $2\frac{1}{2}¢$ a barrel, or $2\frac{1}{2}¢$ per 100 lb.

The margins required for Board of Trade operations vary with the quantity and kind of produce purchased, the credit of the buyer, etc. They are as a rule lower proportionally than stock market margins.

431. The following Board of Trade quotations for April 29, of a recent year were published in the *Chicago Daily Tribune:*

RANGE OF ACTIVE FUTURES

	Open	High	Low	Close	
				April 29	April 28
WHEAT					
May	$97\frac{1}{8}$	$98\frac{1}{8}$	97	$97\frac{3}{4}$	$96\frac{7}{8}$
July	$87\frac{1}{2}$	$88\frac{1}{8}$	$86\frac{7}{8}$	$87\frac{5}{8}$	$86\frac{7}{8}$
Sept.	$84\frac{1}{4}$	$84\frac{5}{8}$	$83\frac{5}{8}$	$84\frac{3}{8}$	$83\frac{1}{2}$
CORN					
May	$66\frac{1}{8}$	$67\frac{1}{4}$	$65\frac{3}{4}$	$67\frac{1}{8}$	$66\frac{1}{8}$
July	$62\frac{5}{8}$	$63\frac{3}{8}$	$62\frac{3}{8}$	$63\frac{3}{8}$	$62\frac{1}{2}$
Sept.	61	$61\frac{7}{8}$	61	$61\frac{7}{8}$	$61\frac{1}{8}$
OATS					
May, old	$53\frac{1}{8}$	$53\frac{1}{4}$	$52\frac{1}{8}$	$53\frac{1}{4}$	$53\frac{3}{8}$
May, new	$52\frac{1}{2}$	$52\frac{3}{4}$	$52\frac{3}{8}$	$52\frac{3}{4}$	$52\frac{1}{4}$
July, old	$45\frac{1}{4}$	$45\frac{5}{8}$	$45\frac{1}{4}$	$45\frac{5}{8}$	$45\frac{1}{4}$
July, new	44	44	44	44	$43\frac{5}{8}$
Sept., new	$36\frac{3}{4}$	37	$36\frac{1}{2}$	$36\frac{7}{8}$	$36\frac{3}{4}$

MESS PORK

	Open	High	Low	Close	
				April 29	April 28
May	12.97½	13.15	12.97½	13.12½	12.95
July	13.32½	13.50	13.30	13.47½	13.27½
Sept.	13.65	13.80	13.62½	13.80	13.60

LARD

May	8.12½	8.37½	8.10	8.37½	8.07½
July	8.32½	8.57½	8.30	8.57½	8.30
Sept.	8.50	8.77½	8.50	8.77½	8.47½

SHORT RIBS

May	6.82½	7.00	6.82½	7.00	6.82½
July	7.12½	7.27½	7.10	7.27½	7.10
Sept.	7.35	7.52½	7.35	7.52½	7.35

WRITTEN PROBLEMS

The following problems are based on the quotations for Apr. 29, in the preceding table. The grain quotations are per bushel, the pork per barrel, and the lard and ribs per 100 lb. Unless otherwise specified, allow the brokerage charges given in (430) both for buying and selling.

8. What would have been gained by buying 15000 bu. of May wheat at the opening and selling at the closing price?

9. On Apr. 10, I bought through my broker, 20000 bu. Sept. corn at 58½. On Apr. 29, I sold at the highest quotations. What was my net gain after deducting interest on purchase at 7%, brokerage for all transactions, and $65.90 for storage and other charges?

10. On Apr. 10, Mr. B, member of the Chicago Board of Trade, bought on his own account, 1000 brl. July mess pork at $13.65, borrowing the money at 6%. He held the purchase till Apr. 29, then sold at closing price, paying charges amounting to $256.45, and his loan with the interest. How much did he lose on the speculation?

11. Mr. A bought through his broker, 40000 lb. May lard at the opening and sold at the closing price. What were his gains?

12. On Apr. 20, Brown Brothers of the Board of Trade, "sold short" 5000 brl. Sept. mess pork at $13.95. On Apr. 29, they "covered" by buying at the lowest quotation. What did they gain?

13. On Apr. 20, Harper & Sayles, Chicago produce brokers, were "long" on 200,000 lb. May lard at 8.05½. How much had they recovered at the closing price of Apr. 29?

14. On the opening of Apr. 29, Mr. C directed his brokers to sell 50,000 lb. July ribs and invest the proceeds in Sept. corn. Had he gained or lost on this deal at the close of the day's trading and how much? (Disregard the fractional part of a bushel in buying the corn, charging the account for the actual amount of purchase. Do not deduct the brokerage on the ribs before buying the corn, simply charge both brokerages against the account, before computing the gain or loss. Brokers never deduct commission *before* making investments.

15. On Apr. 2, I sent my broker a draft for $5000 as a margin on which to buy 20,000 bu. July oats (old crop) at 43¾. On Apr. 29, I wired him to close out the deal at the closing quotation, and send me a statement of the account. The call money rate being 6%, and other charges $52.45, did I lose or gain, and how much?

SECURITIES AND INVESTMENTS

432. Bonds are interest-bearing obligations issued by governments, municipalities, or corporations. In its essential character, a bond is simply a promissory note issued under seal. Bonds are usually issued, to raise the money required for corporation, or governmental enterprises, as the building of railroads, the construction of public works, the erection of buildings, etc. Bonds are generally issued in a series, each bond for the sum of $1000. The bonds of commercial corporations are usually secured by mortgage of the corporation property. Government bonds rest upon the honor of the country or state that issues them. Commercial bonds are classified according to the character of the security, as "first mortgage," "second mortgage," or "third mortgage," bonds. *Debenture bonds* are secured by collateral instead of by mortgage.

433. Coupon bonds are those to which are attached separate contracts, called *coupons,* for the payment of interest, at stated periods, as quarterly, semi-annually, or annually. When the interest payments are due, the bond-holder cuts from the bond the coupons for these payments, and presents them for redemption at the bank or other place designated in the instrument.

434. Registered bonds have no coupons. The holder's name is recorded on the company's books, and the interest payments are sent to him as they mature.

According to class, and to conditions of issue, payment or security bonds are also variously designated as "convertible," "preferred," "first income," "second income," "consolidated," "refunding," etc.

435. Securities. This is a general term for bonds, stocks, mortgages, and other evidences of debt, which represent to the holder a substantial property interest.

436. Collateral. This name is applied to securities that are deposited with a bank as security for loans. When stock is thus deposited, the owner still retaining title and voting power, the security is said to be *in escro*.

437. Investment. This is the laying out of money or its equivalent in some form of property or securities, that will yield a regular income.

438. Speculation, as distinguished from investment, is the purchase of property or securities, with an expectation of gain through a rise in the market value of the thing purchased.

439. Capital is any form of wealth that is or may be used to produce income or profits.

440. To capitalize a property, or source of income, is to appraise, or determine its value on a basis of current interest rates. Suppose, for instance, that a house yields a net rental of \$300 a year, after making all allowances for taxes, repairs, etc., suppose, also, that the current interest rate is 6%. Then, the capitalized value of this property would be \$300 ÷ .06 or \$5000. Since \$5000 put at interest at 6% would yield an annual income of \$300.

441. Market and capitalized value. Owing to the element of *risk*, the market value of a property or security is not always the same as the capitalized value. For example, suppose that a man owned stock in a concern which, owing to exceptional circumstances, pays *now* an annual dividend of 30% or \$30 per share. Capitalized at 6%, these shares would be valued at \$30 ÷ .06 or \$500. But since there is a probability, or at least a possibility, that such a high rate of profits would not be permanent, the market value of

such shares would probably be much less than the capitalized value of $500. The varying judgments of men, as to the future income-producing power of stocks and other properties, causes constant and sometimes extreme fluctuations in the market value of these properties, notwithstanding that interest rates may remain fairly constant. It is this uncertainty as to the earning power of stocks, which causes much of the speculation in our stock exchanges. The directors of a railroad or a manufacturing company, for instance, may declare a dividend when one is not expected, or fail to declare one that is expected, and the market price of the stock will rise or fall accordingly.

442. The bond market. Bonds, like stocks, are bought and sold in the stock exchanges, and daily quotations of the market prices are published. Bonds are designated by the name of the government, state, municipality, or corporation issuing them, also, by the rate of interest and year of maturity. Thus, "N. Y. City 4½s 1957," means bonds of New York City bearing interest at 4½%, and maturing in 1957. Bond quotations are based on the value of a bond of the denomination of $100. Thus, "U. S. 2s registered 1930—104½," means that a United States government registered bond for $100 bearing interest at 2% and maturing in 1930, is worth in the market $104.50. Bond quotations are usually given with less detail than are stock quotations. Bonds are seldom issued in denominations of less than $1000.

443. The following is a part of the bond quotations as recently published in a Washington, D. C., newspaper:

GOVERNMENT BONDS

New York, April 24.—Closing prices of United States government bonds.

	Bid.	Asked.
2s registered, 1930	103¾	104¼
2s coupon, 1930		104
3s small bonds		100½
4s registered, 1925	119½	120½
4s coupon, 1925		122½
Panama 3s, registered	102¼	103

MISCELLANEOUS BONDS

New York, April 24.—Bond transactions on the Stock Exchange:

Virginia def. 5s, Brown Bros., ctfs	36
New York City 4½s, 1957	108
Japanese firsts series 4½s	85⅜
Adams Express 4s	86½
American Tobacco 6s	105⅝
American Tobacco 4s	72
Atlantic Coast Line 4s	87
Baltimore and Ohio 3½s	91½
Baltimore and Ohio Gold 4s	98
Central of Georgia first income 5s	65⅛
Central of Georgia second income 5s	46

444. It should be noted that the market price of U. S. government bonds is much higher than other classes of bonds bearing the same or even a higher rate of interest. This is accounted for, first, because of the absolute reliability of the security, and second, because of the fact that U. S. bonds are not subject to taxation, either national or local.

WRITTEN PROBLEMS

In the following problems, use the data of the preceding bond quotations, when other data are not given, and when brokerage is required, estimate ⅛% of the par value, buying or selling. In each problem the denominations of bonds will be $1000 each unless otherwise specified.

16. On a certain piece of real estate property, the owner receives an annual ground rent of $1600. What is the value of the property capitalized at 4½%?

SUGGESTION.—The annual income is 4½% of the capitalization.

17. U. S. registered 4s of 1925, pay what rate of interest (correct to ? decimal places) on the asked price of the bond quotation list?

18. Find the value of 25 bonds, $1000 each, of Virginia deferred 5s.

19. Which will yield the greater annual income and how much, $25000 invested in Japanese 4½s, or American Tobacco 4s, allowing the usual charge for brokerage?

NOTE.—Compute on a basis of the investment for even thousands, in each case, disregarding the uninvested remainder.

20. If I invest as much as possible of $10000 in Baltimore & Ohio 3½s (with brokerage), what will be the face value of the bonds purchased? What will be my annual income from the investment? What rate of interest will my investment yield? (Give this and similar interest rates correct to two decimal places.)

21. A gentleman has a permanent income of $175 per month. What is the value of his income, capitalized at 5%?

22. By attending a business school, a young man increased his earning power from $30 a month to $65 a month. What is the value of the increase, capitalized at 4½%?

23. A capitalist sold $25000 in U. S. registered 4s, at 119¾, through a broker, and invested the proceeds as part payment for a flat building, which cost $40000, giving his note, secured by mortgage, for the remainder of the purchase price, bearing 7% interest. He then rented the building for $200 per month, with average annual expenses amounting to $375. Was his annual income increased or diminished and how much?

Find the interest rate (correct to four decimal places, representing per cent and hundredths) that will be realized on each of the following investments, allowing for brokerage:

24. Am. Smelt, 6% pfd. stock at 91½.
25. At. Coast Line 4s at 87.
26. Atchison 4½s pfd, at 77½.
27. Am. Tobacco 6s at 105⅝.
28. Japanese 4½s at 85¼.
29. N. Y. City 4½s at 107⅞.
30. Adams Express 4s at 86⅛.
31. Baltimore and Ohio 3½s at 91⅜.
32. Central of Georgia first income at 65⅜.
33. Baltimore and Ohio Gold 4s at 97⅞.
34. Panama 3s registered at 102⅜.
35. U. S. 4s coupon, 1925, at 122⅜.
36. Chicago, Milwaukee & St. Paul 4s at 98⅜.
37. Denver & Rio Grande 5s at 90⅞.
38. Union Pacific 4s at 100⅜.

CHAPTER XIX

MONEY AND EXCHANGE

UNITED STATES MONEY

445. Money is anything used in accordance with law or custom as a standard of value or a medium of exchange.

Each nation has established its own legal standard of value which in nearly all cases is a monetary unit, consisting of a given weight of standard coinage gold.

NOTE.—Pure gold and silver are both too soft to be used economically as coins, since there would be appreciable loss through wear, or abrasion. To prevent this, a percentage (usually $\frac{1}{10}$) of baser metal called *alloy*, is added to the gold or silver to render the coins more durable. Standard coinage gold consists of nine parts pure gold, and one part of an alloy consisting of silver and copper. Coinage silver consists of nine parts pure silver, and one part of copper. There is a slight variation in the alloys used by different governments.

446. The **standard monetary unit** of the United States is the *gold dollar* which weighs 25.8 Troy grains, and contains 23.22 grains of pure gold.

The money of the United States consists of gold and silver coins, and various classes of paper obligations which circulate as money.

447. Legal tender money is any money which, by law, a debtor may require his creditor to receive in payment unless there is a contrary stipulation in the contract or obligation itself.

The following classes of United States money are a legal tender to the extent specified:

Gold coins for all debts public and private to any amount.

Standard silver dollars, for all debts public and private unless otherwise stipulated in the contract.

Subsidiary silver coins (coins less than one dollar), for all dues public or private in sums not exceeding ten dollars.

Minor coins (copper, bronze, or nickel), for any amount not exceeding twenty-five cents in one payment.

United States notes or *"greenbacks,"* for all debts public and private, except duties on imports (payable to the government) and interest on the public debt (payable by the government).

Treasury notes of *1862* and of *1890* are also legal tender to the same extent as greenbacks, and those of 1890 are receivable for customs and other public dues.

448. Not legal tender. Several classes of money circulate freely but are not legal tender, that is, a creditor may refuse to receive them in payment of a debt, if he is so disposed.

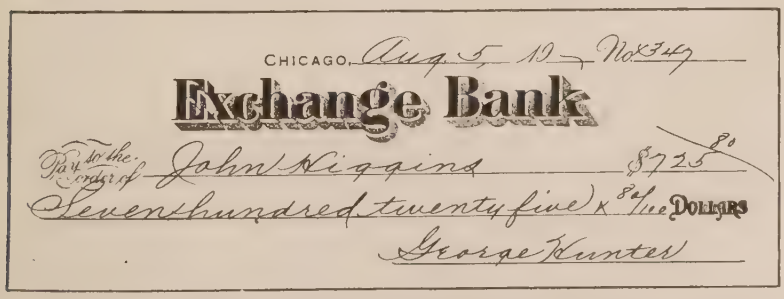
Gold and *silver certificates.* These are not legal tender, but are receivable by the government for all customs, taxes, and all public dues.

National bank notes. These are notes issued, under the authority of law, by national banks. They are receivable by the government for all dues except duties on imports, and are receivable for all dues or demands owed by the government except interest on the public debt and in redemption of national currency.

Foreign coin, especially that of Canada, circulates to some extent in the United States, but it is not legal tender for any purpose.

COMMERCIAL SUBSTITUTES FOR MONEY

But a very small fraction of the business transactions of the world are accompanied by the actual passing of money from the buyer to the seller, or from the debtor to the creditor. To carry on all the commerce of the world by means of the actual transmission of money would involve a vast amount of trouble, expense, risk, and delay. The general process of making purchases or adjusting accounts without the transmission of money itself is called **exchange.** In its simplest form it is illustrated by the use of the ordinary bank check.


CHICAGO, Aug 5 19— No 347

Exchange Bank

Pay to the order of John Higgins $725 80

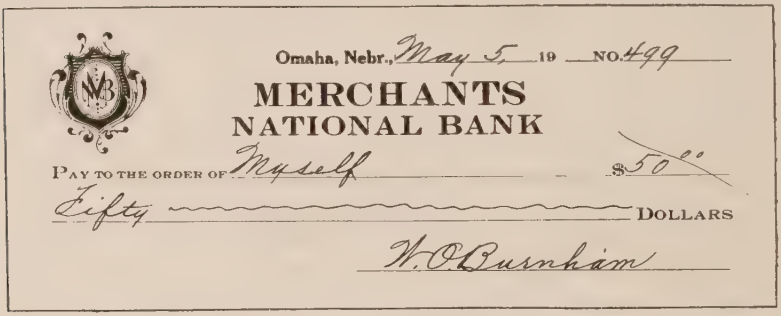
Seven hundred twenty five 80/100 DOLLARS

George Hunter

In the accompanying form, George Hunter, having money deposited in bank, makes payment for purchases or pays his debts

by giving to the seller or to his creditor, orders on the bank called *checks*. Those who receive the checks present them to the bank for payment (or else transfer them by indorsement to others who present them). The nature of a check is the same as that of a draft. (See 381.)

Checks are often drawn payable to the depositor himself, and collected by him, or by anyone to whom he has transferred the check by indorsement. The accompanying form illustrates checks with their indorsements.


Omaha, Nebr., May 5 19 NO. 499

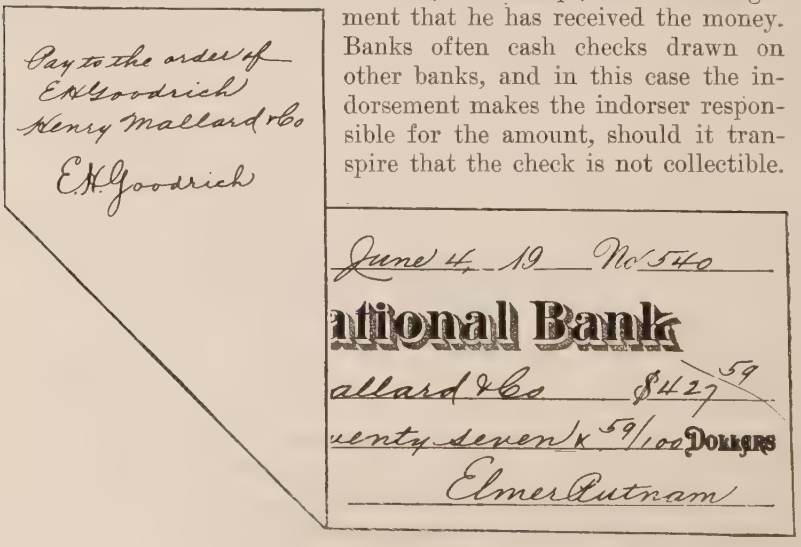
MERCHANTS
NATIONAL BANK

PAY TO THE ORDER OF Myself $50.00

Fifty DOLLARS

W. O. Burnham

449. Indorsement. Before paying the check, the bank requires the payee, or person to whom it is made payable, to write his name across the back of the check, as a receipt, or acknowledgement that he has received the money. Banks often cash checks drawn on other banks, and in this case the indorsement makes the indorser responsible for the amount, should it transpire that the check is not collectible.


Pay to the order of
E. K. Goodrich
Henry Mallard & Co

E. K. Goodrich

June 4, 19 No 540

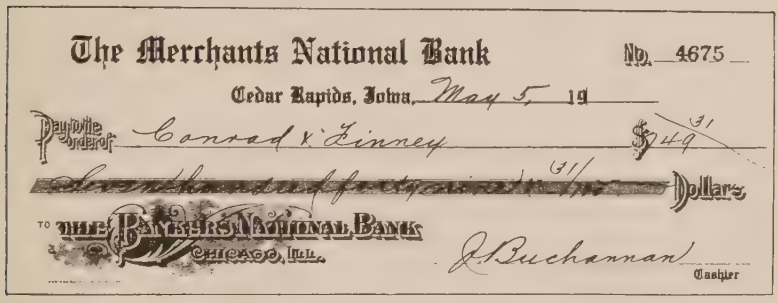
ational Bank

allard & Co $427.59

enty seven 59/100 DOLLARS

Elmer Putnam

450. Bank drafts. These are orders, or checks drawn by one bank upon another, directing the payment of money to the person named in the draft. Most bank drafts are drawn on banks in large cities, like London, Paris, New York, Chicago, and are designated by the name of the city where they are payable, as "Chicago drafts," "New York drafts," etc. They are also spoken of as "Chicago Exchange," "New York Exchange," "Paris Exchange," etc. The following form illustrates "Chicago Exchange" or a draft drawn by a western bank on its "Chicago correspondent" or bank with which it does its exchange business.


The Merchants National Bank No. 4675

Cedar Rapids, Iowa, May 5, 19

Pay to the Order of Conrad & Finney $49.91

Forty-nine and 91/100 Dollars

TO THE BANKERS NATIONAL BANK CHICAGO, ILL.

J. Buchannan Cashier

451. The cost of exchange. Banks usually make a small charge for issuing drafts for ordinary amounts on other banks. Until recently, no charge was made for such drafts, when issued to the bank's customers. For large amounts the cost of exchange varies with the conditions of trade, but *never exceeds the cost of the actual transmission of the money by other means.* For instance, in the fall of the year, or at the time of "the movement of the crops," there is a brisk demand in New York and other eastern cities for Chicago exchange, with which to pay the bills of the eastern merchants who are buying western products. This makes Chicago exchange high or "at a premium" in the eastern cities, while New York exchange is correspondingly low or "at a discount" in Chicago. But in the spring when the western crops have been mostly "moved," and there is a brisk demand for "spring goods" from the east, the movement of money is the other way and the conditions are reversed, New York exchange being at a premium in Chicago, and Chicago exchange being at par, or perhaps at a discount in New York. The same laws govern the price of exchange between the great commercial marts of the different countries.

452. Domestic exchange is the exchange which takes place between cities in the same country. For small amounts, say for \$100 or less, the charge, if there be one, is usually a "flat" or general charge from 5¢ to 10¢ for each draft. For large amounts the charge is a varying amount (from a few cents to a dollar or more) for each \$1000 of the draft, estimated at the nearest even thousands. Thus, if the cost of exchange is quoted at 25¢ per \$1000, the charge for a draft of \$2375 would be 50¢, and for a draft of \$2750 would be 75¢. Sometimes, but not always, the rate is given as a percentage, as $\frac{1}{40}\%$, $\frac{1}{20}\%$, etc.

453. Clearing house. This is an association of banks, formed to facilitate the adjustment of balances among the banks forming the association. There are upwards of one hundred clearing houses in the United States, with a total membership of more than one thousand banks. Banks are constantly drawing drafts against their own deposits in other banks, also they are constantly cashing or receiving the drafts of other banks. It would involve much trouble and expense, for each bank to make daily individual settlements with every other bank whose paper it handles. The clearing house is simply an office where representatives of all the banks in a city can meet daily and adjust their balances without need of making a separate call for this purpose at each bank. A detailed account of the workings of a clearing house would be out of place in a text of this character.

454. Buying and selling exchange. In operations in domestic exchange as carried on by banks, and in counting rooms, the computations are confined to ascertaining *the cost of drafts for given amounts.* Problems involving the accurate conversion of a certain amount of cash into an equivalent amount of exchange, are often given in texts on arithmetic, but rarely or never arise in the counting room, or behind the counters of a bank.

For instance, a man appears at a bank with \$2800 in cash, which he wishes to convert into New York exchange, at $\frac{1}{20}\%$ premium. The bank will issue the draft for \$2800 and charge \$1.50 exchange, or 50¢ a 1000 on \$3000. To be strictly accurate theoretically, the bank should divide the \$2800 by \$1.0005 (the cost of the draft for \$1) to get the face of the draft, but as the difference is so slight, banks never go to the trouble of doing this.

In solving the following problems, the student will compute the exchange on the amount as given correct to the nearest even thousands, whether it be in cash or exchange.

ORAL PROBLEMS

1. At $\frac{1}{10}\%$ for exchange ($1 per $1000) what would be the exchange charge on a draft for $17650?

2. With Chicago exchange $\frac{1}{4}\%$ premium, what is the cost of a Chicago draft for $21370?

3. With New York exchange at a discount of $\frac{1}{2}\%$ in Chicago, what is the cost in Chicago of New York exchange to the amount of $14260?

4. Should a canceled check that has been returned to the drawer be destroyed? Why not?

5. What is meant by "raising" a check or draft? What precautions should be observed in writing checks so that it would be difficult to raise them without detection?

6. Banks usully make a small charge (10¢ to 25¢) for cashing personal checks drawn on banks in other cities. Why?

7. When exchange on New York costs 25¢ a 1000, what is the cost of a draft for $4125?

8. With Chicago exchange $\frac{1}{5}\%$ discount in New York, find the cost of a Chicago draft for $5765.

NOTE.—Here deduct from the amount given the discount on $6000.

9. With St. Louis exchange at a discount of 25¢ a 1000, what will I pay for a St. Louis draft for $23678?

10. With New York exchange at $\frac{1}{4}\%$ premium, what will it cost me to remit $5720 to New York?

11. A Philadelphia grain dealer is going to Chicago with $25000 to invest in cash wheat. He finds that Chicago exchange is selling in Philadelphia at 25¢ a 1000 discount. How much will he gain by investing his money in a draft instead of carrying it with him?

WRITTEN PROBLEMS

1. I have to remit $18659.15 to New York when exchange is $\frac{1}{40}\%$ premium. What must I pay for the draft?

2. A New York broker remits to a Chicago customer a balance of profits due amounting to $13246.50, less cost of exchange at 40¢ a 1000. What is the face of the draft?

SUGGESTION.—The draft will be for the balance less the exchange.

3. When New York exchange is at $\frac{1}{20}\%$ discount in Chicago, a Chicago grain broker remits to his principal in New York $32175.45. What charge should he enter against his principal's account?

NOTE.—The principal is entitled to the benefit of the reduced cost of exchange.

4. When the express rate is 50¢ a 1000 for the transmission of money from a western city to New York, and New York exchange is selling in the same city at $\frac{1}{25}\%$ premium, which will be the cheaper and how much, to send a remittance of $19256 to New York by express or by draft?

5. A Chicago merchant owes a note of $16520 with 60 day's interest at 7%, payable in New York. If he remits by draft at $\frac{1}{20}\%$ premium, what will be the cost of the draft?

6. A St. Louis broker must send to a New York correspondent a balance amounting to $93546.75. Exchange on New York is selling at $\frac{1}{50}\%$ premium, but the broker learns that New York exchange is at a discount of $\frac{1}{40}\%$ in Chicago. Having occasion to go to Chicago, he carries the money with him and buys his draft there. How much does he save by so doing? What does the draft cost in Chicago? In St. Louis?

455. Foreign exchange is that which takes place between cities in different countries. The computation of its cost involves the conversion of the money of one country into terms of another.

456. Par of exchange. This is the intrinsic or coinage value of the monetary unit of one country expressed in the monetary terms of another. The par of exchange is determined by the relative amount of pure gold in the different coins, or standards. For instance, the U. S. gold dollar contains 23.22 grains of pure gold, while the British pound sterling contains 113.0016 grains. The par of exchange between the two, expressed in U. S. money is therefore $113.0016 \div 23.22$ or $4.8665.

457. The following table gives the par of exchange as determined by the director of the U. S. mint, for the standard monetary units of the more important commercial nations:

Country	Standard	Monetary Unit	Commercial Abbreviation	Val. in terms of U. S. gold dollar
Austria-Hungary	Gold..	Krone (kronen)	..Kr.	.203
Belgium	Gold..	Franc	..Fr.	.193
Brazil	Gold..	Milreis	..Milr.	.546
British Possession N. A. except Newf'nd	Gold..	Dollar	..$	1.00
Chile	Gold..	Peso	..P.	.365
China	Silver.	Tael	..Tl.	.561 to .61
Denmark	Gold..	Crown	..Kr.	.268
France	Gold..	Franc	..Fr., F.	.193
German Empire	Gold..	Mark	..M.	.238
Great Britian	Gold..	Pound Sterling.	..£	4.86$\frac{6}{10}$
Greece	Gold..	Drachma	..Dr.	.193
Italy	Gold..	Lira	..Lira	.193
Japan	Gold..	Yen	..Yen	.498
Mexico	Silver.	Peso (dollar)	..P.	.415
Netherlands (Holland)	Gold..	Florin	..Fl., F.	.402
Norway	Gold..	Crown	..Kr.	.268
Philippine Islands	Gold..	Peso	..P.	.500
Portugal	Gold..	Milreis	..Milr.	1.080
Russia	Gold..	Ruble	..R.	.515
Spain	Gold..	Peseta	..Pes.	.193
Sweden	Gold..	Crown	..Kr.	.268
Switzerland	Gold..	Franc	..Fr.	.193

It will be observed that the par of exchange is the same (19.3¢) for the following: The *franc* (Belgium, France, and Switzerland); the *drachma* (Greece); the *lira* (Italy); and the *peseta* (Spain).

The *crown* has the same value (26.8¢) in Denmark, Norway, and Sweden.

A silver coin, approximating the circulation value of the *peso*, is used largely in trade with China, the Philippines, and other islands of the Pacific. It has about the same purchasing value as the Japanese *yen*, and is commonly referred to as the Mexican dollar.

The *guilder* of Holland is the same as the florin.

The German *mark* is also called "reichmark."

In nearly all nations except Great Britain, the monetary standard is for accounting purposes, subdivided into hundredths, as in the United States. Thus, the franc is subdivided into hundredths, called *centimes* (son'teems), the peso, into 100 *centavos* (san tah' vos), the mark into 100 *pfennige* (pfen'nig e) (cents), etc.

458. English or sterling money. In Great Britain and most of the British dependencies, a very different monetary system is used. The following table gives the British monetary units:

TABLE OF ENGLISH MONEY

4 farthings (far.)	= 1 penny
12 pence (d)	= 1 shilling
20 shillings (s)	= 1 pound or soverign (£)
21 shillings	= 1 guinea

The pound (usually called "pound sterling") is the monetary standard and it is equal to $4.8665 in United States gold.

The addition, subtraction, multiplication, or division of sums expressed in *sterling* units, is accomplished as are similar computations in the case of other denominate numbers. (See 265.)

TO REDUCE ENGLISH TO U. S. MONEY

EXAMPLE: Find the value in U. S. money of £12 14s 9d at an exchange rate of $4.87.

OPERATION

£ 12

14s. $= \frac{14}{20} =$ .7

9d $\frac{9}{240} =$ .0375

£ 12.7375

The shillings and pence are changed to decimals of a pound, and the several amounts added. This sum multiplied by the exchange rate, gives the equivalent in U. S. money.

$12.7375 \times \$4.87 = \$60.03+$

TO REDUCE U. S. MONEY TO ENGLISH MONEY

EXAMPLE: Find the value in English money of $125.35, when London exchange is quoted at $4.86.

OPERATION

$125.35 ÷ 4.86 = £ 25.792 +

20

15.84 s

12

10.08 d

4

.32 far.

The U. S. money is first reduced to pounds, then the decimal of a pound is reduced to shillings, the decimal of a shilling to pence, etc., and the several results united.

£ 25 15 s. 10+d.

The computation of equivalents for the money of the other European countries is simply a matter of decimal division, or multiplication, since the several standard units are divided into hundreds.

WRITTEN PROBLEMS

In the following problems, use the par of exchange rates as given on page 279.

7. Find the value in francs of $126.40.

8. Convert fr. 1256.65 (that is, 1256.65 francs) into British monetary denominations.

SUGGESTION.—First convert the sum into American money.

9. An invoice of Sheffield cutlery cost, in English money, £56 15s. What is the cost in U. S. gold?

10. An American arrives in London with $175.75 in Canadian money, which he exchanges for sterling. What sum will he have?

11. An English guinea being equal to 21s, what is the value of 50 guineas in United States money?

12. An American sailor goes ashore in Spain with $25 in U. S. gold, which he has converted in pesetos and centavos. After spending 26.16 pesetos, he converts the remainder into American money and returns to his ship. How much has he then?

13. An Italian laborer saves during his season in America, $142.70. He changes the amount into Italian money, and returns to Italy. If the expenses of his return voyage amount to 46.15 lira, what sum has he on his arrival?

14. An invoice of English woolens cost an American importer £256 14s upon which there is a customs tax of 26% of the cost. If the transportation charges are $13.65, what do the goods cost when laid down in New York?

15. A Norwegian workman returns from America to the old country with $278.40 in U. S. money. What sum will he have in money of his own country?

16. A Russian peasant receives 13 rubles per month for his labor in Russia. If he comes to America and works for $1.25 a day for an average of 275 days in the year, how many more rubles will he receive for his year's work? How many more dollars?

459. Foreign exchange rates. The cost of foreign exchange like that of domestic exchange, depends upon the course of trade and other financial conditions. Daily quotations are wired by large banking establishments in New York to their correspondents in other cities. These quotations give the rates at which exchange will be bought on that day for the leading European commercial centers. The quoted rates are for 100 of the foreign unit in American gold. In the case of the franc, lira, etc., the rate is for the value of $100 in the foreign unit, and for the German reichmark or "mark," the rate quoted is for the value of 400 marks.

460. The following form shows a foreign exchange rate sheet as recently sent out by a New York bank:

NOTE. In the absence of special quotations, the regular or standard exchange rate is assumed.

The National City Bank of New York

Foreign Department

52 Wall Street New York

RATE SHEET NO..... April 29, 19___

Special quotations for FOREIGN EXCHANGE at which settlement will be made by Banks or Bankers for their sale of checks on foreign countries drawn under arrangements made with this bank, for amounts not exceeding $5,000.

Applications for rates for the equivalent of $5,000 or more will elicit prompt telegraphic reply at our expense.

These rates are net to us and remain in force until further notice.

FOREIGN COUNTRY	DRAW DRAFT IN	RATE
Great Britain and Ireland	Pounds Sterling	£ 100= 487.10
Austra-Hungary	Kronen	K 100=
Belgium	Francs	515 7/8
Denmark	Kroner	Kr 100=
France	Francs	515 5/8 less 1/32
Finland	Finmarks	Fm 100=
Germany	Reichsmarks	95 3/8 less 1/32
Holland	Guilders	Fls 100=
Italy	Lire Italiane	515 5/8
Norway	Kroner	Kr 100=
Russia and Poland	Roubles	Ro 100=
Sweeden	Kronor	Kr 100=
Switzerland	Francs	Fr 100=

It will be noted that in rate sheet, page 282, the rate for France and Germany is quoted "less $\frac{1}{32}$." This is a more convenient means of "splitting" a quotation, or giving a very close rate, than would be effected by using a large fraction or an extended decimal. In quoting foreign exchange, the fractions used are halves, fourths, eighths, sixteenths, and thirty-seconds.

WRITTEN PROBLEMS

In the following problems, use the quotations of the foregoing rate sheet.

17. Find the cost of a London draft for £156 9s.

18. Convert $3500 into Italian exchange.

19. Find the cost of a Paris draft for fr. 18000.

NOTE.—Find the cost at $515\frac{19}{32}$.

20. What is the face of a draft on Brussels (Belgium) that can be bought for $5000?

21. Find the cost in New York of a Berlin draft for 4500 reichmarks.

NOTE.—Observe that the quoted rate is for 400 marks, hence, multiply the face of the draft by the net rate as quoted and divide the result by 400.

22. Convert $4850 into London exchange.

23. How much Paris exchange can I buy for $2500?

461. Bills of exchange. This is a term usually applied to bank or other drafts drawn in one country and payable in another. They are often spoken of in financial circles as "bills."

Banker's bills are the foreign bills drawn by one bank upon another.

Commercial bills are those drawn by one merchant upon another.

Documentary bills are those that are secured by an insurance policy, or bill of lading, for goods on the way to the city in which the bill is to be paid. The bill of lading or insurance policy is usually attached to the bill. Domestic drafts are also often similarly secured.

462. Bills of exchange are often made in duplicate and sometimes in triplicate, to save trouble in case there should be delay in transmission. The duplicate or triplicate bills are so drawn that the payment of one of them cancels the others.

The following forms illustrate duplicate bills of exchange:

Exchange for £500 Cedar Rapids, Iowa, June 14, 19

Three days after sight of this FIRST

of Exchange (Second unpaid) pay to the order of

Ourselves

Five hundred Pounds Sterling

Value received and charge the same to account of

To Messrs James Martin & Co. T. M. Sinclair & Co Limited

London.

No 1496 Edward Long Treas.

Exchange for £500 Cedar Rapids, Iowa, June 14, 19

Three days after sight of this SECOND

of Exchange (First unpaid) pay to the order of

Ourselves

Five hundred Pounds Sterling

Value received and charge the same to account of

To Messrs James Martin & Co. T. M. Sinclair & Co Limited

London.

No 1496 Edward Long Treas.

463. Bills of exchange, especially commercial bills for large amounts, are often drawn payable a certain number of days after sight (usually 60 or 90 days). They often pass through many hands before they are paid, and as each indorsement adds to their security, are a favorite form of banking investment.

POSTAL AND OTHER MONEY ORDERS

464. Other forms of exchange are provided by the various money order systems in current use.

465. Postal money orders. These are either *domestic* or *international*. A domestic money order is one that is to be paid in the country in which it is issued.

466. Domestic money orders of the United States are issued for any amount not exceeding $100. They are drawn at one office and made payable upon presentation at another. They may be transferred once only, by indorsement.

467. The following rates are charged for domestic money orders of the United States and their possessions, also for orders payable in Cuba, Canada, and certain other British possessions in North and South America and in the West Indies:

	FOR ORDERS			
From	$ .01	to	$ 2.50	3 cts.
From	2.51	to	5.00	5 cts.
From	5.01	to	10.00	8 cts.
From	10.01	to	20.00	10 cts.
From	20.01	to	30.00	12 cts.
From	30.01	to	40.00	15 cts.
From	40.01	to	50.00	18 cts.
From	50.01	to	60.00	20 cts.
From	60.01	to	75.00	25 cts.
From	75.01	to	100.00	30 cts.

468. International money orders are those payable in Europe and other foreign countries. They are charged for at higher rates than domestic money orders. These rates may be had on application at any money order office.

469. Express money orders. Both domestic and foreign money orders are issued by several of the express companies. They are payable at any of the agencies of the company, and are charged for at the same rates as postal money orders. Express orders are not drawn for amounts greater than $50. There are no restrictions as to the number of indorsements.

470. Bank money orders. To meet the competition in exchange business occasioned by the extensive use of postal and express money orders, the banks have recently established a system of money order issues, but the system is as yet but little used.

471. Telegraphic money orders. This is a system provided by the telegraph companies for the quick transmission of exchange. The order is wired by means of a special code. The charge is usually 1% of the amount transmitted, plus double the cost of a ten-word message to the same place.

472. Traveler's checks and letters of credit. These are other forms of exchange issued for the convenience of travelers, who may

wish to supply themselves with funds at points where personal identification would be inconvenient or difficult. The traveler's signature is placed upon the check or letter, when it is drawn, and thus supplies means of identification to the bank making payment. The letter of credit is so drawn that the holder can draw any part of the credit at any bank in correspondence with the bank of issue. Traveler's checks and letters of credit are issued for both foreign and domestic use. The charge for letters of credit is usually 1% of the face, and for travelers' checks, $\frac{1}{2}\%$.

WRITTEN TEST PROBLEMS

24. Find the total cost of the following postal money orders: \$13.50; \$2.35; \$21.45; \$16.25; \$34.80; \$72.49.

25. What will be the total cost of the following express money orders: \$14.29; \$26.50; \$42.80; \$37.56?

26. What will it cost to wire \$75 from Philadelphia to Boston, the ten-word rate being 25¢?

27. If the ten-word rate from Boston to Galveston is 75¢, what will it cost to wire \$175 from the one city to the other?

28. A firm in Indianapolis wishes to send its traveling representative in Chicago \$150. The ten-word rate being 35¢, how much more will it cost to wire the money than to send it in the form of two postal orders of \$75 each?

29. When New York exchange is at $\frac{1}{40}\%$ premium in Chicago, and Paris exchange is quoted in New York at $516\frac{1}{4}$, what will be the cost in Chicago of a draft on New York sufficient to buy Paris exchange amounting to fr. 15000?

30. When the San Francisco banks are selling New York exchange in amounts less than \$100 at a flat rate of 25¢, how much less will it cost to remit \$742.70 to New York, by means of drafts than by postal money orders?

31. What will be the cost of a draft on Amsterdam for 1275 guilders, when Holland exchange is quoted at $41\frac{1}{4}$?

32. When Japanese exchange is quoted at $50\frac{7}{8}$, what is the face of a Tokio draft that can be purchased for \$875?

33. An official in the Philippine Islands drew his pay amounting to \$629.40 Mexican, and converted the same into American money at the par rate. What sum in American money has he?

34. If the rate for St. Louis exchange is 25¢ a thousand, how much exchange will I have to pay on a St. Louis draft for \$28825?

35. When exchange was $\frac{1}{20}\%$ premium, an agent remitted his principal $12340. What was his total charge?

36. What would have been the total charge in the last problem had exchange been $\frac{1}{20}\%$ discount?

NOTE.—As the principal is charged for the premium, he is entitled to the discount if there be any.

37. At a time when there are more goods being shipped from Chicago to New York than there are from New York to Chicago, what would be the course of exchange between the two cities?

38. If an express company charges 50¢ a thousand for transmitting coin to a given point, what will I save on an amount of $1200 by sending it by draft bought at $\frac{1}{4}\%$ premium, instead of sending it by express?

TAXES AND CUSTOMS DUTIES

473. The expenses of the government, also of the several state, county, and municipal (town and city) governments are paid by means of money collected from the people by the general process called **taxation.**

Taxes are levied in many ways in the various states, and any detailed account of them would be out of place in a text of this kind.

474. Customs or duties, are assessments levied by the government on goods imported from foreign countries.

Ad valorem duties are levied on imported goods at a certain per cent of the foreign valuation.

Specific duties are levied on imported goods according to weight or bulk.

Tare is allowance made for weight of material used in packing and casing.

A *tariff* is a list of the goods subject to import tax, and the rate of charge for each class of goods.

A *custom house* is a government office established for the transaction of business relating to duties and the collection of the same.

A *bonded warehouse* is a warehouse under the control of government customs officials where imported goods are stored until the duty is paid.

Internal revenue is the proceeds of taxes levied by the government on certain goods manufactured in the United States. Such

goods are frequently stored in bonded warehouses until sold or exported.

Drawback is money refunded for duties paid on imported goods which are afterwards exported.

WRITTEN PROBLEMS

39. A imported 1000 bu. wheat bran at 30¢ a bu. What is the duty at 20 per cent?

40. A merchant imported 1200 yd. of Wilton carpets invoiced at £1 per yd. Find the total cost to the merchant, the specific duty being 60¢ per sq. yd., and ad valorem duty 40 per cent, the carpet being 24 inches wide.

NOTE.—1£ = $4.8665.

41. The following were imported from France:

1 gross cotton embroidered gloves at 10 fr. per doz., duty 60 per cent.

6 doz. India rubber gloves at 45 fr. per doz., duty 30 per cent.

1 gross woolen gloves, weight 14 lb., at 20 fr. per doz., duty 44¢ per lb., and 60 per cent.

½ gross gloves, 17 in. long, at 120 fr. per doz., duty $4.50 per doz.

Find the duty and also the total cost to the importer, including freight of $24.60.

NOTE.—1 fr. = $ .193.

42. Brown & Co. import from Germany merchandise invoiced at 3125 marks, a discount of 5 per cent being allowed. The goods weigh 1200 pounds. Transportation charges, $175; specific duty 8¢ per lb., and ad valorem duty 45 per cent. What is the total cost to the importer?

NOTE.—1 mark = $ .2385.

43. A merchant imported from Sheffield:

1 gross pen knives at 2s. per dozen, duty 40 per cent.

1 gross pruning knives at 6s. per doz., duty 5¢ apiece and 10 per cent.

2 gross scissors 8s. per doz., duty 75¢ per doz. and 25 per cent.

1 gross razors 15s. per doz., duty $1.00 per doz. and 15 per cent.

Find the total duty and cost of goods to the merchant.

44. An importer of tobacco receives an importation of cigars weighing 960 lb., invoiced at $1.75 per lb. This price was raised

10 per cent by the custom officials. What duties must be paid by the importer, specific duty $1.50 per lb., and ad valorem 25 per cent.

45. Find the duty on an importation of 250 gal. of liquors invoiced at 5 fr. per gal., 2 per cent being allowed for leakage, duty $1.50 per gal. Also find the total cost to the importer, including charges amounting to $24.60.

46. Find the total duty to the importer:

11240 lb. of pig iron, duty $4 per ton.

8360 lb. pig lead, duty $2\frac{1}{2}¢$ per lb.

2146 lb. of pig zinc, duty $1\frac{1}{2}¢$ per lb.

NOTE.—The long ton of 2240 lb. is used at the Custom House.

47. Invoice of carpets received by Lyons & Co., Chicago, Ill.:

Marks	No.	Articles	Quan.	Price	Duty	Cost
L. C.	61	Aubusson	600 yd.	9s, 4d	60c sq. yd. 40 per cent	.....
L. C.	62	Axminster	752 "	5s, 6d	60c sq. yd. 40 per cent	.....
	62	Brussels	1240 "	8s, 6d	44c sq. yd. 40 per cent	.....
	92	Moquette	450 "	12s,	60c sq. yd. 40 per cent.	.....
L. C.	63	Tapestry Brussels.	620 "	5s,	28c sq. yd. 40 per cent.	.....
L. C.	64	Wilton	800 "	£1,	60c sq. yd. 40 per cent.	.....

Cases $18.00

What is the duty.....

What is the total cost.....

NOTE.—Above carpets 18 inches in width.

48. The goods imported as per description in problem forty-seven were exported to the City of Mexico, less the Wilton carpets. What is the amount of drawback the importer is entitled to, deducting the one per cent withheld by the government?

CHAPTER XX

FINANCIAL SETTLEMENTS AND ADJUSTMENTS

475. Partial payments. It often happens in business that part payments are made from time to time, on a note or other debt, the full settlement being deferred in some cases until several years after the debt was first contracted. In case of a non-interest bearing debt, as, for instance, an ordinary open account, the settlement may consist simply of finding the difference between the total charges and the total payments. But in the case of other obligations, as interest bearing promissory notes, or wholesalers' book accounts in which goods have been sold at definite terms of credit, the matter of interest both on charges and payments must be taken into account.

We shall first consider the settlements of notes.

476. Notes on which partial payments have been made. There are two general methods in use for determining settlements involving partial payments of notes. We shall first consider the method known as "The United States Rule," which is commonly applied to long time notes and which rests upon decisions rendered many years ago by the United States courts. The rule is as follows:

I. Find the amount of the pricipal to the time when the payment, or the sum of two or more payments, equals or exceeds the interest due; subtract the payment or the sum of the payments from the amount.

II. Consider the remainder as a new principal, and proceed in the same manner with other payments to the time of the settlement.

The principle on which this rule is based, is that money paid must first apply on the interest accrued, and the remainder, if any, applies on the principal; and that the interest unpaid cannot be added to the principal so as to compound the interest.

EXAMPLE: Find the amount due Jan. 1, 1910, on the following note:

Find the amount due Jan. 1, 1910, on the following note:

$800.00 *Chicago, June 29, 1908.*

One year after date, I promise to pay to The C. E. Marble Printing Co., or order, Eight Hundred Dollars, value received, with interest at 6 per cent.

N. A. Barrett.

On the back of this note are written the following indorsements: Nov. 1, 1908, $100; Jan. 1, 1909, $50; June 16, 1909, $150; Sept. 21, 1909, $250.

SOLUTION BY U. S. RULE			6% = RATE
'08- 6-29	$800		= Principal and date.
4- 2	16	27	= Interest for 4 mo. 2 da.
	816	27	= Amount of principal.
'08-11- 1	100		= First payment and date.
2	716	27	= New principal.
	7	16	= Interest for 2 mo.
	723	43	= Amount of new principal.
'09- 1- 1	50		= Second payment and date.
5-15	673	43	= New principal.
	18	52	= Interest for 5 mo. 15 da.
	691	95	= Amount of new principal.
'09- 6-16	150		= Third payment and date.
3- 5	541	95	= New principal.
	8	58	= Interest for 3 mo. 5 da.
	550	53	= Amount of new principal.
'09- 9-21	250		= Fourth payment and date.
3-10	300	53	= New principal.
	5	01	= Interest for 3 mo. 10 da.
'10- 1- 1	$305	54	= Balance due Jan. 1, 1910.

PROOF OF TIME	ADD PARTIAL DIFFERENCES
	4 mo. 2 da.
	2 " 0 "
10-1- 1 = Date of settlement.	5 " 15 "
08-6-29 = Date of note.	3 " 5 "
1-6- 2 = Total time.	3 " 10 "
	1 yr. 6 mo. 2 da.

EXPLANATION.—The principal and indorsements should be placed in a vertical column, allowing three spaces between the items for: 1. Principal, or difference brought down. 2. Interest. 3. Amount, before deducting the payment. Subtract each date at the left from the following date, to find the time for which the principal will draw interest before the next payment.

WRITTEN EXERCISE

1.

$2000. *Indianapolis, Ind., July 1, 1907.*

One year after date, we jointly promise to pay William M. Whigam, or order, Two Thousand Dollars for value received, with interest at the rate of 6 per cent until paid.

J. V. Farwell & Co.

On this note were the following indorsements:

March 1, 1908.....$500 Jan. 17, 1909.....$600
Aug. 15, 1908.....$450

Find the balance due July 1, 1910.

2.

$980.00. *Phoenix, Ariz., May 1, 1908.*

One year after date, I promise to pay F. S. McDaniel, or order, Nine Hundred Eighty Dollars for value received, with interest at the rate of 10 per cent per annum.

E. M. Lamson.

The following were the payments:

June 16, 1909.....$250
Sept. 14, 1909.....$200
Nov. 16, 1909.....$175

Find the balance which was paid Jan. 2, 1910.

3.

$3200.00. *Minneapolis, Minn., Aug. 4, 1907.*

On or before three years after date, we promise to pay Curtiss & Chapman, or order, Thirty-two Hundred Dollars, with interest at the rate of 8 per cent per annum.

Ed. F. Luce & Co.

On this note were indorsed the following payments:

Oct. 2, 1907.....$200 Nov. 6, 1908.....$350
March 3, 1908..... 260 March 15, 1910..... 650
Aug. 5, 1908..... 500

Find the amount due on the date of maturity.

4. A note of $6500, at 7½%, dated March 1, 1908, bore the following payments:

Sept. 13, 1908.....$500 Oct. 17, 1909.....$500
April 15, 1909..... 100 June 12, 1910..... 200
Aug. 18, 1909..... 150

What was the balance due Sept. 1, 1910.

5. What was the amount due Aug. 1, 1910, on a note of $1560, dated Burlington, Iowa, November 1, 1907, bearing the following indorsements, interest at 5 per cent?

Dec. 2, 1907	$75	April 14, 1909	$200
Aug. 30, 1908	30	July 2, 1909	20
Oct. 7, 1908	40	Sept. 23, 1909	25
Feb. 16, 1909	50	March 5, 1910	750

6. A note of $2400, at 4%, dated May 14, 1905, bore the following payments:

Oct. 20, 1905	$500	Dec. 15, 1906	$60
Aug. 16, 1906	50	March 3, 1907	50
Nov. 4, 1906	60		

What was due Nov. 1, 1908?

7. A mortgage of $5000, dated June 1, 1907, bore the following indorsements:

Dec. 1, 1907	$150	June 1, 1909	$150
June 1, 1908	150	Dec. 1, 1909	150
Dec. 1, 1908	150	June 1, 1910	150

What sum would pay off the claim Dec. 16, 1910, at 6%?

8. Find the amount due Aug. 1, 1909, on a note of $3500, dated Sept. 4, 1907, and indorsed as follows:

Dec. 19, 1907	$300	April 4, 1908	$ 100
Feb. 14, 1908	450	July 3, 1909	1000

At 4½%.

476. The merchants' rule. This is a method commonly used in banks and other business offices, for notes or open accounts, running less than one year. The rule is as follows:

Find the amount of the pricipal to the date of settlement; find the amount of each payment from the date of payment to the date of maturity; take the sum of these amounts from the amounts of the principal; the remainder is the balance due.

The merchants' rule is also frequently used for obligations running for more than one year. In this case proceed as follows:

Find the balance due at the end of one year, as in the preceding rule, and treat this as a new principal; proceed in the same manner for each entire year that follows, and for the portion of a year that may occur before the time of settlement.

The merchants' rule is generally used in computing interest on open accounts.

EXAMPLE

Find the balance due June 29, 1910, of an account of $800 contracted June 29, 1909, with interest at 6%.

Payments have been made as follows: Nov. 1, 1909, $100; Jan. 1, 1910, $50; June 16, 1910, $15.

SOLUTION BY MERCHANTS' RULE

'10–6–29 Date of settlement.				6%
'09–6–29			$800	
1			48	
'09–11–1	$100		848	
7–28	3	97		
'10–1–1	50			
5–28	1	48		
'10–6–16	15			
13		03	170	48
			$677	52

EXPLANATION.—Arrange the principal and indorsements in the order of their dates in two vertical columns, the right for the principal, and the left for the indorsements, allowing one space to intervene between the items. Subtract each date from the settlement date, to find interest period for each item, insert the interest beneath each item, and from the amount of principal and interest, subtract the sum of the indorsements and interest. The difference is the required balance due.

WRITTEN EXERCISE

Solve the following problems by the merchants' rule:

9. A note for $2500 dated July 1, 1909, had the following indorsements:

Sept. 3, 1909.....$200 March 19, 1910.....$300
Jan. 16, 1910..... 500 June 25, 1910..... 400

What was the amount due July 1, 1910, at 6%?

10. A note for $3250, dated Aug. 16, 1908, was indorsed as follows:

Oct. 14, 1908.....$200 April 1, 1909.....$1300
Jan. 2, 1909..... 375 July 3, 1909..... 750

Find the balance due Aug. 10, 1909, at 6%.

11. Find the amount due Oct. 1, 1910, on a note of $2450, dated Oct. 4, 1908, at 5% interest, which bore the following payments:

Nov. 1, 1908, $1350; Jan. 14, 1909, $100; March 4, 1909, $900.

12. A note for $6000, dated Feb. 1, 1908, for 3 months, without interest, was indorsed as follows:

June 16, 1908.....$1000 Sept. 14, 1909.....$760
Aug. 19, 1908..... 600 Jan. 20, 1910..... 850
March 24, 1909..... 350

Find the amount due March 15, 1910, at 8%.

13. A note dated July 1, 1906, due in 6 months without grace, for $5000, with interest after maturity at 8%, has indorsements of $375 every three months, beginning with April 1, 1907, as the date of the first payment. What amount remained after the last payment, which was made Oct. 4, 1910?

477. The equation of accounts. This is the process of finding the date on which the balance of an account should be paid, in order that neither party will suffer a loss of interest on his credits.

The equitable basis in all such settlements is that the debtor is to be credited for interest on all payments from the time they are made until the time of settlement, while the creditor is allowed interest on all items, from the time they are due until the date of settlement.

478. The average term of credit is the time to elapse from a given date, before the whole of several charges maturing on different dates may be equitably paid in one sum. Suppose, for instance, I owe A $100, due in 1 month and another $100 due in 2 months. Here the average term of credit for the whole amount would be $1\frac{1}{2}$ months, since the use (or interest) of $100 for 1 month and the use of $100 for 2 months are equal to the use of $100 for 3 months, or of $200 for $1\frac{1}{2}$ months.

479. The **equated date** of an account is the date on which it may be equitably paid, as determined by the average term of credit.

To *equate* an account is to find its equated date.

480. The focal date is any date which is taken as a basis or date from which to compute the average term of credit. In equating an account, any date may be taken as a focal date, but the most convenient one is usually some date (either the earliest or the latest) upon which some item of the account falls due.

481. The general principles upon which accounts are equated may be stated as follows:

I. *When a debt is paid before it is due, the debtor loses and the creditor gains the use of the money for the time before the due date.*

II. *When a debt is paid after it is due, the debtor gains and the creditor loses the sum for the time after the due date.*

III. *An account is equitably settled or "equated," when the interest on the items paid or maturing before the due date balances the interest on the items paid or maturing after the due date.*

482. Simple accounts are those consisting entirely of debits or entirely of credits.

ORAL EXERCISE

1. If I work 10 days for a neighbor, how many men of equal ability should he send to work for me 2 days, in return for my services?

2. If I use $5 of A's money for 3 months, how much of my money should A use for one month, in return for the accommodation?

3. If I give a certain amount for the use of $100 for 6 months, for how long a time should I have the use of $200 for the same consideration?

4. The use of $20 for 5 months is equivalent to the use of what sum for 1 month?

5. If I am entitled to 30 days' credit on $200, how long a term of credit should I have on the remainder if I pay $100 at once?

6. A owes B two payments of equal amount, one due in 4 months, and the other due in 6 months. In what time may he pay both in one payment?

7. C owes $200 due in 2 months, and $400 due in 5 months. In what time may he pay both sums in one payment?

The use of

8. $300 for 4 mo. = $600 for what time?
9. $600 for 2 mo. = $300 for what time?
10. $800 for 3 mo. = $1200 for what time?
11. $1200 for 2 yr. = $800 for what time?
12. $800 for 3 yr. = the use of what sum for 2 yr.?
13. $500 for 4 yr. = the use of what sum for 5 yr.?

EXAMPLE

A merchant sold goods as follows: $500 on 7 months credit; $600 on 3 months credit; $700 on 5 months credit; $800 on 2 months credit. Find the equated time for the payment, all these purchases having been made on the same date.

SOLUTION

$500	for	7 mo. =	$3500	for	1 mo.
600	for	3 mo. =	1800	for	1 mo.
700	for	5 mo. =	3500	for	1 mo.
800	for	2 mo. =	1600	for	1 mo.
$2600	for	? mo. =	$10400	for	1 mo.

$10400 ÷ 2600 = 4 times, or 4 mo.

The average term of credit or equated time is 4 mo.

WRITTEN PRACTICE

14. A wholesale dealer sold merchandise as follows: \$600 on 3 mo. credit, \$800 on 3 mo. credit, \$800 on 4 mo. credit, and \$100 on 6 mo. credit. Find the equated time of credit.

15. On Aug. 1, 1910, an agent received a consignment of 100 brl. of flour which he sold as follows: 75 brl. on 30 da., 50 brl. on 60 da., 25 brl. on 10 da., 120 brl. on 30 da., and the remainder on 20 da. What is the equated date?

SUGGESTION.—Aug. 1 + no. of days credit = equated date of payment.

16. An importer sold the following bill of goods Sept. 1, 1910: \$750 on 10 days, \$600 on 20 days, \$300 on 30 days, \$800 on 30 days, \$900 on 60 days, and \$1000 on 90 days. What is the equated date of payment?

17. On Feb. 10, 1910, a wholesale dealer made purchases as follows: \$600 on 10 days, \$750 on 20 days, \$625 on 30 days, \$675 on 30 days, \$725 on 60 days, and \$850 on 90 days. He wishes to give his note for the entire amount. Find the due date of same.

EXAMPLE: In the following, the purchases were made on different dates:

I bought of Marshall Field & Co., the following invoices of goods:

Jan.	6,	1908	an	invoice amounting to	\$960.
Jan.	24,	"	"	" "	840.
Feb.	8,	"	"	" "	600.
Feb.	16,	"	"	" "	520.
Mar.	4,	"	"	" "	320.

What is the average date of purchase?

SOLUTION—PRODUCT METHOD

1908.

Jan. 0=Focal date.

Jan.	6	6	960	5760
"	24	24	840	20160
Feb.	8	39	600	23400
"	16	47	520	24440
Mar.	4	64	320	20480
			3240	94240

94240÷3240=29

Jan. 0+29=Jan. 29, the required date.

SUGGESTION.—Any date may be assumed as the focal date. Count the exact time from the focal date to the date of the item, to find the credit on each item.

By assuming the payment to be made on Jan. 0 (an earlier date than any item) we will lose the time between that date and the date of the item, i. e., 6 days in the first, 24 days in the second, etc.

We find by averaging our payment on the focal date Jan. 0, it would produce a loss of the use of the total, $3240 for 29 days, consequently the focal date is extended 29 days, or Jan. 29, the equated date.

SOLUTION—INTEREST METHOD

1908.

Jan, 0=Focal date.

Jan. 6	6	960		96	=	Interest on	$960	for	6	da.	at	6%
Jan. 24	24	840	3	36	=	" "	840	"	24	"	"	6%
Feb. 8	39	600	3	90	=	" "	600	"	39	"	"	6%
Feb. 16	47	520	4	07	=	" "	520	"	47	"	"	6%
Mar. 4	64	320	3	41	=	" "	320	"	64	"	"	6%
		3240	15	70	=	" "	3240,	required	No.	of	days.	

Int. on $3240 for 1 day at 6%=$ .54

15.70 ÷ .54 = 29

Jan. 0 + 29 days = Jan. 29

SUGGESTION—The interest method is short and practical, at the same time giving the student an excellent review of interest.

WRITTEN PRACTICE

18. J. V. Farwell & Co., sold to H. E. Jones:

Jan. 1 a bill of $375.
Jan. 16 " " " 260.
Feb. 20 " " " 175.
Feb. 28 " " " 360.
Mar. 15 " " " 800.
May 20 " " " 240.

What is the equated date of payment?

NOTE.—When the year date is not given in the problem, supply a common year dating, and give February 28 days.

19. A sold to B the following bills of goods:

Mar. 16 a bill amounting to $840.
Apr. 24 " " " " 375.
May 21 " " " " 250.
May 30 " " " " 264.
June 25 " " " " 365.
July 23 " " " " 480.

What is the equated date of sale?

20. On Apr. 1, a commission merchant received for account and risk of shipper, 600 brl. of flour, which he sold as follows:

On Apr. 10, 150 brl. at $6.25.
Apr. 24, 145 " " 6.50.
May 16, 75 " " 6.60.
May 28, 125 " " 6.75.
June 4, Balance " 5.50.

What is the equated date of sale?

21. Hibbard, Bartlett, Spencer & Co., made the following sales:

Aug.	20,	bill	amounting	to	$ 360	less 10%
"	30,	"	"	"	1260	" 12½%
Sept.	15,	"	"	"	850	" 10 and 10
"	24,	"	"	"	600	" 20% and 25%
Oct.	15,	"	"	"	320	" 33⅓%
"	24,	"	"	"	480	" 10%, 5% and 5%

Find the equated date.

22. The John Spry Lumber Co. made the following sales:

Apr.	4,	12400 ft.	flooring	at	$21	per M
"	16,	3200 "	"	"	18	" M
"	24,	2400 "	"	"	16	" M
May	15,	4870 "	"	"	18.50	" M
June	20,	3260 "	"	"	14.75	" M
"	26,	4500 posts,	round	"	25.00	" C

Find the equated date of payment.

23. L. E. Goodyear bought of Brown & Co.:

Aug. 11, Mdse. amounting to $465.
Aug. 23, " " " 924.
Sept. 10, " " " 840.
Oct. 16, " " " 379.
Oct. 27, " " " 540.

What is the equated date of payment, each item being allowed a credit of 30 days?

SUGGESTION.—Extend each date 30 days, or add 30 days to each term of credit; i. e., Aug. 0 to Aug. 11 is 11 days, plus credit of 30 days is 41 days, term of credit for first item; or, find the equated date, as before, and extend it 30 days, since all have the same term of credit.

Compound accounts are those having both debits and credits.

EXAMPLE: Find the equated date of the following account:

Dr. E. C. LAMSON *Cr.*

19—					19—			
Aug. 4	Mdse.	420	00		Aug. 20	Cash	280	00
" 24	"	360	00		Sept. 5	"	200	00
Sept. 3	"	225	75		Oct. 10	"	50	25

Dr. SOLUTION (INTEREST METHOD) *Cr.*

19—						19—					
Aug.	4	4	420		28	Aug.	20	20	280		93
"	24	24	360		1 44	Sept.	5	36	200		1 20
Sept.	3	34	225	75	1 28	Oct.	10	71	50	25	59
			1005	75	3 00				530	25	2 72
			530	25	2 72						
			475	50	28						

Int. on \$475.50 for 1 da. at 6% = \$ .079.
\$ .28 ÷ .079 = 4 times, or 4 days.
Aug. 0 + 4 da. = Aug. 4, the equated date.

Dr. SOLUTION (PRODUCT METHOD) *Cr.*

19—						19—					
Aug.	4	4	420		1680	Aug.	20	20	280		5600
"	24	24	360		8640	Sept.	5	36	200		7200
Sept.	3	34	225	75	7684	Oct.	10	71	50	25	3550
			1005	75	18004				530	25	16350
			530	25	16350						
			475	50	1654						

1654 ÷ 475.50 = 4 times, or 4 days.
Aug. 0 + 4 da. = Aug. 4, the equated date.

EXAMPLE: Equate the following account:

Dr. H. E. IVES *Cr.*

19—				19—			
Mar. 4	Mdse.	750		Apr. 16	Cash	300	

Dr. SOLUTION (PRODUCT METHOD) *Cr.*

19—					19—				
Mar.	4	4	750	3000	Apr.	16	47	300	14100
			300						3000
			450						11100

11100 ÷ 450 = 25 days.
Mar. 0 — 25 days = Feb. 3, the equated date of payment.

SUGGESTION.—If the balances both fall on the same side, count forward from the focal date, to find the equated date; if they fall on opposite sides, count back from the focal date.

It is evident from the above solution that the payment of the balance could not be made on the equated date, it being earlier than either debit or credit item.

WRITTEN PROBLEMS

Find the equated dates in the following accounts:

24.

Dr.	CARL ZUETTEMEISTER		*Cr.*	
1908			1908	
Apr. 24	Mdse.	$425.00	May 15	Cash $425
May 20	"	375.50	June 10	" 200
June 30	"	269.00		

25.

Dr.	J. S. GRIDLEY		*Cr.*	
1910			1910	
June 20	Mdse.	$425.40	June 24	Cash $225.40
July 5	"	247.65	July 16	" 324.75
Aug. 24	"	364.70	Aug. 1	" 100.00
Sept. 10	"	248.35	" 31	" 200.00

26.

Dr.	J. W. BUTLER PAPER CO.		*Cr.*	
1909			1909	
Aug. 1,	Mdse. 1 mo.	$324.50	Sept. 20,	Cash $300.00
" 30,	" 2 "	247.30	Oct. 14,	Draft 10 da. 100.00
Sept. 20,	" 3 "	463.40	Nov. 1,	Note 2 mo. 250.00
Oct. 10,	" 2 "	247.80	Dec. 4,	Cash 348.50

27.

Dr.	FRED J. MAGERTSTADT		*Cr.*	
1909			1909	
Mar. 3,	Mdse.	$480.75	Mar. 9,	Cash $420.75
Apr. 16,	" 30 da.	325.60	May 10,	" 300.00
May 1,	" 20 "	248.36	June 1,	Note 60 da. 240.00
" 31,	" 60 "	426.42	July 5,	Draft 2 mo. 500.00
June 24,	" 10 "	364.00	Aug. 20,	Check 200.00
Aug. 20,	" 30 "	249.65	Sept. 19,	Acceptance 3 da. 248.90
Sept. 4,	" Net	624.35	Oct. 20,	Note 2 mo. 365.20
Oct. 31,	" 60 da.	387.63	Nov. 24,	Cash 387.63
Dec. 24,	" 60 "	426.38	Dec. 20,	" 150.00
1910			1910	
Jan. 5,	" 30 "	263.21	Jan. 9,	" 200.00
Jan. 31,	" Net	427.95	Feb. 5,	" 300.00

28.

Dr. GEO. BAUMANN *Cr.*

1909			1909		
Aug. 24,	Mdse. 30 da.	$240.75	Aug. 30,	Cash	$200.00
Sept. 10,	" 60 "	360.25	Sept. 16,	Note 1 mo.	250.00
Oct. 20,	" 90 "	365.00	Nov. 30,	Acceptance 30 da.	400.00

29. Find equated date for settling the net proceeds of the following account sales:

Chicago, Apr. 30, 1910.

Account Sales of Flour.

Sold for account of WELLS & WENTWORTH,

Alta, Iowa

1910								
Mar.	3	124 brl. Superfine Flour	@	$6.10 net	....	...		
"	8	48 " do	@	6.20 less 2½%				
"	28	120 " do	@	5.95 30 da.				
Apr.	2	60 " do	@	6.20 30 da.				
"	12	80 " do	@	6.15 10 da.			....	...
		Charges						
Mar.	4	Freight I. C. R. R.			124	50		
"	20	Advertising			18	25		
"	30	Advanced on Flour			450			
Apr.	1	Cooperage			13			
"	14	Storage			25	80		
.....	..	Commission 3%			....	...	....	...
.....	..	Net Proceeds					....	...

SUGGESTION.—Equate the sales, credit side, to find the date of commission; then equate the average date of sale commission date, with charges, debit side, to find the date of net proceeds.

483. The *cash balance* of an account is the amount of cash required to settle the account equitably on any given date.

Since the *equated date* is the date upon which payment of the face balance of the account may be made without loss of interest to either party, it follows that if payment is made on a later date, interest should be added, and that if payment is made on an earlier date, interest (discount) should be deducted.

WRITTEN PROBLEMS

30.

Find the cash balance of the following account on Sept 1, 1910, at 6%:

Dr. A. B. JONES *Cr.*

1910			1910		
Mar. 8	Mdse.	160 75	Apr. 2	Cash	300 00
" 24	"	375 46	May 5	Sight Draft	200 00

SUGGESTION.—Find the equated date as before, and count the exact time between the equated date and settlement date. If the settlement date is later, add the interest; if earlier, subtract the interest, called discount.

Or, since the settlement date is given, count the exact time from each date to the settlement date as the credit for that item, i. e., from Mar. 8 to Sept. 1, 177 da. The balance of the debit and credit interest to be added or subtracted will produce the amount due at the settlement date.

31.

Dr. C. E. TUTTLE *Cr.*

1910				1910			
Apr. 16	Mdse.	562	75	May 4	Cash	372	24
May 24	"	374	25	June 10	"	276	83
June 5	"	247	38	July 14	"	427	38
July 2	"	426	83	Aug. 5	"	327	91

Find the cash balance settled Aug. 20, 1910, at 6%.

32.

Dr. EMIL DAVIS *Cr.*

1909			1909		
May 16, Mdse.	60 da.	$324.60	June 4, Cash		$400.00
" 24, "	30 "	147.83	" 23, Note	30 da.	250.00
June 5, "	45 "	465.00	Aug. 1, Acceptance	1 mo.	147.83
July 6, "	60 "	328.25	Sept. 5, Cash		600.00
Aug. 4, "		476.80			

Find the cash balance settled Oct. 1, 1909, at 6%.

33.

Dr. HINZ & HOEKENDORF *Cr.*

1910			1910		
June 4, Mdse.	4 mo.	$842 55	July 1, Note	3 mo.	$842.55
July 1, "	3 "	372.60	July 31, Acceptance	2 mo.	372.60
" 29, "	2 "	427.36	Aug. 20, Cash		100.00
Aug. 4, "	Net	328.63	Sept. 1, Mdse. returned		426.45
" 31, "	2 mo.	426.45	Oct. 20, Check		250.00
Sept. 5, "	60 da.	375.28			
" 30, "	30 "	167.83			

Find the cash balance paid Nov. 1, 1910, at 8%.

34.

Dr. TRIPP BROS. & CO. *Cr.*

1909			1909		
Oct. 10, Mdse.	30 da.	$175.80	Nov. 16, Cash		$175.80
" 29, "	40 "	362.40	Dec. 12, Draft	10 da.	200.00
Nov. 5, "	10 "	360.00			
Dec. 6, "	$240. Less 10 & 10%	....			
1910			1910		
Jan. 5, "	$475.60 Less 20 & 10%	....	Jan. 19, Cash		450.00
			" 24, "		250.00
Jan. 10, "	$472.50 Less 5 & 2%	....			

Find the cash balance settled Feb. 1, 1910, at 5%.

SAVINGS BANKS

484. A **savings bank** is a bank of deposit for the accumulation of small savings belonging to the industrious and thrifty. Interest is allowed on deposits and is credited as a deposit at the end of the interest term. The depositor is supplied with a book in which he is credited with amounts deposited, and in which he is debited as he withdraws.

485. The **interest term** is the term that a certain sum or balance must be in the bank to draw interest.

Checks cannot be drawn against an ordinary savings bank account. The depositor must take his bank book with him and have the amount entered, that he withdraws. He generally signs a receipt for the money taken out. The interest terms are not uniform; with some the term is one year, with some six months, and with others it is three months.

The regular time for crediting interest is: If semi-annual, Jan. 1, and July 1; if quarterly, Jan. 1, April 1, July 1, and Oct. 1, of each year.

Banks do not usually allow interest on fractional parts of a dollar. There is no general uniformity in the method of finding the interest.

We will explain two methods of estimating the interest on deposits.

In the first method, the interest term will be six months, but interest will be allowed on the smallest balance for each quarter.

In the second method, the interest term will be six months, but interest will be allowed on the smallest average monthly balance.

Interest is not counted on fractional parts of a dollar.

EXAMPLE

What is the balance of the following savings bank account on Jan. 1, 1910, interest being allowed at 6% ? Balance in bank Jan. 1, 1909, $150. Deposits: Mar. 15, $75; Apr. 1, $125; July 5, $160; Sept. 14, $96; Nov. 12, $84; Dec. 9, $50.

SOLUTION BY FIRST METHOD

Date 1909	Deposits	Withdrawals	Balances	Smallest Qr. Bal.	Interest on each Qr. at 6% = 1½%
Jan. 1			$150		$150 for 3 mo.=$2.25
Mar. 15	$ 75		225	150	350 for 3 mo.= 5.25
Apr. 1	125		350	350	Total Int. 6 mo. $7.50
July 1	Cr. Int. 7 50		357 50		
July 5	160		517 50	357 50	$357 for 3 mo.=$ 5.36
Sept. 14	96		613 50		613 for 3 mo.= 9.19
Nov. 22	84		697 50		Total Int. 6 mo. $14.55
Dec. 9	50		747 50	613 50	
1910 Jan 1	Cr. Int. 14 55		762 05	Bal. due	

SOLUTION BY SECOND METHOD

Date 1909	Deposits	Withdrawals	Balances	Smallest Bal. Each Month	Average Monthly Balance
Jan. 1			$150	Jan. $150	
Mar. 15	$ 75		225	Feb. 150	$1500 total÷6=$250, average balance. Int. on $250 for 6 mo. at 6%=$7.50.
Apr. 1	125		350	Mar. 150	
				Apr. 350	
				May 350	
				June 350	
July 1	Cr. Int. 7 50		357 50	July 357 50	
				Aug. 517 50	$3317÷6=$552, average balance. Int. on $552 for 6 mo. at 6%=$16.56.
July 5	160		517 50	Sept. 517 50	
Sept. 14	96		613 50	Oct. 613 50	
Nov. 12	84		697 50	Nov. 613 50	
Dec. 9	50		747 50	Dec. 697 50	
1910					
Jan. 1	Cr. Int. 16 56		764 06	Bal. due	

WRITTEN PRACTICE

Solve the following problems by each of the above methods:

35. On January 1, 1909, my bank balance was $350, interest allowed at 6%. During the year I made the following deposits:

Jan. 14, $125; Feb. 2, $74.50; Mar. 24, $92; Apr. 16, $75; Apr. 24, $12; May 15, $64; June 6, $100; July 7, $25; Aug 23, $124.75; Sept. 23, $86; Oct. 12, $45; Nov. 30, $150; Dec. 15, $90.

What was my balance Jan. 1, 1910?

36. A depositor's balance was $420 on Jan. 1, 1909. He made the following deposits during the year:

Feb. 16, $350; Apr. 4, $450; June 5, $300; June 27, $280; Aug. 3, $800; Oct. 17, $375; Dec. 16, $260.

During the year of 1910, no deposits were made. What was his balance Jan. 1, 1911, interest being allowed at 4%?

37. On January 1, 1908, a merchant had a balance in bank of $600, and during the year he makes the following deposits and withdrawals:

Deposits: Jan. 4, $200; Jan. 21, $250; Mar. 3, $275; Apr. 16, $275; May 5, $196; June 5, $300.

Withdrawals: Jan. 5, $125; Jan. 21, $130; Feb. 16, $75; Mar. 3, $140; Apr. 3, $150; June 4, $50; June 16, $75.

What was the balance due July 1, 1909, at 6%?

38. Find the balance due upon the following savings bank account Jan. 1, 1911, interest at 3%:

Balance in bank Jan. 1, 1910, $396.75.

Deposits: Jan. 4, $125; Jan. 6, $320; Feb. 4, $243.75; Feb. 16, $125; Mar. 5, $327.60; Mar. 16, $142.64; Mar. 30, $325.72; Apr. 15, $243.70; May 25, $327.80; July 5, $378.20.

39. A stenographer opened a savings bank account on Jan. 1, 1909, depositing $10, and deposited $10 on the first day of each succeeding month during the year. The bank allows 4% interest. What will be the amount on deposit after the interest is credited Jan. 1, 1910?

40. On Jan. 1, 1910, a man's savings bank balance was $225, and during the year he made the following deposits and withdrawals:

Deposits: Jan. 20, $75; Feb. 20, $80; March 21, $70; April 20, $60; May 19, $75; June 21, $50; July 19, $70; Oct. 19, $80; Nov. 20, $50; Dec. 20, $75;

Withdrawals: Feb. 5, $25; Aug. 10, $100; Sept. 15, $75.

What was the balance due Jan. 1, 1911, at 4%?

41. A merchant checks his surplus cash each month from his regular bank, and deposits the same in a savings bank that allows 4% interest. He deposits as follows:

Jan. 1, 1909, $1200; Jan. 31, $900; Feb. 28, $500; Mar. 31, $2500; Apr. 30, $1500; May 31, $1800; June 30, $400; Sept. 30, $1600; Oct. 31, $3000; Nov. 30, $1400.

Meeting with an unexpected loss on Dec. 24, he withdraws $10000. What is his balance Jan. 1, 1910? How much interest did he lose by not being able to let the balance remain in bank until Jan. 1, 1910?

CHAPTER XXI

PARTITIVE SETTLEMENTS AND ADJUSTMENTS

486. In many kinds of business, as in partnerships, bankruptcy proceedings, insurance adjustments, etc., gains, losses, allotments, etc., are distributed, or apportioned on a *pro rata* basis, that is, in proportion to the amounts of the several interests involved.

487. Partition, or *partitive proportion,* is the process of dividing a number into parts proportional to two or more given numbers.

EXAMPLE: Divide \$810.54 into parts proportional to 7, 9, and 11.

SOLUTION

7	\$810.54 ÷ 27 =	\$ 30.02
9	30.02 × 7 =	210.14, 1st part
11	30.02 × 9 =	270.18, 2nd part
27	30.02 × 11 =	330.22, 3rd part
	Proof	\$810.54, whole

EXPLANATION. — The sum of the given proportional numbers (27) equals the number of "parts" in the partition. The whole sum divided by the number of parts equals the value of one part (\$30.02). The several parts required are formed by multiplying the single or basal part by each of the several proportional numbers given.

The solution may often be shortened by taking such fractional parts of the number to be apportioned, as the several proportional numbers are of their sum. Thus, to divide a quantity into parts proportional to 2, 4, and 6, the required parts, are respectively $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{2}$, of the whole, since 2, 4, and 6 are respectively $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{2}$ of 12.

In making partitive apportionments, based on sums of money involving dollars and cents, express the amounts to the nearest even dollar. That is, consider \$215.65 as \$216, and \$728.45 as \$728.

PARTNERSHIP

488. A **partnership** is a relation formed by two or more persons, who agree to combine their skill, service, or capital in some lawful business enterprise. The *partners* are the persons entering into such an agreement.

489. In the absence of an agreement, the law directs that the gains or losses of a partnership shall be shared equally, but the agreement of co-partnership usually provides that the losses or gains are to be shared in proportion to the investment, or in some other specified proportion.

NOTE.—In the following 20 oral problems, it is assumed, unless otherwise specified in the problem, that the losses and gains are shared in proportion to the capital and time of investment. In solving the written problems that follow, apply the principle stated in 489.

ORAL EXERCISE

1. If A and B invest the same sum, what part of the gain should each share?

2. If B and C invest the same sum, what part of the loss should each share?

3. If C invests twice as much as D, what part of the gain should each share?

4. If D invests $\frac{1}{2}$ as much as E, what part of the gain should each share?

5. If E invests 3 times as much as F, what part of the gains and losses should each share?

6. If F invests \$2 as often as G invests \$3, what part of the gains and losses should each share?

7. If G invests \$3 as often as H invests \$4, what part of the gains and losses should each share?

8. If H invests \$5 as often as I invests \$4, what part of the gains and losses should each share?

9. If I invests for 1 year and J invests the same amount for 2 years, what part of the gains and losses should each share?

10. If J invests \$2000 for 1 year and K invests \$3000 for 2 years, what part of the gains and losses should each share?

11. If K invests \$300 for 4 months and L invests \$400 for 3 months, what part of the gains and losses should each share?

12. If L invests \$600 for 5 months and \$1000 for 4 months, and M invests \$5000 for 1 month, what part of the gains and losses should each share?

13. Two men investing equally in business gained \$3000 during the year. What part of the gain belongs to each man. How much belongs to each?

14. During a year in business, three partners, having equal investments, gained $7500. What part of the gain belongs to each?

15. A and B form a partnership, each investing $3000; at the end of the year they have gained $4500. To what sum is each one entitled?

16. C and D engage in business for 1 year, each investing $500. At the end of the year their books show the following gains: On merchandise $3000, on real estate $4000, and on shipments $1500. What is each one's share of the gains?

17. E and F engage in business, each investing $4750; at the end of the year their gains and losses are as follows: On merchandise they gained $4100; on real estate they gained $2200; on shipments they gained $1200; and it cost them $2000 to pay salaries and expenses. What was each partner's share of the net gain?

18. Two men, G and H, engage in business for two years, and invest as follows: G invests cash $3000, H invests merchandise invoiced at $2000. At the end of the time, they find a gain of $5000 in merchandise account and a gain of $2500 in real estate. What is each partner's share of the gain, the same being divided in proportion to his investment?

19. I and J engage in business for 1 year, agreeing to share gains and losses in proportion to amount invested by each. I invests $2000 for 3 months and $3000 for 9 months; J invests $4000 for 2 months and $2000 for 9 months. They gained $4200 on merchandise and $800 on grain; the losses being $600. What is each one's share of the net gain?

20. Three men hire a pasture for $45. The first puts in 24 cows for 5 weeks, the second puts in 20 cows for 4 weeks, and the third puts in 100 sheep for 2 weeks. How much should each pay, if the pasturage of one cow is equal to that of two sheep?

WRITTEN PRACTICE

1. A and B commenced business with a cash capital of $12000, and at the end of one year they sold out for $14560. What was the gain?

2. B and C commenced business with a capital of $14500, and at the end of two years had but $12300. What was the average loss per year?

3. C and D commenced business with the following resources and liabilities: Resources:—cash $6000, real estate $4900, bills receivable $2300. Liabilities—bills payable $6860. At the end of the year they sold out for $11500, how much did they gain? What is each one's share of the gain?

4. D and E commenced business with an indebtedness of $5600. At the end of one year they have a net worth of $4860. How much have they gained?

5. E and F commenced business with cash $1200, merchandise $3200, bills receivable $1465; they owed on personal accounts $3600, and on bills payable $7500. At the end of two years they have cash $4600, merchandise $3560, real estate $4760, and bills receivable $6400. What is each one's share of the gain?

6. H and I commenced business Jan. 1, 1909. H invested cash $2000, real estate $4000. I invested merchandise $4500, bills receivable $3000. Jan. 1, 1910, their resources were as follows: Cash $6000, merchandise $7500, real estate $4000, bills receivable $4780. At the same date, there were owing on bills payable $3750, personal accounts $2140. What was the gain or loss for the year? What was each partner's worth at closing?

7. J and K commenced the general merchandise business Jan. 1, 1908. J invested $\frac{1}{3}$ and K $\frac{2}{3}$ of the capital. Jan. 1, 1909, their resources were as follows: Merchandise $6000, cash $3200, real estate $2150, bills receivable $3124, interest receivable $147.50. The loss during the year was $878.50. How much did each one invest?

8. K and L formed a partnership and invested respectively $7000 and $8000. The profits for the year were, on merchandise $3600, on real estate $6900. What was each one's share of the gain, it being divided in proportion to their investment?

SUGGESTION.—K is entitled to $\frac{7}{15}$ and L $\frac{8}{15}$ of the gain.

9. L, M, and N invested $4500, $5500, and $6500 respectively in business, agreeing to share gains and losses in proportion to their investments. The net gain for the year was $15510. What was each one's share?

10. M, N, and P contract to erect an office building for $120,000. M furnished material worth $27,500, N furnished material worth $30,000, and P paid for labor and supervision $45,000. The gain being divided proportionately, how much did each one receive?

11. N, P, and Q engage in business investing respectively $12000, $15000, and $13000. At the end of a year they have the following resources: Cash $21000, real estate $24000, bills receivable $14000, and personal accounts due them $15360. At the same date they owe on bills payable $14750, and for interest on same $450. Q is to receive a salary of $2400 for managing the business. What is each one's share of the gain, divided according to the investment?

12. P, Q, and R conduct a general merchandise business with a joint capital of $26000. How much did they each invest if, at the end of one year, their gains were respectively $3000, $4000, and $6000, distributed according to investment?

EXAMPLE

Q and R formed a co-partnership for the purpose of conducting a renting and general real estate business, investing as follows: Q invested Jan. 1, 1910, $5000, and on June 1, 1910, $3000. R invested Jan. 1, 1910, $7500, and on Aug. 1, 1910, $6000. Q withdrew on July 1, 1910, $3000, and Sept. 1, 1910, R withdrew $2500. How much is each one's share of a gain of $46710, the business being conducted for one year, and profits being divided in proportion to the amount invested by each partner? What is each partner's present worth?

SOLUTION

Q's investments.

Jan. 1, $5000 for 12 mo. = $60000 for 1 mo.
June 1, $3000 for 7 mo. = 21000 for 1 mo.
$81000 for 1 mo.

Q's withdrawal.

July 1, $3000 for 6 mo. = $18000 for 1 mo.
Q's net investment = $63000 for 1 mo.

R's investments.

Jan. 1, $7500 for 12 mo. = $90000 for 1 mo.
Aug. 1, $6000 for 5 mo. = 30000 for 1 mo.
$120000 for 1 mo.

R's withdrawal.

Sept. 1, $2500 for 4 mo. = $10000 for 1 mo.
R's net investment = $110000 for 1 mo.

$\$63000 + \$110000 = \$173000$, total investment for 1 mo.

$\frac{63000}{173000} = \frac{63}{173}$ Q's share of the gain.

$\frac{110000}{173000} = \frac{110}{173}$ R's share of the net gain

$\frac{63}{173}$ of $\$46710 = \$17010$, Q's gain.

$\frac{110}{173}$ of $\$46710 = \$29700$, R's gain.

$\$5000 + 3000 - 3000 = \$5000$, Q's investment.

$\$7500 + 6000 - 2500 = \$11000$, R's investment.

$\$5000 + 17010 = \$22010$, Q's present worth.

$\$11000 + 29700 = \$40700$, R's present worth.

PROMISCUOUS WRITTEN PROBLEMS

13. R and S are partners in the boot and shoe business. R invests \$4000 for 3 months, \$7500 for 6 months, and \$2000 for $1\frac{1}{2}$ months. S invests \$5000 for 10 months, and \$1000 for 2 months. The gains are to be shared in proportion to the average investment of each. The gains for the year are \$8500. Find the gain and present worth of each partner.

14. Three men, A, B, and C, form a partnership for the purpose of buying and selling coal. A invests \$2000, B invests \$3000, C invests \$2500. Their profits amount to \$3000. What is each partner's share of the gain distributed according to investment?

15. D, E, and F form a partnership. D invests \$9000, E invests \$6000, F invests \$500. They purchase a stock of lumber for \$18000, and lose $\frac{1}{3}$ of it by fire and $\frac{1}{6}$ of the remainder in transit. They sell the remainder of the lumber at a profit of 125 per cent of its cost. Their expenses are \$750. What is the net gain of each partner, divided according to investment?

16. G and H engage as partners in buying and shipping stock, sharing gains and losses equally. G pays for sheep \$8750, and freight \$750, shipping them to H. H sells the sheep for \$11250, sending G \$5000 of the cash received. H pays \$6325 for cattle which he ships to G, paying for freight and other expenses of shipping \$825. G sells the cattle at an advance of $33\frac{1}{3}$ per cent of their entire cost, receiving the money. How shall the partners settle with each other?

17. H and I invest in business, each $10000. At the end of three months H draws out $1000, and I invests $1000. At the end of 9 months H invests $3000, and I invests $1000. They gain during the year $3600. What is each partner's present worth, after dividing the gain according to their average investments?

18. L and M are partners, sharing equally in gains and losses. L invests $3000 Jan. 1, 1909. M invests $2500 Feb. 1, 1909. Each partner is to receive 6% interest on his capital invested and to pay interest at the same rate on all sums withdrawn. L draws out $500 and M invests $1500 June 1; L invests $1000 and M draws out $500 Sept. 1. On the first of January, 1910, their books show the following. Resources: Ledger accounts $6000, 5% of which are uncollectible; notes receivable $3300; cash $2200; merchandise unsold $6800. Liabilities: Ledger accounts $1500; notes payable $2500. What is each partner's interest, after the division of the gain or loss?

19. N and O enter into partnership to build a house. N works 10 days and furnishes two hands who each work 10 days; O works 20 days, and also spends 3 days in buying material for the building. A day's work of each is counted of equal value. At the completion of the job they receive $1200 in payment. They owe for lumber $550; stone, lime, and sundries $220; mason work $125. O has paid for hardware used $27.50. What is due each partner from the proceeds?

20. P, Q, and R rent a pasture for three months, paying $225, of which each one advances $75, with the understanding that sheep are to be estimated at one-half the price of cattle and horses. P puts in 25 head of cattle, Q puts in 20 horses, and R puts in 100 sheep. After one month, P puts in 15 head of cattle, and R removes 50 sheep. After two months, P removes 10 head of cattle, and Q removes 5 horses. How shall they settle at the close of the three months?

21. S, T, U, and V engage in partnership in the milling business. S and T run the mill, devoting their entire time to the business, instead of investing capital. U is a provision merchant, and sells the breadstuffs and feed, ground at the mill, for which the firm allows him a commission of 10%. U and V invest $10000 each. At the end of one year, S and T have each drawn out $800,

and V has loaned the firm $3000, which has been at interest for three months at 6%. U has sold breadstuffs for the firm to the amount of $12500. They sell the mill and stock on hand for $24000; they have uncollected accounts amounting to $2450, which V agrees to take at 80% of their face. The firm have on hand, cash $2750, notes receivable $1550. They owe farmers on account for grain $2125. What is each partner's interest, after the gain or loss is divided?

22. Five partners in business invest equal amounts and share gains and losses equally. At the end of two years they close out the business, collecting and paying all notes and accounts; each partner being debited for all amounts collected by him, and credited for all amounts paid by him. The following are the results of the partnership accounts, including their investments:

A—Dr. $5296.26, Cr. $9625.48. B—Dr. $7623.84, Cr. $4287.35. C—Dr. $4823.15, Cr. $3643.28. D—Dr. $1985.34, Cr. $4949.25. E—Dr. $10271.41, Cr. $7494.64.

How shall they settle with each other?

23. G and H begin business, agreeing to share losses and gains, in proportion to amount invested by each. G invests $7000 and H invests $8000. They buy a section of land for $25000, paying cash $9000 and giving a mortgage to secure the balance due in one year with interest at 6%. They immediately subdivide the land into lots, paying for surveying, platting, and grading $2500. One-fifth of the lots are sold at a profit of 20%, one-half of the remainder at a profit of 75%, and the balance at a profit of 33⅓%. What is each one's share of the gain, after paying the mortgage and interest at the end of one year?

24. A firm made the following investment at the beginning of the year: Assets: Cash on hand and in bank $3900, merchandise $4000, bills receivable $3500, accrued interest on bills receivable $240. Liabilities: Accounts payable $3125, bills payable $1750. At the close of the year the resources were: Cash $2780, merchandise $4560, bills receivable $4120, accrued interest $175, personal accounts receivable $4125. Liabilities: Personal accounts payable $3740, bills payable $850, accrued interest on bills payable $110. The firm consists of A, B, and C. They share gains as

follows: A 42 per cent, B 38 per cent, and C 20 per cent. Find the amount of gain due each partner.

25. The assets and liabilities of a firm at closing are as follows: Cash $7500, bills receivable $3250, personal accounts receivable $5120, merchandise $12000, insurance unexpired $75, accrued interest on bills receivable $300. Personal accounts payable $2100, bills payable $3000. The firm decides to write off 5 per cent of personal accounts receivable and bills receivable. The wages unpaid amount to $450. The firm consists of A and B, A having invested $8500, and B having invested $7500. Find each one's share of the gain or loss according to his investment.

26. A, B, and C form a partnership for one year, beginning January 1, 1910. Each one is to receive interest on his investment at the rate of 4 per cent per annum and to be charged interest at the rate of 6 per cent per annum on all withdrawals. On January 1, A invests $6000, B invests $7000, and C invests $7500. On March 1, A invests $1000, B withdraws $1200, and C invests $500. C withdraws $500 at the end of each three months. On June 1, A withdraws $1200, B withdraws $900, and C withdraws $400. On Dec. 31, the loss and gain account shows the following: Merchandise a gain of $10540, interest a gain not including partners' accounts of $1256.40, expense a loss of $3160. The firm decides to reserve $2000 from the profits for a special fund, and decides to write off 10 per cent of the bills receivable account, amounting to $4000. The net profit is to be divided as follows: A 45 per cent, B 30 per cent, and C 25 per cent. What is each one's net profit and net worth at closing?

27. D, E, and F are partners sharing gains and losses according to average investment of each. They are to be charged and credited with interest on their accounts at the rate of 5 per cent per annum. Their accounts are as follows:

D

1909			1909		
Feb. 15,	withdrew	$ 600.00	Jan. 1,	invested	$12000.00
Mar. 15,	"	700.00	Mar. 15,	"	3000.00
June 20,	"	350.00	April 24,	"	2560.00
Aug. 16,	"	1000.00	July 21,	"	840.00
Sept. 18,	"	800.00			
Dec. 20,	"	350.00			

E

1909			1909		
April 1,	withdrew	$ 500.00	Jan. 1,	invested	$10000.00
May 15,	"	300.00	May 28,	"	2000.00
July 25,	"	600.00	July 5,	"	1000.00
Oct. 20,	"	350.00	Sept. 24,	"	2500.00
Nov. 21,	"	124.35			

F

1909			1909		
Feb. 1,	withdrew	$ 200.00	Feb. 1,	invested	$ 2000.00
Mar. 1,	"	200.00	Aug. 15,	"	1000.00
April 1,	"	200.00			
May 1,	"	200.00			
June 1,	"	200.00			
July 1,	"	200.00			
Aug. 1,	"	200.00			
Sept. 1,	"	200.00			
Oct. 1,	"	200.00			
Nov. 1,	"	200.00			
Dec. 1,	"	200.00			

ASSETS AND LIABILITIES EXCLUSIVE OF PARTNERS' ACCOUNTS

Cash	$ 4560.00	Bills Payable	$2100.00
Bills Receivable	7420.00	Per. Acct. Payable	5700.00
Accounts Receivable	6325.00	Accrued Interest	250.00
Real Estate	12000.00		
Bonds	7500.00		
Bond Premium	400.00		
Merchandise	26000.00		
Suspense	840.00		

Prepare special interest account to December 31, the close of the year, and enter interest in each partner's account. Determine each one's net loss or gain and carry the same to his account. Balance each partner's account.

28. X, Y, and Z form a partnership for one year. Losses and gains to be shared as follows: 40 per cent, 37 per cent, and 23 per cent. X is to have a salary of $3000 for managing the business, which sum is to be credited at the end of each six months. Interest at $4\frac{1}{2}$ per cent is to be allowed on average net credit, and charged

at 7½ per cent on average net withdrawal. The following is a trial balance at the end of the year:

X, Net Capital		$ 12,000.00
Y, Net Capital		14,000.00
Z, Net Capital		15,500.00
Mdse.	$120,000.00	144,650.00
Real Estate	10,000.00	
Shipment Account	9,765.00	8,960.00
Cash	12,060.00	
Bills Receivable	9,700.00	
Personal Accounts Receivable	14,600.00	
Personal Accounts Payable		3,620.00
Bills Payable		5,260.00
Expense	5,420.00	
Interest and Discount	524.00	360.00
Cash Discount	351.00	824.00
Plant and Machinery	15,604.00	
Wages	7,150.00	
	$205,174.00	$205,174.00

Additional information given:

Of the notes on hand, one for $1200 has been drawing interest for eleven months at 6 per cent, and one for $800 is non-interest bearing and has nine months yet to run; money is worth 5 per cent.

Of the outstanding notes, one, for $960, dated six months ago, bears interest at 6% for one year; another for $1000, bears no interest, is dated four months ago, and is due in one year from date.

The merchandise on hand was invoiced to the firm for $25161. It is decided to advance the cost 2 per cent to cover transportation charges. Items charged to expense but still on hand valued at $850. Wages not paid $750. The entering of the salary due X was overlooked. Five per cent of the accounts and bills receivable is to be carried to doubtful account. There is a $100 bill in the cash drawer that is counterfeit. The value of the real estate remains fixed. You are to write off 7½ per cent depreciation to plant and machinery. Goods shipped and not sold valued at $3160.

ANALYSIS OF PARTNERS' ACCOUNTS

X

1909		1909	
May 1, withdrew	$2500.00	Jan. 1, invested	$10000.00
Sept. 1, "	1500.00	July 1, "	6000.00

Y

1909			1909		
Mar. 1,	withdrew	$2000.00	Jan. 1,	invested	$12000.00
June 1,	"	1000.00	Oct. 1,	"	7000.00
Nov. 1,	"	2000.00			

Z

1909			1909		
Feb. 1,	withdrew	$1000.00	Jan. 1,	invested	$12000.00
July 1,	"	1500.00	June 1,	"	6500.00
Oct. 1,	"	500.00			

What is the net profit or loss of the business? What is the balance of each partner's account?

INSOLVENCY

490. A **bankrupt** or **insolvent** is one who is unable to pay his debts on the demand of his creditors. The United States Bankruptcy Law provides that an insolvent debtor may, under certain circumstances, turn over his property and resources for the benefit of his creditors, and after these resources have been duly apportioned among the several creditors, the debtor may be discharged from further liability. Some of the states also have bankruptcy laws that are applicable to their own citizens.

491. A **receiver** is a person appointed by a court to take charge of the property and business of an insolvent debtor, and to manage or dispose of the same in the interest of the creditors.

492. The resources of a bankrupt are apportioned among the creditors according to the relation that their several claims bear to the total liability. Suppose for instance, that Mr. X, a bankrupt, owes A $1000, C $2000, and D $5000, or $8000 altogether, and that his total resources amount to but $4000. In this case, each creditor will receive one-half of his claim, since X's total resources are but one-half of his total liabilities, and it will be said that X can pay "fifty cents on the dollar."

WRITTEN PROBLEMS

29. Cook & Co.'s available resources amount to $17956.75, and their liabilities are $28256.15. What sum can they pay on the dollar, correct to four places?

NOTE.—In finding the rate of payment on the dollar, carry the decimal of a cent to as many places as there are figures in the dollars of the largest amount involved. Thus, if there is an item amounting to $27564.16, carry the rate to five places, as for instance, 72.01346.

30. Mr A's debts amount to $9756 and his available resources to $6325. What can he pay on the dollar, and how much will B receive, whose claim amounts to $643.75?

31. A bank fails with liabilities amounting to $275869, and available resources $126575. Allowing 10% of the resources for expenses of the receivership, etc., what sum can be paid on the dollar, and what will be realized by Mr. A, who had $528.60 deposited in the bank?

32. Mr. B, an insolvent, owes debts as follows: Parker & Sons, $275.80; Hanna & Co., $1253.90; Brown & Davis, $672.90; and miscellaneous claims of other persons amounting to $1568.70. B's scheduled resources realized on sale by the receiver $2836.40 in cash. From this sum there is deducted the receiver's fees of 5%, and other charges amounting to $31.50. What sum (correct to five decimal places) can B pay on the dollar, and what will be realized by each of the three principal creditors?

INSURANCE

493. Insurance is a contract under which one person agrees, for a stipulated compensation, to indemnify another for losses to such an amount, and for such causes of loss, as are specified in the contract.

494. Contracts of insurance are of two general classes, *personal* and *property* insurance.

495. Personal insurance provides an indemnity for loss of life, called *life insurance,* or for loss or injury through accidents, called *accident insurance.*

496. Property insurance covers losses of property by fire called *fire insurance,* and loses through accidents or disasters at sea, called *marine insurance.*

497. There are several other forms of property insurance, as insurances against loss by storms, earthquakes, floods, etc., or loss of goods in process of transportation, losses by robbery, etc.

498. The **underwriter** in a contract of insurance, is the *insurer* or the one who agrees to pay the indemnity in case of loss.

499. The **policy** is the written contract of insurance.

500. The **face** of the policy is the amount to be paid in case of loss.

501. The **premium** is the amount as provided by the policy, that is to be paid for the insurance. It is computed as a certain percentage on the face of the policy.

502. The business of insurance, mostly carried on by large stock companies, is strictly regulated by the laws of the several states or countries in which the companies are located. Life, accident, and other forms of personal insurance, are carried on by many fraternal and other societies for the benefit of their members.

503. In marine insurance, as a rule, only that proportion of the loss is paid, which the face of the policy bears to the value of the property. Thus, if a man insures a shipload of grain, valued at $10000 for $5000, and a loss of $8000 occurs, the loss paid would be one-half, or $4000, since the face of the policy is but one-half the value of the property. If in this case, the policy had been for $7500, the loss paid would be $\frac{3}{4}$ of $8000 or $6000. This provision is known as the "average clause."

504. Insurance of property for an amount greater than its value, is not permitted, and any misrepresentation as to the value of the property, or other fraud or concealment of material facts on the part of the insurer, invalidates the policy.

505. In both fire and marine insurance, particularly where large values are involved, it is customary to insure the property in several companies, the loss, should any occur, being shared by the several companies in proportion to the several policies carried. Suppose, for instance, I carry $10000 insurance in three companies as follows: In Co. A $2000; in Co. B $5000, and in Co. C $3000. Suppose a loss of $6000 occurs: Co. A would pay $\frac{2}{10}$ or $1200, Co. B would pay $\frac{5}{10}$ or $3000, and Co. C would pay $\frac{3}{10}$ or $1800.

ORAL PROBLEMS

1. How much must be paid for insuring property for $3000 at $\frac{2}{3}\%$ premium?

SUGGESTION.—The premium is $\frac{2}{3}\%$ of $3000.

2. How much will it cost to insure a vessel for $10000 at $1\frac{1}{2}\%$ premium?

3. What is the insurance on household furniture for $5000 at $\frac{3}{4}\%$ premium?

4. At $\frac{1}{2}\%$, what is the annual premium at $\frac{3}{4}$ valuation of a house costing $8000?

5. At $\frac{2}{3}$ valuation, what is the annual premium at $1\frac{1}{4}\%$ on household furniture, valued at $6000?

6. What is the premium on a house for five years, insured for $5000, at $1\frac{1}{2}\%$ per annum?

7. What premium must I pay for insuring a building for $3600 at $\frac{3}{4}\%$?

8. Insured grain for $5600 at $\frac{7}{8}\%$. What is the premium?

9. An agent receives $80 for insuring a vessel at 2%. What is the value of the vessel?

SUGGESTION.—$80 is .02 of the policy.

10. An agent's premium is $25 for insuring a house at $\frac{2}{3}\%$. What is the face of the policy?

11. If a man insures his store for $\frac{2}{3}$ of its value at $1\frac{1}{2}\%$, paying a premium of $150, what is the value of the store?

12. A merchant pays $75 annually for insurance on his stock of goods at $\frac{3}{4}\%$. For what amount does his policy call?

13. An agent received $20 for insuring a stock of grain at $\frac{2}{3}\%$. What was the value of the grain?

14. At $\frac{3}{4}\%$, an agent received $37.50 for insuring a house at $\frac{3}{4}$ valuation. What was the value of the house?

15. An agent received $75 as premium for insuring a house for three years at $1\frac{1}{4}\%$. What was the amount of the policy?

16. At $\frac{3}{4}\%$ premium, an agent received $72 for insuring a barn for three years at $\frac{2}{3}$ its value. What was the value of the barn?

17. I insure under the average clause, grain valued at $15000, for $10000. A loss of $7500 occuring, how much will be paid me?

18. Mr. A's store and contents valued at $20000 is insured at $\frac{4}{5}$ of its value, as follows: Co. A $2000, Co. B $1000, and the remainder in Co. C. A loss of $4000 occurs. Find the amount to be paid by each company.

19. Clark & Co. insured a cargo of grain, valued at $24000 at $\frac{3}{4}$ of its value under the average clause. A wharf fire damaged the cargo to the amount of $12000. What sum did they realize from the insurance? What sum would they have realized had there been no average clause?

20. A fire in the warehouse of the Hammond Company, caused a damage of $1800, covered by insurance as follows: Co. A $4500,

Co. B $1500, Co. C $1000, Co. D $2000. What is each company's share of the loss?

WRITTEN EXERCISE

33. A merchant has a stock of goods valued at $1200, and a store building valued at $9000. At $\frac{3}{4}\%$, what amount of premium does he pay on a policy, if the insurance is on a $\frac{2}{3}$ valuation?

34. An insurance agent takes a risk of $20000 on a vessel at $2\frac{1}{4}\%$, and immediately re-insures $\frac{1}{2}$ of the risk at $2\frac{3}{4}\%$. What net premium does he receive?

35. A manufacturing plant carries insurance as follows: On building $12000, on machinery $20000, and on stock $30000, paying $1\frac{3}{4}\%$ premium. What is the net loss of the insurance company, if they pay the following losses: On machinery $\frac{1}{2}$ of valuation, on building $\frac{2}{3}$ of valuation, and on stock a total loss?

36. My agent insures a cargo of wheat costing $6400, at the rate of $2\frac{1}{4}\%$ for an amount that will cover the cost of wheat and premium. What is the face of the policy?

SUGGESTION.—$6400 ÷ .97$\frac{3}{4}$ = face of policy.

37. My agent buys 2000 bushels of wheat at $54\frac{1}{2}¢$ per bushel, 3200 bushels of oats at 23¢ per bushel, and 4100 bushels of corn at $48\frac{1}{4}¢$ per bushel, charging 2% commission for buying. I instruct him to ship to Buffalo, per steamer City of Chicago, taking policy at $1\frac{1}{4}\%$, to cover cost of grain and all charges. What is the amount of insurance?

38. A commission merchant bought 40000 bu. of wheat at 75¢ per bu., which he insured at $20000, taking a policy containing the "average clause." How much does he receive, the wheat being damaged to the extent of $7500?

39. I took $50000 insurance in 5 companies at 85 cents per $100. Upon report of the board of survey I was rebated 18%. What sum was returned to me by the insurance companies?

40. A consignment of cotton which cost $30000 was insured at $1\frac{1}{4}\%$ for that amount, and also to cover premium and a commission of 2%. While in transit the goods increased in value 15%, which sum was covered at 3% premium. What was the total premium paid, and what was the net gain to the shipper, the goods being sold at an increase of 21% on the first cost?

41. My house cost $6000, which was insured at $\frac{2}{3}$ valuation in the Wheeling Mutual, at $1\frac{1}{2}\%$ per annum, for 3 years. The house

was totally destroyed by fire. What was my loss, and what was the loss of the company?

42. A farmer took the following insurance in the Farmers' Mutual: House valued at \$6000 for $\frac{3}{4}$ valuation at $1\frac{1}{4}\%$; barn valued at \$1500 for $\frac{2}{3}$ valuation at $1\frac{1}{2}\%$; like stock for \$5500 for $\frac{3}{4}\%$; grain for \$3000 at $\frac{1}{2}\%$. What was his total premium?

43. The Prairie State National Bank carries \$6000 insurance in the Home Insurance Company, \$5000 in the Galena Insurance Company, \$7500 in the Royal Insurance Company, \$10000 in the Manchester Assurance Company, and \$4000 in the Charter Oak Insurance Company. How much does each insurance company pay, the bank being injured by fire causing a loss of \$18687.50?

44. A cargo valued at \$45000 was insured for \$10000 in the Continental Insurance Company, \$6000 in the Liverpool Insurance Company, and \$14000 in the Hamburg Insurance Company. The cargo was injured during a storm to the extent of \$9600. What sum did each company pay according to "average clause" insurance?

506. Life insurance. There is so much variation in the kinds of policies and the methods of computing the costs of and returns from life insurance, that any adequate presentation of the subject would be out of place in a text of this class. The computations are made by means of tables and involve no arithmetical processes not already covered in preceding chapters.

GENERAL REVIEW PROBLEMS

1. A commission merchant sold 75 bales of cotton averaging 475 lb. @ $15\frac{3}{4}¢$ a lb. and charged $12\frac{1}{2}¢$ a bale commission; other charges were \$165.10. He bought for the consignor 8000 bushels of corn @ 45¢ per bushel, charging 3% commission. What amount should be remitted to the consignor?

2. A man 36 years of age takes a 10 year endowment policy October 10, 1902, for \$5000 at an annual premium of \$62.29 on \$1000; he dies May 5, 1909. How much more is his estate worth than it would have been, had he put the premiums in a savings bank at 4% interest compounded annually?

3. What is the present worth of a debt which would amount to \$1595.40 in 73 days from date at 4%?

4. A warehouse insured for \$20,000 contains goods insured for \$17,500. The risk is distributed as follows: Company A, building \$6000, goods \$4000; company B, building \$5500, goods \$1500;

company C, building $5000, goods $2500; company D, building $3500, goods $1500; company E, $5000 on goods. Fire occurs and the damage is appraised as follows: Building $14,750, goods $12,358. Find the amount of insurance to be paid by each company.

5. A Boston importer buys 5246 pounds of wool from a London merchant, @ 1s 3d a pound. What would be the duty in United States money if the specific duty is 10¢ a lb., and the ad valorem duty 11%?

6. The inside dimensions of a rectangular fort are 240 feet and 190 feet; the masonry surrounding this open space is 10 feet high and 4 feet thick. Find the exact contents of the masonry and its cost @ $2.70 a cubic yard.

7. A and B form a partnership January 1 and agree to share gains and losses in proportion to their average investments. A invests Jan. 1, $5000; May 20, $1000; Sept. 10, $1600; and withdraws July 25, $1200. B invests Jan. 1, $3000; Mar. 12, $2000; and withdraws Oct. 1, $800. The net profits of the year are $3600; find each man's share.

8. A speculator buys 10,000 bushels of May wheat at 83½¢ a bushel and is obliged to sell 5000 bushels at 82¼; the remainder he holds and sells at 84¾; the brokerage is ⅛ of a cent per bushel for buying and the same for selling. Does he gain or lose and how much?

9. Coffee costing 18¢ a pound has lost 10% of weight in roasting; how much must it sell for in order to gain 15%?

10. An apartment house was insured as follows: Company A $3000, company B $9000, company C $12000. The building was partially destroyed by fire. The damage was repaired by the companies at a cost of $9600. What amount is paid by each company?

11. A plumber buys fittings listed at $1200 with the option of successive discounts of 20, 10, 10, and 5 with 90 days credit or 40% discount for cash. How much would he gain or lose by borrowing the money at 6% to pay cash?

12. Find the cost of a draft for (a) £500 on London, exchange at 4.84, (b) 2500 francs on Paris, at 5.20, (c) 2500 marks on Berlin, at 94.

13. A merchant is retailing a certain kind of cloth at $4.75 per yard and realizing a profit of 33⅓%. Having a large stock, he sells to a dealer 198¾ yd. at a discount of 20%. What is his net gain or loss on the wholesale transaction?

14. What sum of money must be placed at compound interest at 4% for a boy 16 years old, that he may have $1000 when he is 21 years old?

15. Which investment of $5000 pays the better and how much, bond and mortgage bearing 5%, bought at par; or railway stock paying 6% annual dividends, bought at 113?

16. Two men engage in trade; A puts in $300 for 6 months and B puts in $250 for 8 months; they gain $85.50. What is each man's share of the profits?

17. Bought 3 building lots, each 25 feet front, at $50 a front foot, and leased them at $6 a front foot. Paid $5625 for a house and lot from which I realized 8% on the cost after allowing for expenses, taxes, etc. What is my annual income? What is the rate of income on my total investment?

18. By the provisions of a will $17,032 was bequeathed to three brothers, to be so divided that being invested at 5% simple interest, the three should receive equal amounts at the age of 21 years. What sum should the executor invest in each case, the ages being respectively 12, 15, and 16 years?

19. A retired merchant wishing to invest his capital, $25,256, bought $9000 of U. S. 5's @ 107⅞, and $8500 of U. S. 4½'s @ 106½; the remainder he invested in mining stock @ 78¼, which paid a dividend of 10% on the par value. What is the annual income? (Brokerage not considered.)

20. A grocer bought a cask of vinegar containing 52 gal. 2 qt. @ 18¢ a gal. On transferring the vinegar to another vessel, he found that 13 gal. 1 pt. had leaked away. How much must he charge a quart to cover the cost?

21. The owner of a corner lot 60x100 feet wishes to lay a brick sidewalk 5 feet 4 inches wide on the two exposed sides. What will be the cost of the brick at $9.50 a 1000, if the face of one brick is 8x4 inches?

22. The freight charges on 14,700 pounds of merchandise were $88.20 for 288 miles; at the same rate, what amount would be charged for 33,900 pounds of the same class of goods for 108 miles?

23. J. R. Hartwell buys today lumber of Smith & Bacon as follows:

24 pieces joists 2″x4″x12′	@	$14.00 per M
40 boards ⅞″x12″x14′	@	16.50 per M
3 timbers 8″x6″x16′	@	12.00 per M

Make out a receipted bill in proper form.

24. For what sum must a note payable in 90 days be drawn so that when it is discounted at 6%, the proceeds may be $725. (1 year = 360 days.)

25. A note for $1950 bearing interest at 5%, dated July 11, 1909, and payable one day after date, has the following indorsements: Sept. 23, 1909, $300; Dec. 12, 1909, $500; May 15, 1910, $50. How much is due July 11, 1910? (Solve by United States rule.)

26. A bill of goods amounting to $975 list price was bought at discounts of 10, 10, and 5, and sold at a discount of 12½% from the list price. What was the gain?

27. Holmes and Oliver find that their bank account is overdrawn to the amount of $2936.10. They have in their possession the following negotiable papers: A note for $750 due in 90 days, a note for $925.40 due in 60 days, a note for $500 due in 20 days, an accepted draft for $1025.60 due in 10 days. How will their bank account stand if this paper is all discounted at 6 per cent, and the proceeds placed to their credit?

28. A farmer builds a fence 4 boards high around a field 16 rods square. The posts are set 8′ apart and the boards are 7″ wide. Give cost of material @ $24 per M for boards and 40 cents each for posts.

29. A commission merchant in Milwaukee buys for a Buffalo dealer 32,000 bu. wheat @ 75¢. The charges are: Insurance ¼% on cost of wheat, commission ¾¢ per bushel, freight 2¢ per bushel. If the dealer sells the cargo at 80¢ per bushel, taking in payment a note at one month without interest which he immediately discounts at 5%, how much does he gain?

30. A manufacturer's list price for goods is 25% above cost. If he allows a merchant discounts 5 and 5%, what is the manufacturer's per cent of gain?

31. Find the cost at $1.75 a roll, for papering the walls and ceiling of a room 16′ by 15′ and 12′ high, making no allowances for openings.

32. The prices on a certain quantity of standard hardware as quoted by J. H. Harper amount to $546.84, 20% and 5% off, and as quoted by H. A. Warner & Co., they amount to $575.30, 25% and 10% off. Which is the better offer and by how much?

33. A broker sold certain railway stock @ 90⅝ and with the proceeds purchased $55,500 in U. S. 4% bonds @ 101⅜. Allowing

the usual brokerage of $\frac{1}{8}\%$ on each transaction, find the par value of the stock sold.

34. A grain dealer in Buffalo has in store 13,050 bu. wheat, 10,860 bu. corn, 5625 bu. oats, and 10,875 bu. barley for shipment by rail to New York. How many cars with a carrying capacity of $14\frac{1}{2}$ tons each will be required, estimating the following standard weights per bushel: Wheat 60 lb., corn 56 lb., oats 32 lb., barley 48 lb.?

35. Find (a) the time in which \$350 will yield \$70 at 6%, (b) the rate at which \$500 will yield \$50 in $2\frac{1}{2}$ years, (c) the amount that will yield \$108 in 3 years at $4\frac{1}{2}\%$.

36. Brown & Co. accepted a draft for \$762.75, drawn on them at 90 days sight. What was the proceeds of the draft, if discounted two days after acceptance at 5%?

37. A dealer buys 1364 long tons of coal @ \$2.98 a ton f. o. b. at shipping point. The freight is 7¢ a 100 lb., and the cost of screening and hauling is \$109.20. At what price a short ton must this coal be sold to gain 20% on the dealer's entire expenditures?

38. A broker wishes to close up Henry Miller's account. From the following memoranda complete the account and show how much is due Miller; also his gain, estimating interest at 6%.

HENRY MILLER

19—					19—			
June	5	To 100 shares Pa. R. R. @ 50¼			June	5	By cash (margin)	5000
		To 400 shares Lake Shore @ 98¾					By 25 days interest on margin	
		To 200 shares W. U. Tel @ 102				30	By 100 shares Pa. R. R @ 52⅝	
		To ⅛% brokerage, 700 shares (buying)					By 400 shares Lake Shore @ 99¼	
		To 25 days interest on cost and brokerage					By 200 shares W. U. Tel. @ 105¼	
		To ⅛% brokerage, 700 shares (selling)						

39. In Albany the annual tax bills are subject to a rebate of 1% if paid before Feb. 9, and of $\frac{1}{2}\%$ thereafter through February; in March they are payable at par, but after March $\frac{1}{2}\%$ is added to the bill for each month or part thereof. A owning four houses valued at \$9000, \$5000, \$3000, and \$1500 respectively, pays the taxes on these houses in the order mentioned and on the following dates: Feb. 7, Feb. 12, March 28, and June 29. The tax rate is $2\frac{1}{2}\%$ on valuation. How much does A pay?

40. An importer bought 185 yards of English suiting @ 12s 6d a yard; the duty was 40% ad valorem, freight and other charges £9 3s, rate of exchange 4.88. He sold the entire invoice at a gain

of 20% of the total cost. What was his net gain in United States money?

41. A business firm whose assets were $11,805 failed and made an assignment, in which B was by mutual consent made a preferred creditor. They owed A $1200, B $2240, C $8000, D $9300. What amount should each creditor receive if the expense of settling is $1240?

NOTE.—A preferred creditor receives 100% of his account before any payments would be made to creditors not preferred.

42. A young man deposited $2000 in a savings bank @ 3½% interest, payable semi-annually and compounded; if there were no withdrawals, what amount should his pass book show at the end of three years?

43. The following invoice was given by Mr. Forbush to his clerk with the remark: "John, we want to clear 15% on this invoice. I pay cash." Complete the invoice and state the figure at which each chair should be marked:

Wilmington, Del., May 12, 19—

Peter H. Forbush,

Newark, N. J.

Bought of BOSWORTH & Co

Terms: $^{2}/_{10}$ $^{n}/_{30}$

40 No. 376 A chairs @ $7.75.....

Less special discount 10%.....

Freight prepaid..... 10.45

44. A commission merchant sold 250 bales of cotton, weighing 500 pounds each, @ 12½¢ a pound. The charges were freight 20¢ per 100 pounds, cartage 10¢ a bale, and commission 2½%. He then bought for his principal 500 boxes bacon, 150 pounds each, @ 9¢ a pound, charging 1½% for buying. How much is due the principal?

45. What taxes must a man pay who owns property worth $22,500, assessed at ⅔ value, when the rates are as follows: State tax 1.9 mills, city 2.01 mills, school 1.2 mills?

46. On May 20, a speculator deposited with a broker $6000 as a margin, directing him to buy stocks as follows: 200 shares American tin plate at 40; 200 shares New Jersey Central at 112; 200 shares Missouri Pacific at 43¼. May 27, the stocks were sold as

follows: American tin plate at 35⅛. New Jersey central at 115⅞. Missouri Pacific at 41. Find the speculator's loss or gain, brokerage being ⅛% and interest 6%.

47. An exporter sold through a broker the following bills of exchange: 8650 francs on Paris @ 5.18⅜; 15,000 marks on Hamburg @ 94⅝. What were his net proceeds, brokerage ⅛%

48. A $1000 bond bearing interest at 3½% and payable at the end of 40 years is bought for $1103.60. What is the average annual income?

49. A capitalist invested $50,088.25 as follows: $25,080 in U. S. 4's @ 104½; $7680 in Mex. Cent. 4½'s @ 96; the remainder in manufacturing stock @ 85⅜ paying an annual dividend of 8%. What is his annual income from the investment?

50. A railway company with a share capital of $250,000 preferred stock and $4,000,000 common stock, has a bonded indebtedness of $750,000 bearing 5% interest, payable annually. For the second fiscal year after the bond issue, the net earnings are $360,000. The directors declare a dividend of 6% on the preferred stock and of 4% on the common stock. What is the surplus?

51. A New York merchant imports from Hamburg 400 pieces of linen at 30 marks a piece; he pays $45 freight and an ad valorem duty of 35%; exchange on Hamburg is at 94½. At what price must he sell the linen to gain 25%?

52. Find the equated time for the payment of the balance of the following accounts; all of the dates being in the same year:

Dr.		*Cr.*	
Apr. 20	Mdse. 4 months.. $518.50	May 2	Mdse. 90 days.... $585.60
May 17	" 30 days... 475.00	June 16	Cash 350.00
" 28	Cash 200.00	Aug. 6	Mdse. 30 days.... 268.47
June 14	Mdse. 60 days.... 318.34	" 24	" 60 " 192.25
July 6	Cash 175.00	Sept. 12	Cash 225.00

53. A New York merchant received from Germany an invoice of toys valued at 8436 marks, duty 50% ad valorem; from France 300 meters of silk invoiced at 4 francs a meter, duty 60%; from England 840 yards of woolen goods invoiced at 6s 6d a yard, duty 35%. Find, in United States money, the whole amount of duty paid by the merchant.

54. How many yards of carpet will be required for a flight of stairs 12 feet high, if each riser is 8 inches high and each tread 10 inches wide, and ¼ yard is allowed for projections? How much will the carpet cost at $1.75 a yard?

55. You owe $900 at a bank, and have B's note for $640 due in 60 days. The bank discounts this note for your use, and will also discount your own note at 90 days for an amount sufficient to enable you, with the proceeds of both notes, to discharge your debt today. For what sum should this note, without interest, be drawn?

56. On May 6, a customer deposited $4000, on which margin his broker bought 500 shares of railway stock @ 78; on June 2 following, the stock was sold by the broker @ 76¾. How much is due the customer if the broker is allowed ⅛% brokerage each way and interest on money advanced at 6%?

57. When a New York draft for $50,000 can be bought at New Orleans for $49,950, is exchange on New York at a premium or at a discount? What is the rate? How is the balance of trade?

58. A New York importer buys woolen goods in London as follows: 863 yd. @ 6s 6d; 430 yd. @ 10s; 648½ yd. @ 12s; 234½ yd. @ 15s 6d. The freight and shipping charges amount to $68.25, the duty is 35% ad valorem, and the exchange $4.88. Give total cost of the goods in New York.

59. A certain railway company, whose capital stock consists of 225,000 shares (25,000 preferred) makes the following report at the end of its fiscal year: Gross earnings $20,679,035; operating expenses and taxes $14,548,612; interest paid $1,575,000; added to sinking fund $250,000; dividend on preferred stock 8½%. After the declared dividend on common stock is paid, there is a surplus of $2,492,923. How much did the owner of 125 shares of common stock receive as his share of dividend?

60. A large warehouse, insured with its contents under the "average clause" (which provides that unless the owner insures for the full value of the property the companies shall pay only such proportion of the loss as the whole amount of insurance bears to the total value of the property) is totally destroyed by fire. The amount of insurance is $88,000, distributed as follows: A company $15,000; B company $10,000; C company $7,500; D company $4,500; E company $12,500; F company $17,500; G company $12,000; H company $9,000. Compute amounts due on adjustment from each of the several companies, the actual value of property being $100,000.

61. A bank has on hand $215,000 in specie and $325,500 in other legal tender and the sum is 21.5% of the deposits. Find the amount of the deposits.

62. A coasting vessel valued at $24,000 had on board a cargo worth $20,000, on which the freight was $2400. During a storm goods to the amount of $5000 were thrown overboard to escape wreck; the vessel then put to port and underwent repairs to the extent of $900; the expenses of dentention were $266 and the adjuster's charges $100.40. Make up a general average statement, showing adjustment of loss.

63. A dealer sent a margin of $1500 to his broker, April 16, and ordered him to buy 100 shares of American sugar stock. He filled the order @ 131⅞ and sold the stock, May 1, following, @ 126½, charging ⅛% brokerage each way and 6% interest. How much money should be returned to the dealer?

64. A, B, and C form a partnership, investing $15,000, $10,000, and $8,000 respectively, gains and losses to be shared according to investment. At the end of the year their books showed the following resources and liabilities: Merchandise on hand $15,678.10, value of furniture and fixtures $1500, cash on hand $4710.15, notes and drafts outstanding $3960.50, with accrued interest $83.89, notes and acceptances due the business $5018.75, with interest accrued $157.10, sundry creditors $7213.25, sundry debtors $15,705.90. Show the loss or gain and the present worth of each partner, by making a statement of the business.

65. Make out in proper form, under current date, a bill against H. J. Brown for the following goods sold him by The Excelsior Manufacturing Co., Hartford: 20 doz. punches @ $5.25, discounts 10 and 10; 150 special chucks @ $2.75, discounts 20 and 10; 225 agricultural wrenches @ 30¢, discounts 10 and 5; 18 bundles steel wire, 2750 lb. @ 2¾¢, discounts 5 and 5.

66. Arthur & Hoffman buy in Duluth 32,000 bushels hard wheat @ 72¢. The commission for buying is ½¢, the loading charges 2¢ and the freight to Buffalo 2¾¢ a bushel. When the wheat is weighed out at Buffalo, it falls short 3855 pounds. The wheat is sold in Buffalo @ 81¢ a bushel, the elevator charges being 2¢ a bushel and the brokerage ¼¢ a bushel. What is the gain or loss?

67. The assignee received from the sale of the property of an insolvent debtor $38,112.58; the total liabilities were $48,718.14 and expenses of settling $843.20. How much can he pay on the dollar and what will a creditor receive whose claim is $7526.50?

68. A fruit dealer bought 450 crates of peaches @ 95¢ a crate. He sold 125 crates at a gain of 20%, 75 crates at a gain of 12%, 40

crates at cost, 190 crates at a loss of 10%, and the remainder was worthless. What was his net gain or loss?

69. What amount should be invested in United States 4% bonds *@* 118⅜, in order to realize a quarterly income of $300? What would be the total net income realized at the end of two years if the bonds were then sold at 118? (Brokerage not considered.)

70. The net earnings of a corporation whose capital is $500,000 are $65,000; the corporation wishes to declare a dividend of earnings, after reserving for depreciation of plant 15% of its estimated value of $200,000. There are 1000 shares of the stock unissued. Find the amount of dividend payable to a stockholder who owns 30 shares of stock.

71. By the death of a relative in Philadelphia, a resident of Naples becomes entitled to a legacy of $12,500. He orders the money sent to his agent in London who remits the proceeds to Naples. Exchange on London is quoted at 4.8775 in Philadelphia and the rate between London and Naples is 25.53 lire to the pound sterling. What amount does he realize from the legacy?

72. A stock of goods was insured for one year for $18,500 *@* ¾%. At the end of 4 months the insurance company canceled the policy. What was the return premium?

73. The M. & W. Railroad Co., with a capital stock of $3,750,000, has a mortgage debt of $125,000. Gross earnings for a year were $649,500, and operating expenses $275,000. After expenses and 6% interest on the debt were paid, $1000 was added to surplus. What amount of dividend should be paid a stockholder owning 25 shares?

74. An American importer bought in England 2500 yd. velveteen *@* 9d a yd. The cost of the goods, with £5 for casing, is the amount subject to duty. Freight was $39.50 and duty 47½% ad valorem. The market value of a pound sterling was $4.90. What was the total cost to the importer?

75. A merchant buys hardware amounting at list prices to $965.50, getting discounts of 30, 20, 10, and 5 at 90 days, with privilege of further discount of 3% if he pays spot cash. How much would he gain or lose by borrowing money at 5% to pay the bill? What would be the per cent of gain if he accepted the cash offer and sold the goods at list prices?

76. A commission merchant received a consignment of 460 barrels of flour. He paid freight $56, storage $15.80, cartage $14,

cooperage $3.70. He sold 65 barrels @ $6.40, 185 barrels @ $6.45, 50 barrels @ $6.35 and the remainder @ $6.50. Commission for selling was 2%. What was the balance due the consignor?

77. A New York exporter has matured foreign credits as follows: London, £125 6s; Paris, 625.50 francs; Berlin, 4400 marks. What amount in United States money is due the exporter, if exchange is in all cases at par?

78. The capital of a certain corporation is $650,000, the estimated value of plant is $225,000 and the net earnings for a certain fiscal year are $67,500. From these earnings 10% of the value of plant is added to depreciation fund and $2000 to undivided profits; the remainder is to be divided among the shareholders. How much will the owner of 115 shares of stock receive?

79. The National Novelty Company, a corporation, has an authorized capital of $250,000, represented by 2500 shares of stock, of which 500 shares remain unsold. Net earnings for the fiscal year are $21,318. The directors vote to add $2000 to the permanent depreciation fund; to leave $2318 as undivided profits; and to pay the remainder as a dividend to shareholders. How much should you receive as the owner of 75 shares of the stock?

80. A commission merchant sells 15 tons of hay at $12 a ton, 6 tons of straw at $5 a ton, and 50 bushels of oats at 62 cents a bushel; find the net proceeds after deducting $12 for freight and $2\frac{1}{2}\%$ commission.

81. A merchant bought 6 bushels of clover seed @ $4.25 a bushel; he retailed 5 bushels 6 quarts @ 10¢ a pound and sold the remainder at cost. Find his whole gain and his per cent of gain. (60 pounds clover seed = 1 bushel.)

82. Downs & Morrison fail and make an assignment, assets $82,798.50, liabilities $89,367.10. By consent of all the creditors Henry Johnson's claim of $3500 is made preferential. The assignee's expenses and fees amount to $1325.70. How much should be paid to a creditor whose admitted claim is $7500?

83. The insurance on a certain store room was distributed among the following companies: Germania $2000, Knickerbocker $7000, Hanover $6000, United States $5000. The goods in said store room were insured for $10,000 in the Continental and for $15,000 in the Commercial. Fire damaged the building to the extent of $4750 and the goods to the extent of $6700. How much will each company be required to pay?

84. On Mar. 16, a man was offered $4750 cash, or a note for $5000 due Dec. 22, following, for a certain piece of land. He accepted the cash offer and loaned the money @ 6%. The loan and interest accrued were paid on the date the note would have fallen due. Did he gain or lose by accepting the cash offer and how much?

85. E. S. Foster & Co., New York, import from London 20 cases of cutlery at £48 10s a case. The freight charges amount to $80, the duty is 40% ad valorem and exchange on London is at 4.87½. Find the price per case at which the importers must sell the cutlery to gain 20%.

86. A speculator borrowed money Sept. 24, at 6%, with which he bought 9216 pounds of hops @ 12½¢ a pound. Dec. 6, following, he sold the hops @ 20¢ a pound and paid his debt. What was his gain? (Commission is not considered in this problem.)

87. An agent at Duluth buys for his principal at Buffalo 15000 bushels of oats at 30¢ a bushel: the agent's commission is 2½%, insurance ¼%, freight 2¢ a bushel. If the principal sells the oats at 35¢ a bushel, taking in payment a note at 60 days without interest, which he discounts at once at 6%, what is his profit?

88. A dealer buys hardware listed at $840, getting discounts of 33⅓, 20, 10, and 5 with 90 days credit, or an additional discount of 2% for cash. How much will he gain by borrowing money at 5% to pay the bill?

89. The common stock of a certain railway company is $56,700,000 and the preferred stock $9,500,000: the net earnings at the end of the year are $1,964,135. What is the surplus after payment of a dividend of 4% on preferred stock and 2½% on common stock?

90. The assessed valuation of the property in a certain town is $2,249,260. The combined tax levy is $33,738.90. What is the rate of taxation. What is the tax on property valued at $15,960?

91. Find the difference between the present worth of a debt of $1250, to be paid in 1 year and 3 months without interest, money being worth 6%, and the proceeds of a note for the same time and amount if discounted at a bank at 6%.

92. A stock of goods was insured Jan. 1, for 1 year for $5820 at 75¢ a 100 and the premium paid in full, but the policy was canceled July 1 at the request of the insured, the short rate for 6

months being 48¢. Had the policy been arbitrarily canceled by the company, how much more of the premium would have been refunded?

93. What insurance premiums per annum does a man pay who has insured his life for $15,000 (at $22.85 on $1000), his business block for $20,000 (at $1.95 on $1000,) and his dwelling for $6000 (at .75 on $100 for 3 years)?

94. A Sheffield manufacturer drew a bill of exchange on a New York merchant for £510 8s 6d. What amount must the New York man pay to meet the paper when presented by a local bank, exchange being $4.85\frac{1}{4}$?

95. A owes B $1275, the payment of which he wishes to defer for four months. For what sum must A draw his four months note without interest to cancel this debt? (Interest 6% : 1 year = 360 days.)

96. What must be the face of a draft due 60 days after date, with interest at 6%, that can be bought for $1020.40, when exchange is at a premium of $\frac{5}{8}$%?

97. A tank has a feeder by which it can be filled in 10 hours, and an outlet by which when full it can be emptied in 6 hours. If both feeder and outlet are opened when the tank is full, in what time will it be emptied?

98. Find, by the United States rule, the amount due Jan. 17, 1911, on an interest bearing note for $900, dated July 30, 1907, on which the following indorsements have been made: Apr. 19, 1908, $200; Nov. 29, 1908, $25; July 31, 1909, $100; July 5, 1910, $400. Estimate interest at the rate of 6%.

99. A merchant sells goods at a discount of 20% from the marked price and still makes a profit of 20%. At what per cent above cost must he mark them?

100. A broker received from a customer, Oct. 26, a remittance of $1000 as a margin, and purchased for him 100 shares of the National Salt Company at 65; the broker sold the stock, Nov 2, at 70, charging $\frac{1}{8}$% brokerage each way and 6% interest. What was his customer's net profit?

APPENDIX

The following pages include a treatment of several topics of advanced arithmetic that ordinarily would not be presented in a book of this character, but which are given here for the convenience of teachers who may wish to have their classes take up these topics, and without making use of another text.

INVOLUTION

507. Involution is the process of finding the required powers of a number.

508. A **power** of a number is the product obtained by using it a number of times as a factor.

509. The **square** of a number is the second power.

510. The **cube** of a number is the third power.

511. The **exponent** is a figure written to the right of the factor and a little above, indicating the number of times the factor is to be used.

WRITTEN PRACTICE

EXAMPLE: What is the square or second power of 25?

SOLUTION.—$25 \times 25 = 625$.

1. What is the second power of 33?
2. What is the third power of 75? 85? 95? 65? 55?
3. What is the fourth power of 4? 8? 16? 15?
4. What is the third power of $\frac{3}{4}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{6}{7}$?
5. Expand the following: 5^3, 6^2, 8^4, 10^5, and 25^3.
6. Find the required power of the following: $(\frac{7}{8})^3$, $(\frac{5}{6})^4$, $(2\frac{1}{2})^3$, $(14\frac{2}{7})^2$, $(5.06\frac{1}{4})^3$.
7. Find the difference between 25^3 and 40^2.

EVOLUTION

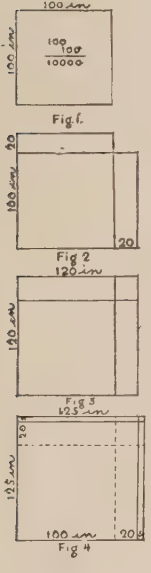
512. Evolution, the reverse of involution, is the process of finding the repeated factor.

The root of a number is one of the equal factors.

513. The **square root** of a number is one of the two equal factors.

514. The **cube root** of a number is one of the three equal factors.

EXAMPLE: What is the largest square than can be formed of 15625 sq. in.?



SOLUTION BY ANALYSIS

			5
	1′	56′ 25	20
	1	00 00	100
200		56 25	125
220		44 00	
240		12 25	
245		12 25	

SOLUTION BY RULE

	1′ 56′ 25	125
	1	
20	56	
22	44	
240	1225	
245	1225	

EXPLANATIONS

The square of any number having but units place will not occupy more than two places.

The square of any number having units and tens will not occupy more than four places, etc.

Therefore, point off the number into periods of two figures each, beginning at the right.

The number of periods shows the number of places in the root.

15625 is the square; 125 is the root; 20 or 200 is the first trial divisor; 22 or 220 is the first complete divisor, etc.

Fig. 1. Shows the first assumed square.

Fig. 2. Shows the two additions, the additions that produce the trial divisor.

Fig. 3. Shows the first complete divisor.

Fig. 4. Shows the second complete square, or a square containing 15625 sq. in.

RULE FOR EXTRACTING THE SQUARE ROOT

I. *Separate the number into periods of two figures each, beginning at the right.*

II. *Find the greatest square in the left hand period, and place its root on the right. Subtract this square from the period, and to the remainder annex the next period for a new dividend.*

III. *Double the root figure already found, add a cipher, and write it at the left for a trial divisor; find how many times it is*

contained in the dividend, writing the trial figure at the right and adding it to the trial divisor for a complete divisor.

IV. *Multiply the complete divisor by the trial figure in the root, subtract and bring down the next period, with which proceed as before.*

NOTE.—1. If the number is not a perfect square, periods of ciphers may be annexed and the root continued as a decimal.

2. If the trial divisor is not contained in the dividend, the next period must be brought down, and ciphers added to both the divisor and the root.

3. To find the square root of decimals, there must be an even number of places; if the decimal has an odd number, a cipher must be added.

4. To find the square root of fractions, find the root of the numerator for a new numerator, and of the denominator for a new denominator, or find the root decimally.

MENTAL PRACTICE

Find the square root of the following:

1.	9	*5.*	36	*9.*	225	*13.*	$\frac{1}{4}$
2.	16	*6.*	49	*10.*	625	*14.*	$\frac{9}{16}$
3.	25	*7.*	64	*11.*	169	*15.*	$\frac{25}{49}$
4.	81	*8.*	144	*12.*	256	*16.*	$\frac{26}{81}$

WRITTEN PRACTICE

Find the square root of the following:

8.	1225	*12.*	5625	*16.*	53361	*20.*	$\frac{225}{625}$
9.	2025	*13.*	7225	*17.*	17424	*21.*	$\frac{169}{289}$
10.	3025	*14.*	15129	*18.*	97344	*22.*	.1225
11.	4225	*15.*	103041	*19.*	45369	*23.*	.007225

EXTRACTION OF THE CUBE ROOT

A **cube,** or third power, is the product obtained by multiplying together any three equal factors. Thus, the cube of 2 equals $2 \times 2 \times 2$, or 8. The cube of $\frac{2}{3}$ equals $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, or $\frac{8}{27}$, and the cube of .4 equals $.4 \times .4 \times .4 = .064$.

Conversely, the **cube root** of a number is one of its three equal factors. Thus, the cube root of 8 is 2; the cube root of $\frac{8}{27}$ is $\frac{2}{3}$; and the cube root of .064 is .4.

The cube root is indicated by the sign ($\sqrt[3]{\ }$); thus, $\sqrt[3]{125}=5$.

NOTE.—Comparatively few numbers are perfect powers, that is, numbers that are separable into equal and ascertainable factors. Thus, the number 36 is separable into the two equal factors, 6×6; the number 343 into the three equal factors $7 \times 7 \times 7$, and the number 81 into the four equal factors, $3 \times 3 \times 3 \times 3$; but the numbers 20, 46, 57, 72, etc., are not thus separable and are therefore not exact powers, and their roots can only be found approximately.

The root of a number that is not a perfect power in the degree of the root required, is called a *surd*. Thus, the square roots of 12, 15, or 20, and the cube roots of 25, 38, or 100, and the fourth roots of 27, 40, or 75, are *surds*, and can only be found approximately.

Before proceeding further, let the student commit to memory *thoroughly* the following table of perfect cubes and their roots:

$1^3 = 1$	$7^3 = 343$
$2^3 = 8$	$8^3 = 512$
$3^3 = 27$	$9^3 = 729$
$4^3 = 64$	$10^3 = 1000$
$5^3 = 125$	$100^3 = 1{,}000{,}000$
$6^3 = 216$	$1000^3 = 1{,}000{,}000{,}000$

From an inspection of the foregoing table it is evident:

1. That the cube root of any number consisting of three figures, or less, consists of *one figure*.

2. That the cube root of any number consisting of more than three figures and less than seven, will consist of *two figures*.

3. That the cube root of any number consisting of more than six figures and less than ten, will consist of *three figures*, and so on.

Hence, to find the number of figures in the cube root of any number:

Separate the figures into groups of three figures each. The number of groups thus formed will equal the number of figures in the cube root.

EXAMPLE: The cube root of 32|768 contains two figures (32); and the cube root of 12|812|904 consists of three figures (234).

EXERCISE

State the number of figures in the cube root of each of the following numbers:

1674	21764321
42796	1207658247
3479160	24765872647962

PROCESS OF FINDING THE CUBE ROOT

It may be shown universally by algebra and geometry, and specifically by arithmetic, that the *cube of the sum* of any two numbers is equal to four united quantities, viz.:

1. The cube of the first number.
2. Three times the square of the first number, multiplied by the second number.
3. Three times the first number, times the square of the second number.
4. The cube of the second number.

We may illustrate this by the following arithmetical operation: Required the cube of $5 + 4$.

$$
\begin{array}{r}
5+4 \\
5+4 \\
\hline
20+16 \\
25+20 \\
\hline
25+40+16 = 81 \text{ (the square of } 5+4) \\
5+\ 4 \\
\hline
100+160+64 \\
125+200+\ 80 \\
\hline
125+300+240+64 = 729 \text{ (= the cube of } 5+4)
\end{array}
$$

Summary:

$$
\begin{array}{l}
125 = 5^3 \\
300 = 3\times(5^2\times 4) \\
240 = 3\times(5\times 4^2) \\
\ 64 = 4^3 \\
\hline
729 = (5+4)^3
\end{array}
$$

Every number consisting of two figures may be considered as the *sum of its tens and its units.*

Thus, $25 = (20 + 5)$, $12 = (10 + 2)$, $79 = (70 + 9)$, etc.

If, therefore, we represent the tens by the letter t and the units by the letter u, we may formulate the cube of every such number as follows.

$(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3$. That is, the cube of every number consisting of tens and units is separable into the four quantities, viz.:

1. The cube of the tens.
2. Three times the square of the tens, times the units.
3. Three times the tens, times the square of the units.
4. The cube of the units.

Thus, in the case of the number 25, the cube may be found as follows:

$$
\begin{aligned}
t^3(20\times20\times20) &= 8000\\
3t^2u(3\times(20\times20)\times5) &= 6000\\
3tu^2(3\times20\times5\times5) &= 1500\\
u^3(5\times5\times5) &= 125\\
25^3 &= 15625
\end{aligned}
$$

EXERCISE

Using the foregoing method, find the cubes of the numbers: 16, 27, 38, 56, 95.

We will now apply this formula to the reverse process of finding the *cube root* when the cube is given.

EXAMPLE: Required the cube root of 42875.

$t^3+3t^2u+3tu^2+u^3=42|875 \qquad (30+5$

$t^3=27$

$3t^2=2700)\,15\ 875$

$3tu=\ 450\ \ 15\ 875$

$u^2=\ \ 25$

3175

EXPLANATION.—First, we perceive that the number contains two "cube root periods," and that its root therefore contains two figures (tens and units).

Since the cube of tens consists of thousands, the largest cube contained in the thousands (27) is the cube of the tens of the root, and the cube root of the 27 (thousands) is 3, written in ten's place. This root we place at the right, as the first figure of the required root, and subtract its cube (27) from the thousands, obtaining a remainder of 15, to which we bring down the next period.

We now know that this remainder (15875) comprises the remaining three parts of the cube formula ($3t^2u+3tu^2+u^3$). And since the first of these terms ($3t^2u$) usually comprises the larger part of the remaining number, we may *assume* that it comprises the *whole* of it, and obtain the value of u by *dividing* the remainder, 15875, by $3t^2(3\times30\times30)=2700$. Performing the division by this "trial divisor" we obtain 5 for the units figure.

It now remains to complete the divisor by adding to the trial divisor the omitted values of t and u in the third and fourth parts of the fomula. These are: $3tu(3\times30\times5)=450$, and $u^2=25$. The sum of these gives the complete divisor, 3175, which multiplied by the unit figure of the root, 5, equals the remainder of the cube, 15875, and proves the correctness of the operation.

Out of the foregoing demonstrations we derive the following

GENERAL RULE FOR EXTRACTING THE CUBE ROOT

I. *Point off the number into periods of three places each, beginning at the decimal point and counting to the left for integers, and to the right for decimals.*

II. *Find the largest cube in the left-hand period, and place its root at the right. Subtract the cube from the left-hand period and annex the second period to the remainder.*

III. *Find three times the square of the first term of the root, considered as tens, and place it at the left as a trial divisor. Divide the second dividend by the trial divisor, and place the quotient as the second term of the root.*

IV. *Find three times the product of the first and second terms of the root, considering the former as tens, and write it under the trial divisor. Square the second term of the root, and write the result under the preceding product. Find the sum of these three results, and multiply it by the second term of the root. Subtract the product thus found from the partial dividend and to the remainder annex the next period.*

V. *Find three times the square of the root already found, considered as tens, and write it at the left as a second trial divisor. Find the third term of the root and complete the divisor as before.*

WHERE THE ROOT CONSISTS OF MORE THAN TWO FIGURES

EXAMPLE: Required the cube root of 34,328,125.

OPERATION

1st trial divisor $(30\times30)\times3=2700$	
$(30\times\ 2)\times3=\ 180$	34|328|125(325
$2\times2=\ \ 4$	27
1st complete divisor 2884	7328
	5768
2d trial divisor $(320\times320)\times3=307200$	1560 125
$(320\times5)\times3=\ 4800$	1560 125
$(5\times5)=\ \ 25$	
2d complete divisor 312025	

EXPLANATION.—The first two figures of the root are found by the process already explained. The third period is then brought down and the portion of the root already obtained (32) is treated as tens, and a second

trial divisor is found and completed as before. If there are other periods, they are in turn brought down and the process continued, the portion of the root already found being each time treated as tens in finding the next required figure.

FINDING APPROXIMATE ROOTS

If the number of which the cube root is sought is not a perfect cube, its root is a surd and may be found approximately by annexing successive periods of three ciphers each, the resulting root figures being decimals.

Find the approximate cube root of 635.

OPERATION

1st trial divisor $(80\times80)\times3=19200$	
$(80\times5)\times3=\ 1200$	635.000(8.59
$5\times5=\ \ 25$	512
1st complete divisor 20425	123.000
	102.125
2d trial div. $(850\times850)\times3=2167500$	20.875000
$(850\times9)\times3=22950$	19.714779
$(9\times9)\ =\ 81$	1.160221
2d complete divisor 2190531	

WRITTEN PROBLEMS

Find the cube root of each of the following perfect cubes:

24. 262144. *26.* 1860867.

25. 884736. *27.* 15252992.

Find the cube roots of the following numbers correct to one decimal place:

28. 196. *30.* 19.875.

29. 2147. *31.* 248.7.

NOTE.—In forming decimal periods, annex enough ciphers for the period to have three places. Thus, the last number should be written in this form: 248.700.

Find the cube root of the following correct to two decimal places:

32. 13. *34.* 3.568.

33. 147.

35. What is the cube root of $5\frac{7}{8}$ correct to hundredths?

SUGGESTION.—Reduce the fraction to a decimal and find the cube root of the resulting decimal number.

36. Find the cube root of $15\frac{3}{4}$, carried to two decimal places.

37. If 160103007 cubical blocks of stone, each one foot square, were placed in a cubical pile, what would be the length of each edge?

38. There are 231 cubic inches in a gallon. Find the inside dimensions, correct to tenths of an inch, of a cubical tank that would hold 1000 gallons.

39. If a bar of iron 25 ft. 4 in. long, 1 ft. 2 in. wide, and 5 in. thick, were melted and cast in the form of a cube, what would be the length of each edge? Give the answer correct to tenths of an inch.

40. Find one of the three equal factors whose product equals 373248.

NOTE.—The cube root of a perfect cube may be obtained by resolving the cube into its prime factors, and then arranging these into equal groups; the product of each root will be the required cube root.

41. What is the cube root of $\frac{343}{729}$?

SUGGESTION.—The cube root of any common fraction may be expressed by writing the cube root of the numerator over the cube root of the denominator. Solve the above problem by inspection.

42. What is one of the three equal numbers that, multiplied together, will produce 28.934443?

43. Find the dimensions of a cubical bin that will have the same capacity as a bin 8 ft. 9 in. long, 5 ft. 3 in. wide, and 6 ft. 4 in. deep. Answer correct to tenths of an inch.

METRIC SYSTEM OF DENOMINATE NUMBERS

In France, and other parts of Europe, and by scientific institutions generally, a system of denominate measures is used, which is known as the Metric System.

The system takes its name from the fact that all standards of weight and measurement are derived primarily from the unit of length called the *meter*.

Theoretically, the meter is one ten millionth part of a meridian quadrant, or the distance from the earth's equator to either pole, but owing to an error in the calculations at the time the meter was established in France, it varies slightly from this length.

The meter is somewhat longer than the English yard, being 39.374 in. It is divided into decimal divisions, decimeters (tenths), centimeters (hundredths), millimeters (thousandths), etc.

In the tables of the metric system the Latin prefixes *deci., centi., milli.,* indicate respectively *tenths, hundredths, thousandths,* while the Greek prefixes *deka, hecto, kilo,* respectively indicate *ten, one hundred, one thousand.*

The application of these prefixes will be seen in the following table of the linear units.

METRIC UNITS OF LENGTH

TABLE

1 kilometer (Km. = 1000 meters) = .621 (about $\frac{5}{8}$) statute mi.
1 hectometer (Hm. = 100 meters) = 109.4 yd.
1 dekameter (Dm. = 10 meters) = 1.988 rd.
1 meter (m) = 39.37 in.
1 decimeter (dm. = $\frac{1}{10}$ of a meter) = 3.937 in.
1 centimeter (cm. = $\frac{1}{100}$ of a meter) = .393 in.
1 millimeter (mm. = $\frac{1}{1000}$ of a meter) = .039 in.

In practice the hectometer and dekameter are rarely or never used.

The myriameter (100 kilometers, or 1000 meters) is sometimes employed to indicate long distances.

In microscopic measurements the *micron.,* or one millionth of a meter (about $\frac{1}{25400}$ of an inch), is used.

REDUCTION OF METRICAL DENOMINATE NUMBERS

The reduction of metric numbers to higher or lower terms is very simply accomplished by merely moving the decimal point to the left or to the right. This simplicity of calculation is one of the chief advantages of the system. Thus, if it be desired to change 25678 meters to kilometers, all that is necessary is to point off three places, and we have 25.678 kilometers. Conversely, kilometers may be changed to meters by simply moving the decimal point three places to the right, annexing ciphers if necessary; thus, 26.71 kilometers = 26710 meters.

To add a series of metrical quantities, all that is necessary is to place the decimal points so that all the quantities will be of the same denomination required in the answer, and then add the numbers as in addition of decimals.

EXAMPLE: How many meters in 16712 decimeters, 34127 centimeters, 29.6 kilometers, and 47625 millimeters?

OPERATION

16712 dm.	=	1671.2	meters.
29.6 Km.	=	29600.	meters.
34127 cm.	=	341.27	meters.
47625 mm.	=	47.625	meters.
		31660.095	meters. Ans.

In reducing linear metrical measurements to English standards, and *vice versa,* when exact results are required, the equivalents given in the table should be used, but for approximate calculations it is well to remember the following:

1 meter is about $3\frac{1}{4}$ ft.
1 kilometer is about $\frac{5}{8}$ of a mile.
1 centimeter is about $\frac{4}{10}$ of an inch.
1 millimeter is about $\frac{1}{25}$ of an inch.

WRITTEN EXERCISE

Use the above equivalents in solving the following practical problems:

44. The distance between two French cities is given as 122 Km. What is the distance in miles?

45. A table is 146 cm. wide and 258 cm. long; give its dimensions in inches.

46. A work on entomology gives the length of a certain beetle as 27.6 mm. What is its length expressed in decimals of an inch?

47. An American automobile manufacturer has a vehicle that can run at the rate of 46 miles an hour. A French customer writes to know its speed per hour in kilometers. What answer should be returned?

48. An account of a European battle states that the artillery was planted at a distance of 1600 meters from a given fort. What was the distance in yards?

49. The draught of a Belgian war vessel is given at 9.68 meters, her length at 126.15 m., and her beam 15.27 m. Give the dimensions of the vessel in feet and inches.

50. A Paris paper gives the speed of a new French mail train at 72.46 Km. per hour. What is the speed of the train in miles?

In the folowing problems make exact calculations, using the equivalents given in the table.

51. A Paris firm places with an American foundry an order for a cargo of steel bridge frames 17.128 m. in length. What is the required length in feet and inches, correct to hundredths?

52. A Chicago picture dealer wishes to order from Paris a frame for a canvas 42x38 inches. Required the dimensions of the frame in centimeters, correct to hundredths.

53. A bolt of imported silk is marked as containing 72.28 m. A dressmaker purchased it at $1.25 per yard. What is the amount of the bill?

54. A scientific journal gives the focal distance of a microscopic lens at 7 mm. What is the distance in decimals of an inch?

SURFACE MEASUREMENTS

The area of ordinary surfaces is expressed in the metric system in square meters, square decimeters, square centimeters, etc.

The standard unit for measurement of land is the *Are,* which is a square, each side of which is 10 meters, and its area is, therefore, equal to 100 square meters.

The *Hectare* is also used; it is a square, each side of which is 100 meters, and its area is 10000 square meters.

TABLE OF EQUIVALENTS

1 sq. cm. = .154 + sq. in.
1 sq. m. = 1.196 + (about $1\frac{1}{5}$) sq. yd.
1 are = 3.95 + (about 4) sq. rd.
1 hectare = 2.471 + (about $2\frac{1}{2}$) acres.

In calculating areas, the dimensions are multiplied together, and the result is reduced to the desired unit by moving the decimal point.

EXAMPLE: What is the area in ares, of a lot 28.2 meters wide by 65.6 meters long ?

OPERATION

28.2
65.6
1692
1410
1692
1849.92 sq. m. = 18.49 ares.

WRITTEN PROBLEMS

55. At 25 francs per are, what is the value of a lot 75 m. long and 32.6 m. wide?

56. What would the same land be worth at $100 per acre?

NOTE.—In this and subsequent problems involving French exchange, consider one franc as being equal to 19.3 cents U. S. money.

57. How many square yards in a room 4.15 m. by 6.21 m.?

58. What is the area in square inches of a piece of sheet metal 21.1 centimeters in length and 8.5 centimeters wide?

59. How many square yards of surface can be covered by a bolt of Belgian carpeting 70 cm. wide, containing 100 meters?

60. A lot 15 by 25 rods would contain how many ares?

61. A Paris paper represents that a tract of land, 521x475.5 meters, was sold at 1500 francs per hectare. Considering a hectare as $2\frac{1}{2}$ acres, how many acres of land were in the tract? What was the cost in U. S. money?

VOLUME MEASUREMENTS

In the metric system, volumes are expressed in cubic meters, cubic centimeters, etc., depending on the extent of the object to be measured.

For measuring excavations, stonework, wood, etc., the *stere,* or cubic meter, is used.

EQUIVALENTS

1 cu. centimeter = .0607 cu. in.
1 stere (cu. meter) = 1.3 cu. yards or 35.1 cu. ft.

WRITTEN PROBLEMS

62. How many steres is in an excavation 4.5 m. by 6.2 m. by 2.8 m.?

63. How many cubic feet in a piece of masonry 5.7 m. long, 2.15 m. high, and 42 cm. thick?

64. When wood is sold in the Paris market at 3 francs per stere, what is the price per cord in United States money?

65. At 17 francs per stere, what will be the cost of the coal required to fill a bin 5.16 m. long, 1.8 m. wide, and 76 cm. deep?

66. An exporter ships to France a cargo of 650 cords of tan bark which cost $11.50 per cord. The cargo was sold in Paris at 18 francs per stere. What was the gross gain?

MEASUREMENTS OF CAPACITY

The standard measurement of capacity in the metric system is the *Liter* (pronounced *leeter*), or cubic decimeter, the volume of which is 1000 cubic centimeters.

TABLE

1 kiloliter (1000 liters) = 264 gal.
1 hectoliter (100 liters) = 26.4 gal.
1 dekaliter (10 liters) = $2\frac{5}{8}$ gal. (very nearly).
1 liter (1 cubic decimeter) = 1.057 qt.
1 deciliter ($\frac{1}{10}$ of a liter) = .845 gi.

WRITTEN PROBLEMS

67. Find the capacity in dekaliters of a box-tank 2.5 m. long, 1.6 m. wide, and 2.8 m. deep.

68. A 200-liter cask of oil was purchased in Paris at 1.5 francs per liter, and retailed in New York at $1.95 per gal. Allowing $12.95 for duties and transportation, what sum was gained?

69. Find the capacity in liters (correct to hundredths) of a box 10x15x8 inches.

70. When the price of milk in Paris is 6 sous per liter, what is the price (correct to tenths of a cent) per quart in U. S. money?

NOTE.—A sou is equal to $\frac{1}{100}$ of a franc.

71. A 30-gallon cask of American alcohol is worth how much in Paris, if sold at 1.6 francs per liter?

METRIC WEIGHTS

The standard is the *gram* which is the weight of a cubic centimeter of distilled water at a temperature of greatest density, or 4 degrees centigrade (about 39 degrees Fahrenheit).

TABLE

1 tonneau (t) 1,000,000 g. or 1000 Kg. = 2204.621 lb.
1 quintal, 100000 g. or 100 Kg. = 220 lb. 7 oz.

1 kilogram (Kg.) or "kilo." 1000 g. = 2.204 practically $2\frac{1}{5}$ lb.), Avoir.

1 hectogram (Hg.) 100 g. = 3.537 oz. Avoir.

1 dekagram (Dg.) 10 g. = 154.32 gr.

1 gram (g.) = 15.432 gr.

1 decigram (dg.) $\frac{1}{10}$ of a g. = 1.54 gr.

1 centigram (cg.) $\frac{1}{100}$ of a g. = .154 gr.

1 milligram (mg.) $\frac{1}{1000}$ of a g. = .015 gr.

The units in most common use are the tonneau or "ton," the quintal, the kilo, the gram, and the centigram.

The fact that the standards of weights are derived from a given volume of water, renders the weights of given volumes of other substances most easy of computation, when specific gravity tables are used.

NOTE.—By the specific gravity of a substance is meant the ratio of its weight to that of an equal volume of pure water.

ILLUSTRATION.—The weight of a cubic foot of water is 62.5 lb. Avoir., while the weight of a cubic foot of gold is 1206.25 lb.

Dividing the weight of the gold, 1206.25, by the weight of the water, 62.5, gives 19.3, the specific gravity of the gold.

The number of cubic centimeters in the volume of any given body is equal to the number of grams in the weight of an equivalent volume of water. Hence, to find the weight of any substance, multiply its volume in cubic centimeters by its specific gravity; the result will be its weight in grams.

In solving problems of this character, the accompanying reference table of specific gravities will be found convenient:

Metal	Sp. gr.
Platinum	21.5
Gold	19.3
Mercury	13.6
Lead	11.3
Silver	10.5
Copper	8.9
Iron	7.6
Tin	7.2
Zinc	7.
Aluminum	2.7

WRITTEN PROBLEMS

72. Find the weight in pounds avoirdupois, of the water contained by a tank 2.5 m. long, 1.8 m. wide, and 1.2 m. deep.

SUGGESTION.—The number of cubic decimeters equals the weight in kilos.

73. An importer bought an invoice of 129.6 kilos of gum at 28.5 francs per kilo. After adding $28.75 for duties and transportation charges, what is the cost per Troy ounce (480 grains) in U. S. money?

74. What is the weight in Troy ounces and grains, of a bar of silver 15 cm. long, 7 cm. wide, and 42 cm. thick?

75. The dimensions of an aluminum paper weight are 11.2x 6.4x4.1 cm. Give its weight in pounds avoirdupois, correct to three decimal places.

NOTE.—There are 7000 grains in one pound avoirdupois.

76. A rectangular block of Italian marble is 85 cm. long, 56.4 cm. wide, and 35 cm. thick. Find its weight in pounds avoirdupois, the specific gravity of the marble being 4.8.

77. When wheat is quoted in the Brussels market at 21.4 francs per quintal, what is the equivalent price per bushel of 60 lb.?

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ANSWERS
TO WRITTEN PROBLEMS

Inductive Commercial Arithmethic

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1914

Answers to Written Problems

ADDITION—PAGES 19-24

1. 2660. **2.** 2509. **3.** 3639. **4.** 3665. **5.** 3449. **6.** 24,655. **7.** 6,361,729. **8.** 5,883,784. **9.** 10,292. **10.** 183,719. **11.** 237,766. **12.** 211,213. **13.** 184,108. **14.** 275,274. **15.** $3331.41. **16.** $535. **17.** 20,692 lb. **18.** 5973. **19.** 625. **20.** 650. **21.** 356,453. **22.** 44,650. **23.** $371,197. **24.** $24,006.34. **25.** 668,925,142. **26.** 287,171. **27.** 6,058,872. **28.** 52,970.48. **29.** 8,680,041. **30.** 349,320,665. **31.** 434,528,573. **32.** 196,459,835,043. **33.** $38.17. **34.** 3440. **35.** 4144. **36.** 3503. **37.** 3113. **38.** 2182. **39.** 1747. **40.** 2396. **41.** 1606. **42.** 2123. **43.** Final total $23,453.45. **44.** Total Attendance 68,591. **45.** (a) Milltown 3902; Fairview 2620; Piedmont 3585; Pine Valley 2462; Springville 4085; (b) Monday 3312; Tuesday 3492; Wednesday 2589; Thursday 3134; Friday 1721; Saturday 2406; (c) Total output 16,654.

SUBTRACTION—PAGES 27-38

1. 8267. **2.** 718. **3.** 15,197. **4.** 52,858. **5.** 893,329. **6.** 14,047. **7.** 334,438. **8.** 74,999,263. **9.** $5097.50. **10.** $25,539.83. **11.** $910.33. **12.** 2,184,234. **13.** 41,972,315. **14.** 3317. **15.** 7538. **16.** $34,804.73. **17.** $353,148. **18.** 2,891,741. **19.** 24,674,546. **20.** 62,376. **21.** $1101.60 **22.** 1215 acres. **23.** $2467.95. **24.** Wilson's balance $6594.49; Davis's balance $5780.46; Brown's balance $833.20. **25.** Total of debit and credit total columns $23,204.37; Totals of Dr. and Cr. Bal. columns $5404.82. **26.** Dec. 31, Cameron owes us $938.27; Feb. 20, Cameron owes us $1179.20. **27.** Gorman owes us $35.30. **28.** Erickson owes us $2333.06. **29.** Cash balance $1801.75. **30.** Cash balance $442.57. **31.** Cash balance $1668.06. **32.** Mdse. loss $378.28. **33.** Gain on Gladbrook Farm $1286.31. **34.** Gain on house and lot $26.16. **35.** Bal. due from J. C. Farmer $501.30. **36.** Bal. due from Chambers & Lee $9.75. **37.** Bal. due from Johnson & Stevens $647.79.

MULTIPLICATION—PAGES 44-51.

1. Five answers (1) 3856, (2) 29,784, (3) 36,964 (4) 98,252, (5) 39,072. **2.** Five answers (1) 10,230, (2) 37,435, (3) 462,535, (4) 354,530, (5) 99,335. **3.** Five answers (1) 446.28, (2) 125,436, (3) 232,320, (4) 508,614, (5) 2,232,402. **4.** Five answers (1) 300,132, (2) 628,145, (3) 341,166, (4) 635,348, (5) 410,746. **5.** Five answers (1) 77,400, (2) 55,664, (3) $781.20, (4) $1515.84, (5) $75,810.24. **6.** Five answers (1) $7.83, (2) $88.65, (3) $5378.58, (4) $88,555.14, (5) 420,822. **7.** 216,456. **8.** 542,184. **9.** 664,652. **10.** 139,482. **11.** 669,768. **12.** 699,678. **13.** $86,323.02. **14.** $250,709.67. **15.** 50,112. **16.** 45,360. **17.** 306,816. **18.** 367,416. **19.** 382,270. **20.** 3,391,152. **21.** 768,000. **22.** 1,591,600. **23.** 92,550,000. **24.** 3,120,300,000. **25.** 3,222,400,000. **26.** 20,720,000,000. **27.** 572. **28.** 1353. **29.** 288. **30.** 744. **31.** 351. **32.** 594. **33.** 378. **34.** 396. **35.** 1118. **36.** 408. **37.** 864. **38.** 594. **39.** 1008. **40.** 798. **41.** 2074. **42.** 1,022,868. **43.** 6,644,400. **44.** 5,914,048. **45.** 15,012,000. **46.** 7,022,366,120. **47.** 2,488,201,600. **48.** 19,116,000,000. **49.** 2,944,861,920. **50.** 34,202. **51.** 7216. **52.** $1414.35. **53.** $929.26. **54.** $1268.25. **55.** No. of lots 560, value $98,000. **56.** $3139.20. **57.** 247,632. **58.** $101,616. **59.** $3506.25. **60.** 41,280 rods. **61.** 76,960 lb. **62.** $4425. **63.** $3412.50. **64.** Gain $531.25. **65.** $19.55. **66.** $176.20. **67.** $79.10. **68.** $426. **69.** $15.96. **70.** $502.15. **71.** $1547.35. **72.** $11.30.

DIVISION—PAGES 60-62

1. 282—4R. **2.** 1496. **3.** 1384—2R. **4.** 1,965,311—3R. **5.** 4,152,240—1R. **6.** 1,352,549—5R. **7.** $2891.19. **8.** $45,248.05. **9.** 882,401—10R. **10.** 16,802,883. **11.** 1146—12R. **12.** 3123—19R. **13.** 70,349—132R. **14.** 1,406,084.—30R. **15.** 1,129,470—443R. **16.** 131,287—354R. **17.** 91,439—11R. **18.** 111,925—371R. **19.** 569,283—245R. **20.** 5274—3469R. **21.** 17—878R. **22.** 2030—6201R. **23.** 85—266,730R. **24.** 167—42013R. **25.** 27,403—6367R. **26.** Per minute 11,562,500 mi., per second 192,708⅓ mi. **27.** Average daily attendance 32; pupils average 16. **28.** $16.33— **29.** 19,000 bu. **30.** 124.

31. 2358 lb. **32.** 57 minutes. **33.** 795 bu. **34.** 12 da. 10 hr. **35.** 20 yr. 10 mo.

GENERAL PROPERTIES OF NUMBERS—PAGES 69-77

1. 17. **2.** 13. **3.** Prime. **4.** 36. **5.** 95 ft. long 4 ft. wide. **6.** 2640. **7.** 6300. **8.** 504. **9.** 4900. **10.** 60. **11.** 600. **12.** 720. **13.** 2520. **14.** 1125. **15.** 315. **16.** $22\frac{1}{2}$. **17.** 720. **18.** 320. **19.** $25\frac{1}{2}$. **20.** 405 lb. **21.** 96 lb. **22.** 32 bu. **23.** $6\frac{1}{4}$ pieces. **24.** 50 cents. **25.** \$50. **26.** \$1. **27.** \$4.50. **28.** \$32. **29.** 620 bu. **30.** 45 miles. **31.** \$432.60. **32.** \$37.49. **33.** $13\frac{7}{10}$ cents. **34.** \$411. **35.** 250 bu. **36.** 26 hr. 40 min. **37.** \$375. **38.** \$1050. **39.** \$312. **40.** \$40. **41.** \$5510. **42.** \$1415. **43.** 160 yd. **44.** \$135. **45.** \$16.50 **46.** 50. **47.** 105 brl. **48.** \$630. **49.** 18 bu. **50.** \$4737. **51.** 12 mi. **52.** \$12.85. **53.** \$58,850.

COMMERCIAL METHODS—PAGES 83-90

1. \$117.15. **2.** \$160.10. **3.** \$330.20. **4.** \$325.63. **5.** \$793.80. **6.** \$225.50. **7.** \$111.48. **8.** \$330.42. **9.** \$21.48. **10.** \$525.60. **11.** \$1479.41. **12.** \$1234. **13.** \$456.91. **14.** \$361.69. **15.** \$305.20. **16.** \$709. **17.** \$69.24. **18.** \$265.70. **19.** \$358.50. **20.** \$209.52. **21.** \$1944. **22.** \$95.10. **23.** \$1200. **24.** \$3.50. **25.** \$2050. **26.** \$7001.35. **27.** \$1.44. **28.** \$12.04+. **29.** \$20.10 (exact \$20.096). **30.** \$32.80 (exact \$32.80$\frac{2}{7}$). **31.** \$14. **32.** 84 lb. **33.** \$3.72 (exact \$3.7164). **34.** \$3625. **35.** \$71.55. **36.** \$610.45. **37.** \$2.25. **38.** 4000 lb. **39.** \$13. **40.** 65 cents.

COMMON FRACTIONS—PAGES 96-117

1. $26\frac{3}{4}$. **2.** $40\frac{5}{24}$. **3.** $11\frac{6}{25}$. **4.** $19\frac{31}{32}$. **5.** $28\frac{2}{7}$. **6.** $68\frac{1}{48}$. **7.** $3455\frac{1}{4}$. **8.** $74\frac{77}{125}$. **9.** $1\frac{5}{6}$. **10.** $602\frac{11}{16}$. **11.** $87\frac{7}{8}$. **12.** $5\frac{725}{1728}$. **13.** $693\frac{2}{3}$. **14.** $58\frac{23}{625}$. **15.** $28\frac{117}{160}$. **16.** $\frac{297}{16}$. **17.** $\frac{999}{8}$ ds. **18.** $\frac{2037}{16}$. **19.** $\frac{873}{7}$ wk. **20.** $\frac{219}{4}$ gal. **21.** $\frac{437}{12}$ hr. **22.** $\frac{2167}{30}$ min. **23.** $\frac{18509}{100}$ rd. **24.** $\frac{4903}{20}$ ton. **25.** $\frac{6343}{16}$ lb. **26.** $\frac{252655}{27}$ cu. yd. **27.** $\frac{505549}{60}$ min. **28.** $\frac{941009}{25}$ dollars. **29.** $\frac{9279411}{1728}$ cu. ft. **30.** $\frac{19663}{8}$ gal. **31.** $\frac{1389695}{144}$ sq. ft. **32.** $\frac{111555}{128}$ cd. **33.** $\frac{29533}{32}$ bu. **34.** $\frac{3}{8}$. **35.** $\frac{17}{20}$. **36.** $\frac{5}{8}$. **37.** $\frac{7}{8}$. **38.** $\frac{1}{80}$. **39.** $\frac{81}{160}$ sq. mi. **40.** $\frac{61}{112}$ L. Ton. **41.** $\frac{39}{40}$ A. **42.** $\frac{7}{10}$ A. **43.** $\frac{9}{20}$ mi. **44.** $\frac{165}{125}$ T. **45.** $\frac{5}{6}$ lb. **46.** $\frac{9}{200}$ lb. **47.** $\frac{3}{4}$ cu. ft. **48.** $\frac{1}{6}$ mi. **49.** $\frac{9}{80}$ L. Ton.

50. $\frac{1}{5}$ mi. **51.** $\frac{9}{20}$ sq. mi. **52.** $\frac{7}{40}$ lb. **53.** $1\frac{3}{8}$ mi. **54.** \$96.30. **55.** $\frac{1}{9}$. **56.** $\frac{19}{100}$. **57.** \$273. **58.** \$25,000. **59.** $28\frac{4}{5}$ ds. **60.** 52 mi. **61.** \$100 loss. **62.** \$247.50. **63.** \$12,000, sister's share; \$15,000, Henry's share. **64.** \$362.20. **65.** \$3378.56. **66.** $158\frac{31}{48}$. **67.** $236\frac{1}{4}$ A. **68.** $147\frac{3}{4}$ lb. **69.** $1953\frac{18}{40}$ bu.; $\$835\frac{25}{48}$. **70.** 1050 yd. **71.** $2663\frac{1}{24}$. **72.** $2512\frac{8}{15}$. **73.** $3641\frac{35}{48}$. **74.** $3667\frac{1}{5}$. **75.** $3451\frac{3}{5}$. **76.** 24,658. **77.** $183,722\frac{15}{16}$. **78.** $237,770\frac{17}{20}$. **79.** $211,216\frac{29}{75}$. **80.** $184,112\frac{31}{36}$. **81.** $275,278\frac{3}{10}$. **82.** $9\frac{5}{7}$. **83.** $125\frac{1}{9}$. **84.** $137\frac{29}{32}$. **85.** $152\frac{9}{16}$. **86.** $70\frac{1}{8}$. **87.** $5\frac{19}{24}$. **88.** $41\frac{3}{14}$. **89.** $228\frac{7}{17}$. **90.** $8\frac{11}{12}$. **91.** $12\frac{13}{16}$. **92.** $60\frac{41}{56}$. **93.** $4\frac{23}{48}$. **94.** $56\frac{11}{12}$. **95.** $51\frac{11}{12}$. **96.** $50\frac{281}{640}$. **97.** $88\frac{47}{112}$. **98.** $121\frac{1}{48}$. **99.** $22\frac{1}{8}$ gal. **100.** $186\frac{11}{16}$ acres. **101.** $547\frac{1}{2}$. **102.** 1356. **103.** 903. **104.** 6150. **105.** $227\frac{1}{4}$. **106.** $1816\frac{1}{14}$. **107.** 2170. **108.** $156\frac{1}{4}$. **109.** $650\frac{1}{4}$. **110.** $1290\frac{3}{20}$. **111.** $412\frac{1}{2}$. **112.** $687\frac{1}{2}$. **113.** $2772\frac{9}{20}$. **114.** $1251\frac{1}{4}$. **115.** $2738\frac{21}{32}$. **116.** $\$147.80\frac{1}{2}$. **117.** $\$271\frac{3}{8}$. **118.** $\$213.15\frac{3}{4}$. **119.** $\$171\frac{7}{8}$. **120.** $\$153\frac{9}{16}$. **121.** $\$81\frac{7}{8}$. **122.** \$49.45. **123.** $\$3.24\frac{37}{32}$. **124.** $\$16.87\frac{1}{2}$. **125.** $\$30.18\frac{3}{4}$. **126.** $\$69.42\frac{21}{32}$. **127.** $\$106.78\frac{1}{8}$. **128.** $112\frac{17}{32}$. **129.** $\$369\frac{37}{32}$. **130.** $21\frac{1}{3}$. **131.** $27\frac{3}{7}$. **132.** $43\frac{1}{5}$. **133.** 128. **134.** $19\frac{5}{31}$. **135.** $4\frac{1}{5}$. **136.** $45\frac{5}{6}$. **137.** $4\frac{2}{3}$. **138.** 8. **139.** $5\frac{5}{7}$. **140.** 16. **141.** 45. **142.** 14. **143.** 20. **144.** 720. **145.** $248\frac{8}{9}$. **146.** 126. **147.** $27\frac{3}{7}$. **148.** $10\frac{1}{2}$. **149.** $9\frac{3}{5}$. **150.** $91\frac{11}{15}$. **151.** $1\frac{1}{9}$. **152.** $\frac{3}{4}$. **153.** 2. **154.** $\frac{1}{2}$. **155.** $1\frac{1}{8}$. **156.** $\frac{9}{40}$. **157.** $\frac{3}{4}$. **158.** $2\frac{1}{4}$. **159.** $2\frac{2}{3}$. **160.** 4. **161.** $8\frac{3}{4}$. **162.** $33\frac{1}{3}$. **163.** 20. **164.** $10\frac{13}{16}$. **165.** $1\frac{1}{14}$. **166.** $1\frac{4}{11}$. **167.** 2. **168.** $3\frac{1}{3}$. **169.** $2\frac{1}{2}$. **170.** 3. **171.** $1\frac{309}{500}$. **172.** $\frac{39}{50}$. **173.** $\frac{94}{131}$. **174.** $\frac{113}{170}$. **175.** $\frac{3723}{5583}$. **176.** $\frac{41}{300}$. **177.** (a) $\frac{855}{976}$, (b) $\frac{433}{877}$, (c) $\frac{64}{75}$. **178.** (a) $\frac{2}{363}$, (b) $1\frac{7}{11}$, (c) $\frac{555}{664}$. **179.** $142\frac{17}{24}$. **180.** $19\frac{1}{8}$. **181.** $67\frac{19}{88}$. **182.** $28\frac{13}{16}$. **183.** $168\frac{3}{4}$. **184.** $9\frac{1455}{4864}$. **185.** $110\frac{5}{8}$. **186.** $20\frac{5}{6}$. **187.** $\$82\frac{7}{8}$. **188.** $8\frac{1}{2}$ds. **189.** $201\frac{3}{5}$ cu. in. **190.** $\$206\frac{1}{4}$. **191.** $\frac{13}{60}$. **192.** $79\frac{9}{176}$. **193.** $17\frac{17}{18}$ cds. **194.** $\$312\frac{1}{2}$. **195.** \$3000 cost of lot, \$4500 cost of house. **196.** \$57,600 total investment, \$19,200 in real estate, \$28,800 in bonds, \$7200 in railway stock. **197.** \$8 loss. **198.** $\$32.06\frac{1}{4}$. **199.** $\$12,389\frac{55}{64}$. **200.** $\$2057\frac{1}{7}$, C's money; $\$5142\frac{6}{7}$, B's money. **201.** $\$949\frac{1}{976}$. **202.** $\$15\frac{5}{8}$. **203.** $156\frac{3}{5}$ bu. **204.** $100\frac{35}{32}$. **205.** 80ds. **206.** $50\frac{13}{60}$. **207.** $\$21\frac{1}{176}$. **208.** $19\frac{1}{2}$ Tons. **209.** $\$17\frac{3}{8}$. **210.** \$60,000. **211.** $10\frac{41}{61}$ cd. **212.** $\$111.96\frac{21}{64}$. **213.** $24\frac{1}{4}$ mi. **214.** $\$193\frac{1}{2}$. **215.** $387\frac{109}{768}$ lb.

DECIMALS—PAGES 119-135

1. .21. **2.** .00007. **3.** .327. **4.** .3028. **5.** .000401. **6.** 106.054. **7.** 40.00019. **8.** .000008$\frac{1}{3}$. **9.** 12.00$\frac{1}{5}$. **10.** .12$\frac{1}{6}$. **11.** 42.0204. **12.** 200.000004$\frac{1}{3}$. **13.** .502. **14.** 500.002. **15.** 2100.17. **16.** 89.09006. **17.** $\frac{4}{25}$, $\frac{1}{8}$, $\frac{19}{20}$, $\frac{1}{400}$, $\frac{3}{40}$, $\frac{9}{625}$, $\frac{1}{160}$, $\frac{18}{625}$. **18.** .6000, .1500, .1250, .0100, .0040, .5670, .0010, .0070. **19.** .00570, .90000, .12345, .65000, .00900, .00560, .00016. **20.** .86, .0012, .7, .85, .0001, .0101. **21.** $\frac{451}{600}$. **22.** $\frac{13}{150}$. **23.** $\frac{9}{1000}$. **24.** $\frac{6}{7}$. **25.** $\frac{7}{12}$. **26.** $\frac{1}{140}$. **27.** $\frac{2}{15}$. **28.** $\frac{89}{90}$. **29.** $\frac{2}{3}$. **30.** $\frac{37}{300}$. **31.** $\frac{11}{40}$. **32.** $\frac{1}{45}$. **33.** Exact decimals, $\frac{17}{25}$, $\frac{18}{24}$, $\frac{49}{64}$, $\frac{51}{85}$, $\frac{33}{44}$, $\frac{36}{60}$, $\frac{71}{80}$, $\frac{29}{40}$, $\frac{129}{500}$, $\frac{19}{250}$, $\frac{11}{16}$, $\frac{23}{32}$, $\frac{52}{65}$. Inexact decimals, $\frac{29}{75}$, $\frac{17}{128}$, $\frac{41}{96}$, $\frac{59}{60}$, $\frac{30}{36}$. **34.** .7142+, .8461+, .6521+, .4166+, .1666+, .3544+, .5201+, .6166+. **35.** .666+, .6363+, .857142+, .384615+, .2+, .416666+. **36.** .05166+, 13.70909+, .00125, 9.00071+, .14083+, .78571+. **37.** $\frac{2}{15}$, $\frac{1}{120}$, $2\frac{19}{25}$, $\frac{33}{2000}$, $\frac{1}{40}$, $\frac{78}{625}$, $\frac{1}{16}$, $\frac{32}{125}$, $1\frac{7}{25}$. **38.** .78$\frac{4}{7}$, .64$\frac{4}{9}$, .42$\frac{6}{7}$, .38$\frac{6}{13}$, .91$\frac{13}{17}$, .70$\frac{10}{11}$, .64$\frac{12}{17}$, .95$\frac{5}{6}$. **39.** .73208+, .68+, .684+, .684210+. **40.** .848+. **41.** .158, .079, .0158. **42.** 340.912962. **43.** 228.845495. **44.** 69.21875. **45.** $58.875. **46.** 4.4375. **47.** 3.0863. **48.** 217.53125 A. **49.** 17.296. **50.** 3.0895. **51.** 31.041975. **52.** .024975. **53.** $11.127. **54.** .099999. **55.** 13.899898. **56.** 35.03125 yds. **57.** 1424.25 Tons. **58.** 11.14066$\frac{2}{3}$. **59.** 128.75625 A. **60.** $207.778725, written $207.78. **61.** $175. **62.** $6.974. **63.** .8983+. **64.** $8206.1975, written $8206.20. **65.** 2010.624+ft. **66.** 302.1165 mi. **67.** 398.94528 ft. **68.** $3642.57525. **69.** $59,440.02825, written $59,440.03 **70.** 184.56288, written 185. **71.** 232.2 gr. **72.** 8.793+. **73.** 9.187+. **74.** .6111+. **75.** 11.307+. **76.** 2.321+. **77.** 11.904+. **78.** 1.928+. **79.** 1.159+. **80.** .381+. **81.** .016+. **82.** .7875. **83.** .3297+. **84.** .6448+. **85.** .9506+. **86.** 3.375. **87.** .38. **88.** 126. **89.** 235,750. **90.** .0017+. **91.** 127,000. **92.** .144. **93.** 360,400. **94.** 3.05. **95.** .0138+ **96.** .3125. **97.** .00042. **98.** 21.286+. **99.** 267,900. **100.** .0457. **101.** $22.95. **102.** $250.096, written $250.10. **103.** $82.18+. **104.** $4866. **105.** $24.42. **106.** $3.39. **107.** $6.25. **108.** $5.029+. **109.** 612.244+lb. **110.** 10,066+ft. **111.** $96.25. **112.** 5145.955106. **113.** 42,261.727035. **114.** .875, .5625, .09375, .109375. **115.** $\frac{1}{8}$, $\frac{1}{16}$, $\frac{7}{800}$, $\frac{1}{15}$, $\frac{1}{90}$. **116.** .999999. **117.** 40.6092.

118. 900, 9000, 900000. **119.** $100. **120.** $877.50. **121.** $2500, profit first year; $2812.50, profit second year. **122.** $7500, real estate; $4000, bonds; $5000, stocks. **123.** $6.375. **124.** $1200, B's money; $2700, total. **125.** $3000. **126.** $6120. **127.** 40 cents. **128.** $7678.50, wife's share; $5119, daughter's share. **129.** $5.426+. **130.** $5\frac{5}{6}$. **131.** .613+. **132.** .15. **133.** $7056.05. **134.** .133+ cu. ft.; 7.480+ gallons. **135.** 7.0686+ sq. ft. **136.** 1.610+ kil. **137.** $55.687+. **138.** 6796.875 lb. **139.** 1.719+. **140.** .047+. **141.** .1297+. **142.** .914+. **143.** .8391+, developed; .1609—, lost. **144.** .7835+. **145.** 10 per cent. **146.** 1362.94+ feet. **147.** .72197+. **148.** $566.46.

DENOMINATE NUMBERS—PAGES 139-156.

1. 2.951+ mi. **2.** 80 ch. **3.** 25.0125 mi. **4.** 400 rd. per minute, 110 ft. per second. **5.** 825 ft. **6.** 92 ft. **7.** 69 mi. **8.** $324. **9.** 88.64 rds. **10.** $59\frac{3}{5}\frac{1}{3}$ rd.; $\frac{41}{2}\frac{1}{6}$ mi. **11.** 924 sq. in. **12.** 15,600 acres. **13.** $60. **14.** 1.95+ tons. **15.** 189$\frac{7}{8}$ sq. ft. **16.** $152,460. **17.** $28,888+ written $28.89. **18.** 841 sq. in. **19.** $2.50. **20.** $696.96. **21.** $134.89+. **22.** 67.32+ gal. **23.** $29.86\frac{2}{3}$ written $29.87. **24.** 13,750 lb. **25.** 70.3125 cu. in. **26.** 420 cu. ft. **27.** 28.42+ hr. **28.** 364.58+ lb. **29.** $3128.888+ written $3128.89. **30.** 2975 cu. ft. **31.** 22.85+ gal. **32.** 4.59+ bu. **33.** .969+. **34.** 167.860+ brl. **35.** $8.02+. **36.** $28.50. **37.** 194.99+ bu. **38.** .8035+. **39.** 7.48+ gal. **40.** 9.309+ gal. **41.** 19.140+ oz. troy. **42.** 1560 gr. = 65 pwt. **43.** $80.31. **44.** $14.40. **45.** $41,100. **46.** .733+. **47.** $116.40. **48.** $31\frac{9}{2}\frac{9}{5}$ cents. **49.** $33.79+. **50.** $1413.48+. **51.** 27.61+ A. **52.** 5 feet 7.79+ inches. **53.** 14.66+ ft. **54.** $36.69. **55.** 8.104 sq. ft. **56.** 61 gal. 2 qt. 1 pt. **57.** 64 T. 1532 lb. **58.** 94 yd. 2 ft. 8 in. **59.** 5 gal. 1 qt. 1 pt. 2 gi. **60.** 12 rd. 3 yd. 3 in. **61.** 52 yr. 22 da.; 56 yr. 2 mo. 3 da.; 4 yr. 1 mo. 11 da. **62.** $5627 salary. **63.** 59 yr. 9 mo. 25 da. **64.** 3 yr. 10 mo. 3 da. **65.** 3 yr. 10 mo. 16 da. **66.** 4 yr. 8 mo. 9 da. **67.** 4 yr. 10 mo. 16 da. **68.** 4 yr. 5 mo. 23 da. **69.** 2 yr. 3 mo. 20 da. 2 hr. 16 min. **70.** 12 yr. 8 mo. 19 da. 11 hr. 52 min. **71.** 4 yr. 2 mo. 10 da. 10 hr. 33 min. **72.** 114 da. **73.** 346 da. **74.** 277 da. **75.** 92 da. **76.** 4 lb. 10 oz.

13 pwt. 9 gr. **77.** 272 rd. 1 yd. 1 ft. **78.** 1101 lb. 9 oz. **79.** 25 gal. 1 qt. $\frac{1}{2}$ pt. **80.** 72 rd. 3 ft. $6\frac{1}{5}$ in. **81.** 1 bu. 2 pk. 3 qt. $1\frac{1}{10}$ pt. **82.** Using table for months and days, 9 mo. 18 da. 12 hr. 2 min. 1 sec.; Using 365 days for the year, 292 da. 12 hr. 2 min. 1 sec. **83.** 1 rl. 17 gal. $1\frac{3}{25}$ pt. or 48 gal. 2 qt. $1\frac{3}{25}$ pt. **84.** 8 cwt. 57 lb. $3\frac{1}{5}$ oz. **85.** 23 rings — 26 gr. remainder **86.** 47 bottles—1 qt. $\frac{1}{4}$ pt. remainder. **87.** 18 rd. 4 yd. 1 ft. $7\frac{1}{17}$ in. **88.** $63\frac{15}{43}$ mi. **89.** 240 rd. **90.** \$109.779+ written \$109.78. **91.** 26.85 + sq. yd. **92.** 81. **93.** \$4356. **94.** \$19.29+. **95.** 552.92 + cu. in.

COMPUTATIONS IN THE TRADES—PAGES 158-185.

In solving the problems involving lumber calculations, we have followed the rule for fractions that is usually observed by lumber dealers, namely: If a quantity of lumber includes a fraction of ½ foot or more, it is considered a foot; if less than ½ foot it is dropped before multiplying by the price.

1. 600 ft. **2.** \$36.17. **3.** \$4314.27. **4.** \$699.60. **5.** \$223.43. **6.** \$40.99. **7.** \$21.92. **8.** \$25.31. **9.** \$100.43. **10.** \$12.67. **11.** \$21.56. **12.** 19 + cents. **13.** \$9.62. **14.** \$125.13. **15.** \$24.49. **16.** \$1382.09. **17.** \$25.42. **18.** 87 cents less Ax. **19.** Second bid lower \$4.15. **20.** \$242.40. **21.** \$87.74. **22.** Plastering \$38.38, calcimining \$7.68. **23.** First coat \$90.47. second coat \$75.39. **24.** \$22.32. **25.** \$101.93, cost paper; \$79.20, cost hanging. **26.** \$27.50. **27.** 960 cu. ft. **28.** \$243. **29.** $72\frac{8}{11}$ perches. **30.** 14.72 cords. **31.** \$80.81. **32.** \$109.09. **33.** \$442.91. **34.** \$435.05. **35.** \$4090.84. **36.** \$4707.68. **37.** \$309. **38.** \$21.54. **39.** \$123.40. **40.** \$22.97. **41.** \$18.07. **42.** \$510. **43.** $1922\frac{9}{10}$ bu. **44.** \$571.73. **45.** 43.68 bu. **46.** \$179.39. **47.** \$44.93. **48.** 257,280 bu. **49.** \$43.25. **50.** 47,856 brl. **51.** 10 hr. 58 min. **52.** 2 hr. 24 + min. **53.** 63,677 bu. **54.** 110 brl. **55.** 406 qts. (not corrected). **56.** 331 brl. **57.** \$5.22. **58.** 65 cents. **59.** First car 43,756 lb.; second car 46,249 lb.; third car 44,568 lb.; freight \$50.49. (Estimate freight on each car separately.) **60.** \$6.93. (The fraction is considered on each ticket separately.) **61.** \$43.49. **62.** \$17.75. **63.** \$5.65. **64.** \$11.50. **65.** \$13.17. **66.** \$15.30. **67.** Freight cheaper 15 cents. **68.** 25 cents. **69.** 10 cents. **70.** 23 cents. **71.** \$1.21. **72.** 8 cents. **73.** 42 cents. **74.** \$7. **75.** 23,040 acres. **76.** 960 acres. **77.** \$12,245. **78.** 200 acres.

79. 520 acres. **80.** Total cost \$16,000; gain on four sections sold \$2680. **81.** \$180.69. **82.** \$131.27. **83.** \$19,134.50. **84.** \$99,637.68+. **85.** 524.368 acres. **86.** 3.57 A. **87.** 572.4 A **88.** 28.21 lb. **89.** Inch pressure at bottom 6.076 lb.; pressure per foot at bottom 875 lb.; total pressure at bottom 68,722½ lb.; average inch pressure on sides 3.038 lb. **90.** Total pressure 18,750 lb.; inch pressure at bottom 5.208 lb. **91.** 6313.88 lb. **92.** Per square foot 21,875 lb.; per square inch 151.9 + lb. **93.** 5034.4 lb. **94.** 262.5 lb. **95.** 1201.6 + lb. **96.** 151.171 + lb. **97.** 28.31 + lb. **98.** 2.285 + spec. gravity. **99.** 14.745 + lb. **100.** 71°—35′ difference in latitude; 8°—55′ difference in longitude. **101.** 272.03 mi. **102.** 1314.04 mi. **103.** 1 hr. 26 min. 40 sec.

PERCENTAGE—PAGES 188-196.

1. Real Estate \$4320; bank stock \$6480; city lots \$6480. **2.** \$225. **3.** \$3562.50. **4.** \$11,760. **5.** \$3784. **6.** \$70. **7.** \$1890. **8.** \$4500. **9.** 476 bu. **10.** \$1760 gain. **11.** \$10,500. **12.** Germans 20%; Russians 32%; Austrians 48%. **13.** First day 33⅓%; second day 25%; third day 41⅔%. **14.** 250 brl. left; 50% sold. **15.** 20% lost. **16.** 37½% or 204 brl. left. **17.** A's capital 40%; B's capital 60%. A's money 66⅔% of B's; B's money 150% of A's. **18.** 25%. **19.** Personal property 53⅓% of real estate; real estate 187½% of personal property. **20.** 25%. **21.** \$2918.25. **22.** \$6000. **23.** First farm \$4200, second farm \$5208. **24.** \$33,000. **25.** Sales this year \$4800; sales last year \$4000. **26.** Machinery \$20,000; building \$12,000; stock \$40,000. **27.** \$47,200. **28.** \$10,800. **30.** 60 sheep. **31.** 4⅕%. **32.** 79⅛%. **33.** 7½%. **34.** 37½% left; 240 acres left. **35.** \$1485. **36.** \$28,000. **37.** \$30,000. **38.** 80%. **39.** \$4800. **40.** Pear trees 900; apple trees 1980. **41.** A's money 66⅔% of B's; B's money 150% of A's. **42.** Profits first year \$7500; profits first year 33⅓% greater than second year. **43.** Younger son \$16,250; elder son \$11,375. **44.** B's asking price \$7390; A's asking price \$11,085. **45.** \$9000 **46.** \$360. **47.** \$9000. **48.** \$50,203.13. **49.** \$7893. **50.** \$9048.

COMMERCIAL DISCOUNTS—PAGES 200-207.

1. $46.17. **2.** $31.40. **3.** $27.02. **4.** $6.14. **5.** $38.10. **6.** $87.55. **7.** $11.97. **8.** $11.49. **9.** $51.47. **10.** $28.45. **11.** 73%. **12.** $54\frac{2}{5}$%. **13.** $37\frac{525}{000}$%. **14.** 76%. **15.** $59\frac{1}{2}$%. **16.** $40\frac{15}{100}$%. **17.** $506.25. **18.** $194.40. **19.** $300. **20.** $400. **21.** $143.43. **22.** $188.03. **23.** $336.80. **24.** $428.40. **25.** $192.25. **26.** $106.70. **27.** $136.80. **28.** $14.20. **29.** $34.02. **30.** $155.52. **31.** $36.19. **32.** $128.30. **33.** $319.34. **34.** $219.90. **35.** $118.12. **36.** $243.72. **37.** $76. **38.** $683.48. **39.** $946.17. **40.** $456.30. **41.** $139.50. **42.** $835.13. **43.** Net, $1290.73; net cash, $1213.29; Sept. 2, $1226.19; Sept. 14-23, $1239.10. **44.** $580. **45.** June 1, $148.81; July 6, $157.08; July 20, $165.35. **46.** $700. **47.** $555.56. **48.** $38.14. **49.** $88.24. **50.** $2.78 in favor of seller. **51.** $451.16. **52.** $247.86. **53.** $36. **54.** Net, $355.69; Mar. 10, $341.46. **55.** Net, $2647.30; Cash, $2514.93; Mar. 11, $2594.35. **56.** $897.30. **57.** Net, $223.24; May 1, $218.93. **58.** $221.86. **59.** $1191.04. **60.** $58\frac{4}{6\frac{2}{3}}$%. **61.** $133.33 **62.** $46.87.

LOSS AND GAIN—PAGES 210-215.

1. 20%. **2.** $11\frac{1}{9}$%. **3.** 205%. **4.** $11,458.33. **5.** $33,250. **6.** $800 loss. **7.** Neither gain nor loss. **8.** $17\frac{3}{8}$%. **9.** $5\frac{5}{8}$ cents. **10.** $33\frac{1}{3}$%. **11.** $4\frac{1}{6}$%. **12.** $1.65. **13.** $8\frac{1}{1\frac{1}{8}}$%. **14.** 95 yds. **15.** $6\frac{1}{4}$%. **16.** $5696. **17.** $7.33\frac{1}{3}$. **18.** Flour $1540; Corn $2002. **19.** $10,800. **20.** 25%. **21.** $37\frac{1}{2}$%. **22.** $11\frac{4\frac{1}{2}}{8}$%. **23.** 20%, **24.** $40. **25.** $2187.50. **26.** $2\frac{1}{4}$% loss. **27.** $2000. **28.** $4.37\frac{1}{2}$. **29.** First lot $3000; second lot $3000. **30.** $17,600.

31. $\frac{\text{h a s}}{\text{o w e}}$ **32.** $\frac{\text{r d s}}{\text{e o e}}$ **33.** $\frac{\text{m o}}{\text{a b}}$ **34.** $\frac{\text{r o}}{\text{h o}}$ **35.** $\frac{\text{h m s}}{\text{o b p}}$ **36.** $\frac{\text{h d x}}{\text{n s a}}$

37. $\frac{\text{r o}}{\text{b a}}$ **38.** $\frac{\text{r m s}}{\text{w a e}}$ **39.** $\frac{\text{h m s}}{\text{o t e}}$ **40.** $\frac{\text{a d s}}{\text{w o b}}$ **41.** $\frac{\text{a x m}}{\text{w p t}}$ **42.** $\frac{\text{n o e}}{\text{r s p}}$

INTEREST—PAGES 219-236.

1. $648. **2.** $384. **3.** $283.50. **4.** $42.12. **5.** $67.20. **6.** $134.61. **7.** $118.47. **8.** $15.83. **9.** $15.82. **10.** $187.50. **11.** $75. **12.** $690. **13.** $497. **14** $392. **15.** $164.80. **16.** $37.35. **17.** $100.28. **18.** $500. **19.** $157.50. **20.** $392.

21. $1.52. **22.** $5.50. **23.** $2.83. **24.** $2.76. **25.** $5.91.
26. $3.17. **27.** $7.20. **28.** $1.14. **29.** 48 cts. **30.** 69 cts.
31. $8.82. **32.** $12. **33.** $5.12. **34.** $25.83. **35.** $35.13.
36. $11.20. **37.** $23.21. **38.** $55.20. **39.** $7.88. **40.** $1.44.
41. $11.68. **42.** $6.44. **43.** $14.73. **44.** $16.31. **45.** $9.38.
46. $9.03. **47.** $4.43. **48.** $2.59. **49.** $3.37. **50.** $2.87.
51. $5.25. **52.** $7.70. **53.** $36.40. **54.** $2.38. **55.** $2.50.
56. $14.51. **57.** $21.02. **58.** $25.71. **59.** $23.60. **60.** $14.79.
61 $12.17. **62.** $93.54. **63.** $150.19. **64.** $65.56 **65.** $9.10.
66. $188.86. **67.** $36.09. **68.** $112. **69.** $76.30. **70.** $160.53.
71. $282. **72.** $160.23. **73.** $17.74. **74.** $9.11. **75.** $40.71.
76. $348.29. **77.** $74.05. **78.** $258.84. **79.** $263.33.
80. $454.88. **81.** $865.97. **82.** $323.73. **83.** $1303.79.
84. $1597.27. **85.** $2648.79. **86.** $471.66. **87.** $972.60.
88. $913.46. **89.** $2687.85. **90.** $1291.77. **91.** $4725.45.
92. $891.20. **93.** $2588.80. **94.** $722.55. **95.** $647.84.
96. $1434.30. **97.** $579.06. **98.** $869.10. **99.** $377.63.
100. $489.12. **101.** $10.10. **102.** $34.11. **103.** $69.52.
104. $11.11. **105.** $25.49. **106.** $4.04. **107.** $6.09 **108.** $4.74.
109. $32.92. **110.** $83.61. **111.** $1530. **112.** $709.70.
113. $4428.20. **114.** $1551.22. **115.** $5401.56 **116.** $4877.40.
117. $55,627.50. **118.** $46,632.78. **119.** $1599.94.
120. $2499.999+ written $2500. **121.** $2600. **122.** $6675.00+.
123. $7200. **124.** $3600. **125.** $2400. **126.** $2684.91.
127. $6615.77. **128.** $1919.68. **129.** $2765.82. **130.** $3122.36.
131. $5852.18. **132.** $5034.96. **133.** $6400. **134.** $7200.
135. $3600. **136.** $2553.10. **137.** $48. **138.** $673.20.
139. In six months worth $135.92 more than cash. **140.** $826.77.
141. 6%. **142.** 7%. **143.** 6%. **144.** $3\frac{1}{2}$%. **145.** 6%. **146.** 4%.
147. 9%. **148.** 6%. **149.** 6%. **150.** 9%. **151.** 9%. **152.** 1 yr. 6 mo. **153.** 1 yr. 6 mo. **154.** 1 yr. 1 mo. 1 da. **155.** 2 yr. 6 mo.
156. 2 yr. 6 mo. **157.** 2 yr. 9 mo. 20 da. **158.** 3 yr. 7 mo. 6 da.
159. 2 yr. **160.** 3 mo. **161.** 3 yr. 2 mo. 17 da. **162.** 4 mo. maturing Dec. 1.

COMMERCIAL PAPERS AND DISCOUNTS—PAGES 240-245.

1. June 15; 60 days. **2.** Aug. 3; 92 days. **3.** Aug. 14; 90 days. **4.** Dec. 14; 183 days. **5.** Nov. 25; 113 days. **6.** Nov. 3; 63 days. **7.** Dec. 23; 69 days. **8.** June 29, 1910; 187 days. **9.** Mar. 1, 1911; 91 days. **10.** Sept. 15, 1911; 184 days. **11.** $709.20. **12.** $973.19. **13.** $922.02. **14.** $1253.38. **15.** $755.36. **16.** $2380. **17.** $5615.63. **18.** $418.32. **19.** $853.64. **20.** $920.41. **21.** $844.41. **22.** $1297.97. **23.** $757.61. **24.** $857.73. **25.** 682.48. **26.** $499.70. **27.** $1257.80. **28.** $5268.22. **29.** $3388.64. **30.** $2078.72. **31.** $4924.59. **32.** $5141.17. **33.** $9915. **34.** $24,841.67. **35.** $585.80. **36.** $462 and $1012. **37.** $3184.58. **38.** $1227.34. **39.** $3369.63. **40.** $4996.34. **41.** $2084.69. **42.** $263.42. **43.** $190.36. **44.** $974.62. **45.** $4602.27.

COMMISSION—PAGES 247-256.

1. $552.63. **2.** $2723.65; Col. Com. $173.85. **3.** $1415. **4.** $3\frac{1}{3}\%$. **5.** $24,978. **6.** $450.69. **7.** $13,459.40. **8.** $148.12. **9.** Com. $226.38; Net Proceeds, $3889.62. **10.** $209.95. **11.** $197.13. **12.** $116.05. **13.** $74.82. **14.** $1182.35. **15.** $39,366.25. **16.** 5434.17. **17.** $183.16 gain. **18.** 1554.97. **19.** $37.50. **20.** 2%. **21.** $3825. **22.** $1650. **23.** $2\frac{1}{2}\%$. **24.** $1250. **25.** $700. **26.** 50%. **27.** $197.38. **28.** $227.78. **29.** $32.16. **30.** Car loads 44; Unexpended balance, $200; Total commission $986. **31.** $1732.75. **32.** $1282.41. **33.** $1060.97. **34.** $80.17.

STOCKS AND BONDS—PAGES 264-271.

1. $664.46. **2.** Check, $3392.04; loss, $4107.96. **3.** $32,310. **4.** $2199.44. **5.** $68\frac{1}{2}$. **6.** $461.12\frac{1}{2}$. **7.** $289. **8.** Brokerage considered, $56.25 gain. **9.** $515.87. **10.** $474.68. **11.** $80. **12.** $1625. **13.** $640. **14.** Gain, $24; relative loss, $38.50. **15.** Gain $381.24. **16.** $35,555.56. **17.** 3.31 + %. **18.** $9000. **19.** Am. Tob. 4's, $55. **20.** Face, $10,000; Annual income, $350; Rate of interest $3\frac{1}{2}\%$. **21.** $42000. **22.** $9333.33. **23.** Increased $318.44. **24.** 6.54 + %. **25.** 4.59 + %. **26.** 5.79 + %. **27.** 5.67 + %. **28.** 5.27 + %. **29.** 4.16 + %. **30.** 4.63 + %. **31.** 3.82 + % **32.** 7.63 + %. **33.** 4.08 + %. **34.** 2.92 + %. **35.** 3.26 + % **36.** 4.06 %. **37.** 5.49 + %. **38.** 3.98 + %.

MONEY AND EXCHANGE—PAGES 277-289.

1. $18,663.90. **2.** $13,241.30. **3.** $32,159.36. **4.** Draft cheaper, $1.90. **5.** $16,721.23. **6.** $42.30. **7.** Fr. 654.92. **8.** £49 16s 9d. **9.** $276.17. **10.** £36 2s 3d 2far. **11.** $255.49. **12.** $19.95. **13.** Lira 693.23. **14.** $1,587.68. **15.** Kr. 1,038.8. **16.** R 511.48, $263.41. **17.** $762.07. **18.** Lira 18,046.87. **19.** $3,491.12. **20.** Fr. 25,793.75. **21.** $1,072.62. **22.** £995 13s 9d 1 far. **23.** Fr. 12,889.84. **24.** $161.59. **25.** $121.70. **26.** $76.25. **27.** $178.25. **28.** $1.70. **29.** $2,906.32. **30.** 28c. **31.** $525.94. **32.** Yen. 1,719.90. **33.** $261.20. **34.** $7.25. **35.** $12,346. **36.** $12,334. **37.** Chicago above par in New York, New York below par in Chicago. **38.** 35c. **39.** $60. **40.** $8,655.72. **41.** $90.48. $375.63. **42.** $1,297.67. **43.** Duty, $64.80; Cost, $178.68. **44.** $6,630. **45.** $628.53. **46.** $229.91. **47.** Duty, $5,498.25; Total cost, $16,411.37. **48.** Drawback less 1% $1,779.31.

FINANCIAL SETTLEMENTS—PAGES 292-306.

1. $655.56. **2.** $503.52. **3.** $1,822.43. **4.** $6,217.90. **5.** $545.43. **6.** $1,948.52 **7.** $5,162.50. **8.** $1896.03. **9.** $1,220.82. **10.** $760.67. **11.** $136.66. **12.** $3,081.22. **13.** $212.70. **14.** 3 mo. 14 da. **15.** Aug. 30, 1910. **16.** Oct. 16. 1910. **17.** Mar. 25, 1910. **18.** Feb. 27. **19.** May 13. **20.** May 7, **21.** Sept. 15. **22.** June 6. **23.** Oct. 13. **24.** May 15. **25.** Aug. 4. **26.** Oct. 25, 1909. **27.** Aug. 30, 1909. **28.** Mar. 10, 1909. **29.** Apr. 9, 1910. **30.** $348.31. **31.** $217.34. **32.** $348.89. **33.** $949.93. **34.** $807.90. **35.** $1,458.88; (2) $1,463.83. **36.** $3,424.06; (2) $3,433.86. **37.** $1,454.01; (2) $1,458.97. **38.** $3,015.93; (2) $3,023.10. **39.** $122.20. (2) $122.60. **40.** $726.72; (2) 729.28. **41.** $5,000.26, Loss $56; (2) $5,071.72. Loss $33.32.

PARTITIVE SETTLEMENTS—PAGES 309-323.

1. Gain $2560. **2.** $1100. **3.** $2580. **4.** $10460. **5.** $12,277.50. **6.** Net gain $2890; H's $7,445; I's $8,945. **7.** J's $5,166.67, K's $10,333.33. **8.** K's $4,900; L's $5,600. **9.** L $4,230; M $5,170; N $6,110. **10.** M $4,695.12; N $5,121.95;

P $7,682.93. **11.** N $5,028; P $6,285; Q $5,447. **12.** P $6,000; Q $8,000; R $12,000. **13.** R $4,553.57; S $3,946.43 **14.** A $800; B $1,200; C $1000; **15.** D $2,177.43; E $1461.61; F $120.96. **16.** G's gain $2,066.67; H's gain $2,066.66; G to pay H $2966.66. **17.** H $13,714.29; I $13,885.71. **18.** L's $7.001.25; M's $6,998.75. **19.** O $120.42; N $157.08. **20.** R pays $15, and P $10.50 to Q. **21.** S $3,685; T $3,685; U $3,235; V $15,570. **22.** A receives $4329.22; D receives $2963.91; B pays $3336.49; C pays $1179.87; E pays $2776.77. **23.** G $5626.45; H $6430.22. **24.** A's $2,181.90; B's $1,974.10; C's $1,039. **25.** A $3,334.39; B $2,942.11. **26.** A Net profit $2,501.61, Net worth $8,532.94; B n.p. $1,667.74, N. W. $6,756.24; C n. p. $1,389.79, N. W. $7,247.22. **27.** D's profit $14,323.11, balance $29, 654.63; E's profit $11,199.32, balance $25,396.95; F's profit $1,087.86, balance $1,943.42. **28.** Net profit $33,609.02; X's balance $28,899.86; Y's balance $26,860.34; Z's $23,806.32. **29.** $.635498+. **30.** $.648318+; $417.35. **31.** $.4129405. A's $218.28. **32.** $.70614+; P & Son $194.75; H & Co. $885.43; B. & D. $475.16. **33.** $57.38. **34.** $175. **35.** $46,915. **36.** $6,547.32. **37.** $3,929.45. **38.** $5,000. **39.** $76.50. **40.** $5,273.28. **41.** My loss $2180; Co's loss $3,820. **42.** $157.50. **43.** Home $3450; Galena $2,875; Royal $4312.50; Manchester $5,750; Charter Oak $2,300. **44.** Continental $2,133.33; Liverpool $1,280; Hamburg $2,986.67.

GENERAL REVIEW PROBLEMS—PAGES 323-335.

1. $1,728.46. **2.** $2,484.05. **3.** $1,582.56. **4.** A $7,249.69; B $7,234.02; C $5,452.93; D $3,640.51; E $3,530.85. **5.** $700.12. **6.** $3,504. **7.** A $2,012.85; B $1587.15. **8.** Loss $25. **9.** 23c. **10.** A $1,200; B $3,600; C $4,800. **11.** Gain $7.92. **12.** $3,488.27. **13.** $118.39. **14.** $821.93. **15.** Stock $14. **16.** A $40.50; B $45; **17.** $900=9$\frac{3}{5}$%. **18.** A $5,200; B $5,800; C $6,032. **19.** $1662.50. **20.** 6c. **21.** $37.70. **22.** $76.28. **23.** $14.23. **24.** $736.04. **25.** $1,172.11. **26.** $102.86. **27.** $241.02. **28.** $111.94. **29.** $553.33. **30.** 12$\frac{1}{16}$%. **31.** $49. **32.** Warner's better $27.27. **33.** $62,900. **34.** 72 cars. **35.** 3 yr. 4 mo. ;4%; $800. **36.** $753.43. **37.** Selling price per short ton $5.19. **38.** Due Miller $5,687.44; Millers gain $687.44. **39.** $460.18. **40.** $166.92 **41.** A $540.

B $2,240; C $3,600; D $4,185. **42.** $2,219.41. **43.** $8.16. **44.** $6,108.12. **45.** $76.65. **46.** Loss $888.62. **47.** $5,210.60. **48.** $32.41. **49.** $2,936. **50.** $147,500. **51.** $12.10. **52.** Exact Product Method Oct. 1, '08. **53.** $1,607.84. **54.** $16.63. **55.** $270.46. **56.** $3,092.50. **57.** (1) At discount, (2) $\frac{1}{10}\%$, (3) Against N. Y. **58.** $7,093.12. **59.** $1,000. **60.** A $15,000; B $10,000; C $7,500; D $4,500; E $12,500; F $17,500; G $12,000; H $9,000. **61.** $2,513,953.49. **62.** Vessel pays cargo $2.285.19 **63.** $908.28. **64.** Loss $1,487.64; A $14,323.80. B $9,549.20; C $7,639.36. **65.** $508.02. **66.** Gain $429.41. **67.** (1) .7650000. (2) $5,757.77. **68.** Loss $4.75. **69.** (1) $35,587.50; (2) $2212.50. **70.** $262.50. **71.** 65,427.99 Lire. **72.** $77.09. **73.** $244. **74.** $753.22. **75.** (1) Gain $8.26; (2) 115+%. **76.** $2,817.91. **77.** $1,777.69. **78.** $760.77. **79.** $637.50. **80.** $222.97. **81.** (1)$9.08.(2)$35\frac{3}{5}\frac{1}{1}\%$. **82.** $6810.48. **83.** G $475; K $1,662.50; H $1,425; U. S. $1,187.50; Con. $2,680; Com'l $4,020. **84.** Lost $31.50. **85.** $402.02. **86.** $677.38. **87.** $273.75 **88.** $2.97. **89.** $166,635. **90.** (1) $1\frac{1}{2}\%$; (2) $239.40. **91.** P. W. greater $6.54. **92.** $6.12. **93.** $396.75. **94.** $2,476.84. **95.** $1,300.50. **96.** $1,024.24. **97.** 15 hours. **98.** $317.94. **99.** 50%. **100.** $468.58.

APPENDIX—PAGES 336-351.

1. 625. **2.** 421875—614125—857375—274625—166375 **3.** 256—4096—65536—50625. **4.** $\frac{27}{64}$—$\frac{64}{125}$—$\frac{125}{216}$—$\frac{216}{343}$. **5.** 125—36—4096—100000—15625. **6.** $\frac{343}{512}$—$\frac{625}{1296}$—15.625—$204\frac{4}{49}$—129.746337890625. **7.** 14025. **8.** 35. **9.** 45. **10.** 55. **11.** 65. **12.** 75. **13.** 85. **14.** 123. **15.** 321. **16.** 231. **17.** 132. **18.** 312. **19.** 213. **20.** $\frac{1}{25}$. **21.** $\frac{1}{17}$. **22.** .35. **23.** .085. **24.** 64. **25.** 96. **26.** 123. **27.** 248. **28.** 5.8. **29.** 12.8+. **30.** 2.7+. **31.** 6.2+. **32.** 2.35. **33.** 5.27+. **34.** 1.52+. **35.** 1.80+. **36.** 2.50+. **37.** 543 ft. **38.** 61.3. **39.** 27.7+. **40.** 72. **41.** $\frac{7}{9}$. **42.** 3.07. **43.** 79.5 in. **44.** $76\frac{1}{4}$ mi. **45.** 58.4×103.2 in. **46.** 1.104 in. **47.** 73.6 km. **48.** $1733\frac{1}{3}$ yd. **49.** draught 31 ft., 5.52 in.; length 409 ft., 10.85 in.; beam 49 ft., 7.53 in. **50.** 45.2875 mi. **51.** 56 ft., 2.32+ in. **52.** 106.87×96.69 cm. **53.** $98.83. **54.** .273 in **55.** Fr. 611.25. **56.** $60.35. **57.** 30.82 sq. yd. **58.** 27.61 sq. in. **59.** 83.72 sq. yd. **60.** 94.93 ares. **61.** 61.933 acres; $7,171.94. **62.** 78.12 steres. **63.** 180.66 cu. ft. **64.** $2.11. **65.** Fr. 119.99. **66.** $759.67. **67.** 1,120 dekaliters. **68.** $32.11. **69.** 19.65 liters. **70.** 10.9 cts. **71.** Fr. 181.6. **72.** 11,901.6 lb. **73.** 17.8 cts. **74.** 1,489 oz., 338.76 gr. **75.** 1.748 lb. **76.** 1,775.083 lb. **77.** Fr. 5.82.

7.05.23
10.00